Metaheuristics for
Clustered Vehicle Routing Problems

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Computational Logistics – Seminar #3

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Joint work with A. Rossi, K. Sörensen and T. Barthélémy
Problem description

- Problem initially introduced by a parcel delivery company
- Routing problem with enormous number of customers
- Applications: parcel delivery
  - Pick-up in milk-runs (dynamic!)
  - Unloading/sorting/loading in depot/sortation centre
  - Full truck load and airplane transport to destination depot
  - Unloading/sorting/loading in destination depot
  - Delivery in milk-runs
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Problem delineation – current situation

- One depot
- Many (thousands) of known customers
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Problem: VRP with clustering constraints
Geographic zones

- Geographic delivery region is divided into clusters
  - Historically often based on postal codes
- Advantages
  - Easy sorting
  - Containers correspond to clusters
  - Last-minute changes are possible
  - Easier routing
- Problem: how to determine clusters
  - Robust
  - Easy to sort
  - Easy to route
  - “Right size”
Clusters are already predefined
A company proposal

- Clusters are already predefined
- Solve the high-level routing
Clusters are already predefined
Solve the high-level routing
The driver solves the low-level routing (within clusters)

or alternatively
A company proposal

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or alternatively

- The low-level routing is solved as a variant of a Hamiltonian path problem (EU/ME 2008)
Zooming in on a cluster

- Determine shortest path between starting node and ending node
- *Shortest Hamiltonian path problem*

**Hamiltonian: each node has to be visited once**

- shortest Hamiltonian path
- shortest Hamiltonian path between two nodes
- shortest Hamiltonian path between two groups of nodes
- shortest *pre*-Hamiltonian path
- shortest pre-Hamiltonian *rural* path problem
A global approach

The driver’s solution can fail!!!

- the driver cannot be easily replaced
- the driver’s solution is not robust

The solution method should take into account the
- high-level routing
- low-level routing

simultaneously
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The big-M approach

Clustering constraint

A simple idea consists in adding an artificial big-M distance between the clusters and then, using the classical VRP solution methods.
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Methods: Clarke & Wright, 2-opt, SA, ILP
Clarke & Wright: good initial solution

Clarke & Wright

- Start with one tour for each customer
- Make savings by merging tours
Clarke & Wright: good initial solution

Clarke & Wright
- Start with one tour for each customer
- Make savings by merging tours

It works because *savings* are more important within clusters

Tours within clusters are of high quality

But...
Clarke & Wright: good initial solution?

Truck capacity = 6
Distance to depot = M
Clarke & Wright: good initial solution?

Truck capacity = 6
Distance to depot = M
Distance between clusters += M
Distance within clusters = $\varepsilon$
Clarke & Wright: good initial solution?

Truck capacity = 6
Distance to depot = M
Distance between clusters += M
Distance within clusters = $\varepsilon$

Cost = $16M$
Clarke & Wright: good initial solution?

Truck capacity = 6
Distance to depot = M
Distance between clusters += M
Distance within clusters = ε

Cost = $8M + 4\varepsilon$
Clarke & Wright: good initial solution?

Truck capacity = 6
Distance to depot = M
Distance between clusters += M
Distance within clusters = $\varepsilon$

Cost = $7M + 4\varepsilon$

STOP (truck capacity)
Clarke & Wright: good initial solution?

Truck capacity = 6
Distance to depot = M
Distance between clusters += M
Distance within clusters = ε

Cost = 6M + 4ε

OPTIMAL sol.
2-opt and Simulated Annealing

The 2-opt method is used after the Clarke & Wright heuristic

Classical moves
- intra-route moves
- inter-route moves

Advanced Simulated Annealing
- Based on the above moves
- Annealing schedule
  - based on rejection rates
  - try to follow a predefined initial scheme
  - may increase during the search
- 2-opt is used during the search to improve solutions
ILP methods

Two different formulations are currently tested

- works only on small instances
- comparison are conducted on
  - big-M approach
  - clustering constraints

Results up to now are rather disappointing

- explore more advanced formulations
- combine ILP with metaheuristics
  → matheuristics
First observations of big-M approach

Advantages

- Easy to implement
- No changes in data (except cluster information)
- Use all existing methods (including exact approaches)
First observations of big-M approach

### Advantages

- Easy to implement
- No changes in data (except cluster information)
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### Drawbacks

- No formal proof yet
- **Does not work** with Boltzmann acceptance criterion
- Same limitations with all local search methods?
Drawbacks

5 Tours; Cost = 901602
Drawbacks

5 Tours; Cost = 901602

5 Tours; Cost = 855782
Dynamic big-M

To overcome the big-M limitation
→ change \textit{dynamically} the big-M value
Dynamic big-M

To overcome the big-M limitation
→ change dynamically the big-M value

4 Tours; Cost = 72.3
Dynamic big-M

To overcome the big-M limitation
→ change dynamically the big-M value

4 Tours; Cost = 72.3

3 Tours; Cost = 69.4
Experiments conducted

A large set of experiments is currently conducted

- two ILP formulations with clustering constraints
- two ILP formulations with big-M approach
- heuristics + SA with clustering constraints
- heuristics + SA with big-M approach
  (with static or dynamic big-M)

Instances are provided by

- literature
- industry
Conclusions

Clustering customers

- important for parcel delivery companies
- need help from researchers
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Two-phase approach
- high-level routing = OK
- low-level routing
  - Hamiltonian path problems
  - Exact and heuristics approaches
Conclusions

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- important for parcel delivery companies
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Two-phase approach
- high-level routing = OK
- low-level routing
  - Hamiltonian path problems
  - Exact and heuristics approaches

Global approach
- handled by setting the clustering constraints
- or by the big-M approach
Remaining research questions

- How to combine efficiently high- and low-level routing?
- Which kind of metaheuristics is most appropriate (EA, VNS)?
- Do we need special moves for clustering constraints?
- Can we combine exact and approximate methods for better solutions?
- At which level?
- How to cluster the customers?
- ...
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