Path Relinking for the Vehicle Routing Problem

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Path Relinking

Reference

- Randomness can be problematic
- “Deterministic crossover”
- Transform an incumbent solution $s_i$ into a guiding solution $s_g$
Path relinking illustrated
Path relinking illustrated

Sörensen, Sevaux, Schittekat
Path Relinking for the Vehicle Routing Problem
Path relinking illustrated

Sørensen, Sevaux, Schittekat

Path Relinking for the Vehicle Routing Problem
The vehicle routing problem

- Capacitated
- Distance-constrained
- Representable as a set of permutations, e.g. |12|345|6789|
Distance-based path relinking

- Based on a *move type*
- Transform $s_i$ into $s_g$ in a *minimal number of moves*
- Requires a distance measure
Using the distance-based path relinking

Experiment setup

- “Deconstruct” one of the best algorithms for the CVRP
- Replace some operator(s) with PR
Variable Record–To–Record Travel

Reference

Components

- Modified Clarke and Wright savings heuristic (construction)
- Variable Neighborhood Descent
- Record-To-Record Travel
- Perturbation
Modified Clarke and Wright savings heuristic

- Construction heuristic
  - Initialize: one route per customer
  - Combine routes that yield largest saving

- Saving

\[ s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij} \]
Modified Clarke and Wright savings heuristic

- **Construction heuristic**
  - Initialize: one route per customer
  - Combine routes that yield largest saving

- **Saving**
  \[ s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij} \]

- \( \lambda \) can be used to generate different solutions
Variable Neighborhood Descent

- Relocate

- Swap

- 2-opt
Record-To-Record Travel

- Similar to VND
- Differences
  - Non-improving moves allowed if
    \[ f(\text{current}) < f(\text{record}) \times (1 + \text{deviation}) \]
  - Search always stopped after \( I \) iterations
  - Different search strategy
- Diversification strategy
Perturbation

- Remove $M$ customers from the current solution
- $M = \min(20, N/10)$
- Remove customers in order of $d_i/s_i$
  
  \[ s_i = \text{dist}(\text{prev}(i), i) + \text{dist}(i, \text{next}(i)) - \text{dist}(\text{prev}(i), \text{next}(i)) \]
- Insert customers in their best position
Path relinking setup

- Eliminate parameters from VRTR algorithm
- Replace RTR and Perturbation with Path Relinking
Distance-based path relinking

- Move operator: relocate
- Distance: minimal number of relocate moves to move from $s_i$ to $s_g$
The edit distance

- Distance between two strings
- Three edit operations
  - Add character
  - Remove character
  - Substitute character
- Edit distance = minimal number of edits required to *transform string 1 into string 2*
- Each edit operation may have a weight

Relocate edit distance
Set weight of substitution to $\geq 2$
- One relocate move = 2
- One remove or one insert = 1
The relocate edit distance

Example: \( d_{\text{relocate}}(346512978, 35412786) = 5 \)
The relocate edit distance

Example: $d_{\text{relocate}}(346512978, 35412786) = 5$

```
  3465-12978-
   |--|--|--|--|--
   3--5412-786
```

- Determines customers to remove/insert/relocate
- Order of relocates is arbitrary
- Can be between routes
Distance measures for routing problems

- Routes can be reversed
- Order of the routes is not fixed

\[ s_i = (abc)(defg)(hij) \text{ and } s_g = (bcdef)(gjiha). \]

<table>
<thead>
<tr>
<th></th>
<th>abc</th>
<th>defg</th>
<th>hij</th>
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</thead>
<tbody>
<tr>
<td>bcdef</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>gjiha</td>
<td>5</td>
<td>5</td>
<td>4</td>
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<tr>
<td>(\Lambda)</td>
<td>3</td>
<td>4</td>
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Path relinking for the VRP

- Transform routes of $s_i$ into routes of $s_g$
- One relocate move at a time

**Issue**

- Which search strategy?
  - In which order?
- When to stop?
Path relinking for the VRP

- Transform routes of $s_i$ into routes of $s_g$
- One relocate move at a time

**Issue**

- Which search strategy?
  - In which order?
    - Lexicographic, Random, Best feasible
  - When to stop?
    - In the middle
Algorithm structure

Path relinking

- Clarke & Wright generates different \( n \) solutions (different \( \lambda \)'s)
- \( n(n-1) \) new solutions are found by Path Relinking
- \( \lambda_i \in [0, 2] \) in equal steps

Structures tested

- CW + VND
- CW + PR
- CW + PR + VND
- CW + VND + PR
- CW + VND + PR + VND

Sørensen, Sevaux, Schittekat
Path Relinking for the Vehicle Routing Problem
Results

- Tested on Christofides instances
- All methods generate the same number of solutions $n(n - 1)$ for $n = 3, 4, 5, 6$
Results - detailed analysis

- Best: CW + VND₁ + PR + VND₂

<table>
<thead>
<tr>
<th>VND₁</th>
<th>VND₂</th>
<th>None</th>
<th>Random</th>
<th>Lex</th>
<th>BF</th>
<th>Total</th>
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<td>5.59%</td>
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## Results - computing times

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<th>VND₁</th>
<th>VND₂</th>
<th>Path relinking</th>
<th>None</th>
<th>Random</th>
<th>Lex</th>
<th>BF</th>
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<tr>
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<td>1</td>
<td></td>
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<td>450</td>
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<td></td>
<td>Total Result</td>
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<td>173</td>
<td>251</td>
<td>236</td>
<td>218</td>
</tr>
</tbody>
</table>
(Preliminary) conclusions — Path Relinking for the CVRP

- “Deterministic crossover”
- Distance based PR — shortest path from $s_i$ to $s_g$
- Based on “normal” VRP moves
- Comparable results to VRTR (but without parameters)
Future work

- Other move types – other distance measures (but: reversal distance (2-Opt) is NP-hard)
- More detailed analysis, testing on larger instances
- Visualization of path
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