Curve fitting for styling application by genetic algorithm

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Curve fitting for styling application by genetic algorithm

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Introduction
Sculptured parts - Styling surfaces

aesthetic appealing

Automotive industry
Car bodies design
Design methodology

a reverse engineering process

measuring device → clay model

→ discrete representation → CAD

→ continuous representation

very dense set of digitized points

Noisy data
Quality Requirements

1: accurate approximation $\text{dist}(S(u_i,v_i),P_i) < \text{error}_{\text{max}}$

2: smooth curvature variation

3: specific curvature variation

4: even control points distribution

low amount of control points
Form fitting

non-linear problems
multi-objective optimization

A curve-based design methodology

1: generation of high-quality section curves

2: surface modeling by interpolation or a blending function

fair curves ⇒ fair surfaces
Form fitting

2D raw data set

Classical methods:
- arbitrary parametrisation
- least squares approximation
- minimisation of the deformation energy

Drawbacks:
- non global optimization
- difficult trade-off problem
- non crisp criteria
Statement of the problem

$P_0$ data points $P_j$ fitting

Common curvature and tangent direction

node

$G^2$ Bézier Composite Curve

“harmonious” curvature variation

monotonous curvature variation

convex curvature variation
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Related works
Typical Curve

Curve with a **constrained definition**
providing a **given shape characteristic**

2D Bézier curve

“involute of circle”

monotonous curvature variation

\[ h > \frac{1}{\cos} \quad \text{or} \quad h < \cos \]

(odd degree: \( m = 3, 5, \ldots \))

Pythagorean-Hodograph curves [Farouki]
Typical Curve as Template Curve

Definition parameters sets

1. Canonical parameters
   \[ l, h, \phi \]

2. Form parameters
   \[ \rho_0, \Delta W, \Delta S \]
A Composite Curve Definition

*eg* : a 3 segments composite curve

initial parameters

shape parameters
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Development
Composite $G^2$ Curve Fitting

Fixed number $m$ of segments

$W_0, \rho_0, \Delta W_1, \Delta S_1, \Delta W_2, \Delta S_2, \Delta W_3, \Delta S_3$

$(2m + 4)$ real variables

$Q_0 = P_0$

$\sum \Delta S_i = S \approx \sum |P_{j-1}P_j|$ for $j = 1..N$

$(2m + 1)$ real variables
Optimization by Genetic Algorithm

Fitness function

\[ \sum_{i=0}^{N} d^2_i = \sum_{i=0}^{N} \left( P_i - C(u_i) \right)^2 \]

Coding

\[ (W_0, \rho_0), (\lambda_1, \mu_1), \ldots, (\lambda_m, \mu_m) \]

\[ \Delta S_k = (\lambda_k - \lambda_{k-1}) S \]

with \( \lambda_k \in ]0, 1[ \) for \( k = 1..m-1 \)

and \( \lambda_0 = 0 \), \( \lambda_m = 1 \)

\[ \Delta W_k = \mu_k \rho_{k-1} \Delta S_k \]
**Coding**

\[(W_0, \rho_0), (\lambda_1, \mu_1), \ldots, (\lambda_m, \mu_m)\]

$k^{th}$ curve segment

**Diagram:**
- \(W_{k-1}\)
- \(\rho_{k-1}\)
- \(\Delta S_k\)
- \(\Delta W_k = \mu_k \rho_{k-1} \Delta S_k\)
- \(\mu_k\): acceleration coefficient
- Tangent direction
- Initial curvature \(\rho_{k-1}\)
- Arc length \(\Delta S_k\)

**Equation:**
\[
\Delta W_k = \mu_k \rho_{k-1} \Delta S_k
\]
Fitness function calculation

\[ \sum_{i=0}^{N} d_i^2 = \sum_{i=0}^{N} (P_i - C(u_i))^2 \]
Implemented Algorithm

Program Curve_Fitting_by_GA
Generate the initial population;
Identify the best and the worst individuals;

While (mean_fitness – best_fitness) > given threshold_value
    Select randomly two individuals;
    Generate two new individuals by crossover and keep the best offspring;
If offspring_fitness < worst_fitness
    Then Mutate it according a given probability_rule;
    If offspring_fitness > worst_fitness
        Then Discard it;
        Else Insert it into the population;
Select randomly an individual for deletion;
    Endif;
Endif;
Endwhile;
End.
Crossover Operator

one point crossover – $s$ randomly chosen

individual $A$ \( (W_{0,A}, \rho_{0,A}), (\lambda_{1,A}, \mu_{1,A}) \ldots (\lambda_{m,A}, \mu_{m,A}) \)

individual $B$ \( (W_{0,B}, \rho_{0,B}), (\lambda_{1,B}, \mu_{1,B}) \ldots (\lambda_{m,B}, \mu_{m,B}) \)

\[
\begin{align*}
\text{if } & k < s \\
(\lambda_{k,A}, \mu_{k,A}) &= w (\lambda_{k,A}, \mu_{k,A}) + (1 - w) (\lambda_{k,B}, \mu_{k,B}) \\
(\lambda_{k,B}, \mu_{k,B}) &= (1 - w) (\lambda_{k,B}, \mu_{k,B}) + w (\lambda_{k,A}, \mu_{k,A})
\end{align*}
\]

\[
\begin{align*}
\text{if } & k \geq s \\
(\lambda_{k,A'}, \mu_{k,A'}) &= (1 - w) (\lambda_{k,A}, \mu_{k,A}) + w (\lambda_{k,B}, \mu_{k,B}) \\
(\lambda_{k,B'}, \mu_{k,B'}) &= w (\lambda_{k,B}, \mu_{k,B}) + (1 - w) (\lambda_{k,A}, \mu_{k,A})
\end{align*}
\]

2 new individuals $A'$ and $B'$
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A result
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Variational least squares fitting

Fitting by genetic algorithm

Number of individuals # 30

Error_max = 0.64

Manually improved fitting

Error_max = 0.54
Fitting by genetic algorithm

number of individuals # 30

$error_{max} = 0.54$
variational least squares fitting
error_max = 0.64

manually improved fitting
Fitting by genetic algorithm

\[ \text{error}_{\text{max}} = 0.54 \]
Fitness = 0.068
2 segments
166 iterations

Fitness = 0.029
3 segments
244 iterations
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Conclusion
+ Promising results

+ Relevant approach in CAD

- Cases of premature convergence

- Predominance of certain genes (epistasis case)?
More reliable determination of the search domain

Diversification - Intensification

Varying number of segments