A Mathematical Formulation for a School Bus Routing Problem

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Motivations

Request of the Flemish region (Belgium)
- Being transported to school is a right
- Public transportation is organised by the Flemish transportation society
- Routes of buses are annually revised

Interesting features
- Students can walk to a nearby bus stop
- Potential bus stop locations are known

⇒ School Bus Routing problem
The School Bus Routing problem

- a school
The School Bus Routing problem

- a school
- a set of students
The School Bus Routing problem

- a school
- a set of students
- a set of potential bus stops
The School Bus Routing problem

- a school
- a set of students
- a set of potential bus stops
- a maximum walking distance (students → stops)
A possible solution
A possible solution

- students are assigned to bus stops
A possible solution

- students are assigned to bus stops
- two potential bus stops are not visited
A possible solution

- students are assigned to bus stops
- two potential bus stops are not visited
- two bus tours are created
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State of the art

Aims

Mathematical formulation

First results

Conclusion

Literature

L. Bodin and L. Berman.
Routing and scheduling of school buses by computer.
Transportation Science, 13:113-129, 1979

L. Bodin and B. Golden and A. Assad and M. Ball.
Routing and scheduling of vehicles and crews. The state of the art.

Scheduling school buses.
Management Science, 30(7):844-853, 1984

A. Corberán, E. Fernández, M. Laguna and R. Martí
Heuristic Solutions to the Problem of Routing School Buses with Multiple Objectives.

M. Spada, M. Bierlaire and Th.M. Liebling.
Decision-Aiding Methodology for the School Bus Routing and Scheduling Problem.
Subproblems

- **Covering Tour Problem**
  - M. Gendreau, G. Laporte and F. Semet.
  - The Covering Tour Problem.

- **Traveling Purshaser Problem**
  - A Branch-and-cut Algorithm for the Undirected Traveling Purchaser Problem.

- **Vehicle Routing Problem**
  - P. Toth and D. Vigo (Eds.)
  - The Vehicle Routing Problem.
Aims of the study

Company related requests
- help the Flemish region in this decision problem
- reducing their costs / keep a high service level

Research related aims
- solve this two-level problem at once
- implement dedicated heuristics and metaheuristics
- take advantages from *similar* problems
  (Location Routing problem - PhD Prodhon Oct. 2006)

Aim of this presentation
- solve small instances to optimality
- provide bounds for large problems
A Mathematical Formulation for SBR

Notations:
- \( c_{ij} \): Cost of traversing arc \((i, j)\)
- \( K \): Number of buses
- \( C \): Capacity of the buses
- \( V \): Set of all potential stops (0 is the depot)
- \( E \): Set of all arcs between stops
- \( S \): Set of all students
- \( s_{li} \): Binary indicator that tells whether student \(l\) can walk to stop \(i\) or not

Variables:
- \( x_{ijk} \): Number of times vehicle \(k\) traverses arc from \(i\) to \(j\)
- \( y_{ik} \): 1 if vehicle \(k\) visits stop \(i\), 0 otherwise
- \( z_{ilk} \): 1 if student \(l\) is picked up by vehicle \(k\) at stop \(i\), 0 otherwise
Constraints

All vehicles start at the depot

\[
\sum_{k=1}^{K} y_{0k} \leq K, \quad k = 1, \ldots, K \quad (2)
\]

If stop \( i \) is visited by vehicle \( k \), then one arc should be traversed by \( k \) entering and leaving \( i \)

\[
\sum_{j \in V} x_{ijk} = \sum_{j \in V} x_{jik} = y_{ik}, \quad \forall i \in V, k = 1, \ldots, K \quad (3)
\]

All stops are not visited more than once

\[
\sum_{k=1}^{K} y_{ik} \leq 1, \quad \forall i \in V \setminus \{0\} \quad (4)
\]
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All students walk to an allowed single stop

$$\sum_{k=1}^{K} z_{ilk} \leq s_{li}, \quad \forall l \in S, \forall i \in V$$ \hspace{1cm} (5)

Capacity of buses is not exceeded

$$\sum_{i \in V} \sum_{l \in S} z_{ilk} \leq C, \quad \forall k = 1, \ldots, K$$ \hspace{1cm} (6)

A student is picked up if a vehicle visits this stop

$$z_{ilk} \leq y_{ik}, \quad \forall i, l, k$$ \hspace{1cm} (7)

All students are picked up once

$$\sum_{i \in V} \sum_{k=1}^{K} z_{ilk} = 1, \quad \forall l \in S$$ \hspace{1cm} (8)
Constraints and Objective

Subtour elimination constraints

\[ \sum_{i \in S} \sum_{j \notin S} x_{ijk} \geq y_{hk}, \quad \forall S \subseteq V \setminus \{0\}, \ h \in S, k = 1, \ldots, K (9) \]

Objective

\[ \min \sum_{i \in V} \sum_{j \in V} c_{ij} \sum_{k=1}^{K} x_{ijk} \quad (1) \]

Other possible objectives are:

- min total walking distance of students
- min combination of walking distance and route length
- min [max] trip duration for students
- \ldots
Practical solving method

Implementation with XPress-MP © Dash Associates
(Mosel modeling language)

Algorithm:
1. Implement formulation without constraints (9)
2. Solve the formulation
3. Detect violated constraints
   if no violated constraints GOTO 6
4. Add specific capacity cut constraints
5. GOTO 2
6. END: the solution is optimal

Example with 50 students, 10 potential bus stops
provided by a real-life style instance generator
Example
Example
Example
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Example
Example
Advantages and drawbacks

At each iteration

- Detecting non-valid subtours can be done efficiently
- Several cuts can be added simultaneously (symmetry, vehicle ID ignorance, . . .)
- The cost of a subformulation is a valid lower bound

But

- The subproblem is still a 0-1 ILP formulation
- Optimal solutions are not always obtained → no lower bounds
- Search can be stopped before optimality
- Cuts can be added anyway
- Optimal solutions can still be found
Conclusion and Future Work

Concluding remarks

- Efficient for small size instances
- Sub-formulation still difficult
- Linear relaxation need to be solved differently
- Lower bounds not always efficient

Future work

- New formulation with two-index variables
- Different types of cuts
- Improved lower bounds
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