Neighborhood Selection in Variable Neighborhood Search

The Pilot Method
A Telecom Application
Linking Ideas to VNS
A Scheduling Application
Numerical Results
Conclusions and Outlook

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The Pilot Method

Local search strategies generally incorporate a myopic evaluation of the local choices

Idea: enhance local search by looking ahead for each possible local choice

Heuristic measure as a unified approach

Idea: repeatedly modify a master solution; each time in a 'minimal' fashion to account for the 'best' choice, where all choices have been judged by means of a separate heuristic result, the 'pilot' solution

[Duin and Voß (1994, Networks 1999), Voß et al. (2005)]

Applicable for construction and improvement methods
Pilot Method: Related Ideas
Pilot Method: Related Ideas

Rollout method

- Bertsekas, Tsitsiklis and Wu (1996)
- Bertsekas and Castanon (1999)
- Meloni et al. (2002)

Miscellaneous

- Iterative Greedy Heuristics: Amberg et al. (1999)
Speeding up a Pilot Method

Restrict the pilot process to a given evaluation depth

The pilot method is performed until an incomplete solution with a given number of objects is reached; this solution is completed by continuing with a conventional heuristic.

Candidate selection

Shortcuts in the calculation of the heuristic measure (incl. approximation)
Which combinations of guiding process and pilot process are meaningful?

<table>
<thead>
<tr>
<th>Pilot Process</th>
<th>Guiding Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steepest Descent</td>
<td>Steepest Descent</td>
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<tr>
<td>Mildest Ascent</td>
<td>Mildest Ascent</td>
</tr>
<tr>
<td>Reactive Tabu Search</td>
<td>Steepest Descent</td>
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</tbody>
</table>
A Telecom Application

SDH/WDM network planning:
- Input data
  - Fiber graph, switching locations
  - Demand prognosis
  - Equipment specifications and prices
- Output data
  - Numbers and types of the used equipment
  - Routing of the demands

Goal: Minimization of the investment in equipment while carrying all demands

Method of resolution:
- Integer model
- Variable Neighborhood Search heuristic

(WDM: Wavelength Division Multiplex; SDH: Synchronous Hierarchy)
Basic Variable Neighborhood Search

Initialization.

Find an initial solution $S$.

Select the set of neighborhood structures $N_k$, for $k = 1, \ldots, k_{\text{max}}$.

Choose a stopping condition.

Iterations.

Repeat the following sequence until the stopping condition is met:

(1) Set $k = 1$.

(2) Repeat the following steps until $k = k_{\text{max}}$:

   (a) **Shaking.** Generate a point $s'$ at random from the $k^{\text{th}}$ neighborhood of $s$ ($s' \in N_k(s)$).

   (b) **Improvement method.** Apply some improvement method with $s'$ as initial solution; denote as $s''$ the obtained solution.

   (c) **Move decision.** If this solution is better than the incumbent, then $s = s''$, and $k = 1$; otherwise, set $k = k + 1$. Goto (a).
### Computational results

<table>
<thead>
<tr>
<th>nodes</th>
<th>edges</th>
<th>demands</th>
<th>CPLEX</th>
<th>VNS+pilot</th>
<th>GRASP+pilot</th>
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<td></td>
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<td>cost</td>
<td>time [s]</td>
<td>cost</td>
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<td>172.55</td>
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</tbody>
</table>

**optimal solutions**
A Telecom Application

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A New Application …

Which neighborhoods can be designed?
How should they be explored and how efficient are they?
How structurally different are they?
In which order should they be applied?
The Single Machine Total Weighted Tardiness Problem is a well-known machine scheduling problem. In this problem, a set of $n$ jobs with weights indicating their relative importance needs to be processed on a single machine while minimizing the total weighted tardiness.

$$1 \mid \sum w_j T_j$$
Investigated Neighbourhoods

APEX it is the most classical neighborhood for permutation encodings. Two adjacent jobs are exchanged in the sequence. For a solution, the size of this neighborhood is $n - 1$.

BR4 consists in taking a block of four consecutive jobs and inverse its internal orientation. The size of this neighborhood is $n - 3$.

BR5 is identical to BR4 but with a block of five jobs. The size of this neighborhood is $n - 4$.

BR6 is identical to BR4 but with a block of six jobs. The size of this neighborhood is $n - 5$.

EX\APLEX is the general pairwise interchange where the APEX is excluded. Two non-adjacent jobs are exchanged. The size of this neighborhood is $n(n - 3) + 2$.

FSH\APLEX takes a job and moves it to a position further in the sequence, resulting in a forward shift for the jobs in between these two positions. APEX is excluded from this move. The size of this neighborhood is $\sum_{i=1}^{n-2} i = (n - 2)(n - 1)/2$.

BSH\APLEX works as FSH\APLEX but the jobs are shifted backward. The size is identical to FSH\APLEX.
Approaches for Comparison

VND-R  When a local optimum is reached, the next neighborhood is selected randomly among possible alternatives. The search terminates when all the neighborhoods cannot produce a better solution than the incumbent.

VND-F  This is the classical version of VND. In our experiments, we choose the order in which neighborhoods are introduced above.

VND-A  This is an adaptive version of VND. Every time a new neighborhood has to be selected, an investigation of all neighborhoods is run for a short number of iterations (say, 100) and the neighborhood producing the best solution at this time is selected for the descent method. This neighborhood is used until no more improvements are obtained. The investigation phase is run again, etc. The stopping condition is the same as before.
Numerical Results

VND-A improves VND-R
VND-A improves VND-F

End of VND-A
End of VND-F
End of VND-R
## Numerical Results

<table>
<thead>
<tr>
<th>Description</th>
<th>adaptive VNS, 10000 Evaluations before neighbourhood choice</th>
<th>10000 Evaluations before neighbourhood choice (UFC Un-fair Comparison)</th>
<th>fixed sequence of neighbourhoods</th>
<th>random sequence of neighbourhoods</th>
<th>Smallest average (10 runs)</th>
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</thead>
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<td>218133,46</td>
</tr>
</tbody>
</table>
Conclusion

During our work with different methods (Pilot, VNS) and different problems (incl. Single Machine Total Weighted Tardiness Problem) we found a “nice” research topic for VNS.

Idea: Looking Ahead for the Neighbourhood Choice

Good results for one application; indifferent results for another. Proof of concept?
Remarks? Questions?