Application of bubble-check algorithm to non-binary LLR computation in QAM coded schemes

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The generation of intrinsic LLR messages in non-binary (NB) coded schemes associated with quadratic amplitude modulation (QAM) is considered. It is shown that the intrinsic LRR message generation corresponds to the same kind of computation as the one performed at the elementary check nodes in extended min-sum (EMS) [4] decoding algorithms. In our study, index $k$ is omitted ($y = y_k$). Each $L_k$ is defined as

$$L_k = \ln \left( \frac{P(y|a)}{P(y|\bar{a})} \right) = \frac{d_i^2(C(a_k, y_k) - d_i^2(C(\bar{a}), y_k))}{2\sigma^2} \quad (1)$$

where $\bar{a}$ is the GF(q) symbol associated with the nearest point to $y$ in the QAM constellation, i.e. the one that maximises $P(y|a_k)$ for $i \in \{1, ..., n_m\}$. The Euclidean distance between two points in signal space is represented by $d$.

Demapper: The function of the demapper is to generate the intrinsic message for each received symbol $y_k$. For the sake of simplicity, let $d_i^2 = d_i^2(a, y)$ and $\bar{d}^2 = d_i^2(\bar{a}, y)$. Moreover, $d_i^2 = d_i^2 + d_i^2$ and $\bar{d}^2 = \bar{d}^2 + \bar{d}^2$, as we decompose the M-QAM into two $2m^2$-ary PAMs for distance calculation. Then, we can write

$$d_i^2 - \bar{d}^2 = (d_i^2 - \bar{d}^2) + (d_i^2 - \bar{d}^2) \quad (2)$$

Finally, the objective is to select the $n_m$ smallest distances (sorted in increasing order) and their associated symbols $a_k$ in order to generate the intrinsic EMS message for $y$.

Finding the minimum distances with the bubble-check algorithm: In [10], Boutillon and Conde-Canencia presented a low-complexity algorithm for extracting $n_m$ minimum values in the set defined as $U(i) + V(j)$. This set is represented as a matrix $T_\Sigma$, where $T_\Sigma = U(i) + V(j)$. The elements in $U = [U(1), U(2), ..., U(n_m)]$ and $V = [V(1), V(2), ..., V(n_m)]$ are sorted in increasing order. Then, we can directly apply the bubble-check algorithm to generate the intrinsic message, as illustrated in Fig. 2.

$$\begin{align*}
(U(i), \pi(p))_{i=1, ..., n_m} & \rightarrow \text{bubble-check circuit} \\
(V(j), \pi(q))_{j=1, ..., n_m} & \rightarrow (L_p, a)_{i=1, ..., n_m}
\end{align*}$$

**Fig. 2 Application of bubble-check circuit to generate LLR intrinsic message**

**Example for $M = q = 64$ and $n_m = 8$:** Let us consider the case of a 64-QAM associated with a GF(64)-LDPC code with a Gray mapping as in the IEEE.802.11 standard (Fig. 4). Let $G$ be $[-7, -5, 1, -3, 7, 3, 5, 1, 3]$, then $\pi(p(G)) = [p(8)]$ and $\pi(q(p(G)) = \{p(8) \text{ mod } 8\}$. This way $C(G(6), G(4)) = ((+1, +7) \text{ or } C(G(3), G(0)) = (+7, -7)$.

**Fig. 3 Sequential distance computation on I-axis**

| $i$ | $\pi(p)$ | $|p|$ | $d_i^2$ | $U(i)$ |
|-----|-----------|------|--------|--------|
| 1   | 5         | 5    | 0      | 0      |
| 2   | 7         | 4    | 2.8    |        |
| 3   | 7         | 4    | 5.2    |        |
| 4   | 1         | 6    | 18.4   |        |

Let us now illustrate the demapping with the example that the received noisy signal is $y = (5.3, -3.2)$. Then, $C(a) = ((+5, +3), -3) \text{ and } \bar{d}^2 = 0.32^2 + 0.2^2$. Let us focus first on the I-axis. The calculation of the sorted values in $U(i)$ can be performed with a state machine.
the intrinsic messages are \((3, 0.2, \ldots)\), the closest points to \(y\) for EMS (Fig. 5) to obtain

\[
\begin{align*}
(U_i, \pi_i(p)) &= \{(0, +5), (2.8, +7), (5.2, +3), (18.4, +1), \ldots\} \\
(V_j, \pi_j(p)) &= \{(0, -3), (3.2, -5), (4.8, -1), (14.4, -7), \ldots\}
\end{align*}
\]  

The same procedure for the \(Q\)-axis generates

\[
\begin{align*}
(U_i, \pi_i(p)) &= \{(0, +5), (2.8, +7), (5.2, +3), (18.4, +1), \ldots\} \\
(V_j, \pi_j(p)) &= \{(0, -3), (3.2, -5), (4.8, -1), (14.4, -7), \ldots\}
\end{align*}
\]

Finally, Table 2 illustrates the generation of the EMS intrinsic message \((L_i, \alpha_i)\) through the bubble-check circuit (Fig. 2). For \(n_m = 8\), the intrinsic messages are \((0, a_{23}), (2.8, a_{43}), (3.2, a_{14}), (4.8, a_{13}), (5.2, a_{34}), (6, a_{33}), (7.6, a_{43})\) and \((8.4, a_{17})\) which correspond to the \(n_m\) closest signals to \(y\).

\[
\text{Table 2: Application of bubble-check algorithm}
\]

\[
\begin{align*}
(U_i, \pi_i(p)) &= \{(0, +5), (2.8, +7), (5.2, +3), (18.4, +1), \ldots\} \\
(V_j, \pi_j(p)) &= \{(0, -3), (3.2, -5), (4.8, -1), (14.4, -7), \ldots\}
\end{align*}
\]

**Fig. 4 Performance comparison of BP and EMS for GF(64)-LDPC associated with 64-QAM**

**Fig. 5 Zoom on 64-QAM constellation and \(n_m\) closest points to \(y\) for EMS intrinsic message generation**

**Results:** Fig. 4 presents the simulation results obtained for the ultra-sparse protograph-based NB-LDPC on GF(64) associated with a 64-QAM as in Fig. 5 for frame sizes of \(N = 192\) symbols (1152 bits) and \(N = 384\) (2304 bits) with a code rate of 1/2 over the AWGN channel. The BP curves correspond to the belief propagation decoding, simulated on the floating point with 100 decoding iterations (see [11]). The EMS curves consider the EMS NB-LDPC decoder described in [12] with \(n_m = 12\), 20 decoding iterations, 6 bit quantisation [note that this decoder design was implemented on a field programmable gate array (FPGA) [12]] and the intrinsic LLR generation presented in this Letter. A performance gap of about 0.4 dB is observed between the BP and EMS curves, which confirms the interest of our approach from both the performance and the low-complexity implementation aspect.

**Conclusion:** This Letter focuses on low-complexity intrinsic LLR generation for high-order NB-coded QAM designs. The originality is in the use of the bubble-check algorithm for the computation of the intrinsic message. The simulation results show the interest of this work in terms of performance. The FPGA implementation of the NB-LDPC EMS decoder and the bubble-check architecture design considered in [12] is proof of the implementation feasibility of both the QAM demodulator and the decoder (Fig. 1).

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**References**


