

# A Generalization of the Pignistic Transform for Partial Bet

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**Abstract.** The Transferable Belief Model is a powerful interpretation of belief function theory where decision making is based on the pignistic transform. Smets has proposed a generalization of the pignistic transform which appears to be equivalent to the Shapley value in the transferable utility model. It corresponds to the situation where the decision maker bets on several hypotheses by associating a subjective probability to non-singleton subsets of hypotheses. Naturally, the larger the set of hypotheses is, the higher the Shapley value is. As a consequence, it is impossible to make a decision based on the comparison of two sets of hypotheses of different size, because the larger set would be promoted. This behaviour is natural in a game theory approach of decision making, but, in the TBM framework, it could be useful to model other kinds of decision processes. Hence, in this article, we propose another generalization of the pignistic transform where the belief in too large focal elements is normalized in a different manner prior to its redistribution.

## 1 Introduction

The Transferable Belief Model [1] (TBM) is based on the decomposition of the problem into two stages: the **credal level**, in which the pieces of knowledge are aggregated under the formalism of belief functions, and the **pignistic level**, where the decision is made by applying the Pignistic Transform (PT): It converts the final belief function (resulting from the fusions of the credal level) into a probability function. Then, a classical probabilistic decision is made.

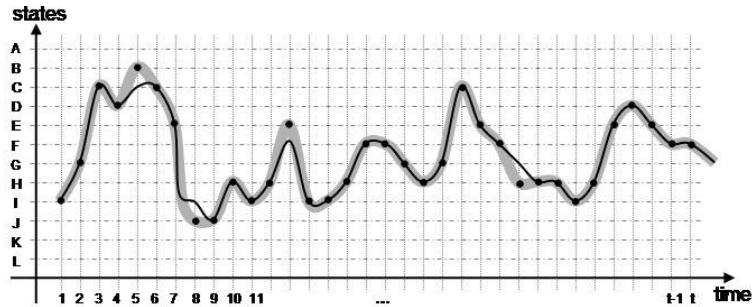
The manner in which belief functions allow to deal with compound hypotheses (i.e. set of several singleton hypotheses) is one of the main interests of the TBM. On the other hand, the decision making in the TBM only allows betting on singletons. Hence, at the decision making level, part of the belief function flexibility is lost. Of course, it is made on purpose, as, betting on a compound

hypothesis is equivalent to remain hesitant among several singletons. It would mean no real decision is made, or equivalently, that no bet is booked, which seems curious, as the PT is based on betting (“pignistic” is derived from the Latin word for “bet”).

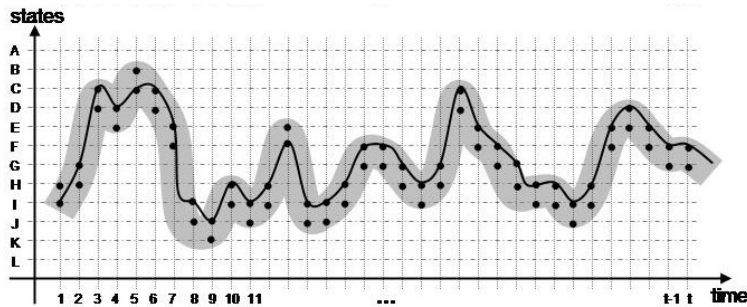
Nevertheless, there are situations in which it could be interesting to bet on compound hypotheses. From the TBM point of view, it means generalising the PT so that it can handle compound bets. Smets has already presented such a generalisation [2], and it appears [3] to correspond to the situation of a “*n*-person games” [4] presented by Shapley in the Transferable Utility Model in 1953. This work on game theory considers the case of a coalition of gamblers who wants to share fairly the gain with respect to the involvement of each. Once the formula is transposed to the TBM, the purpose is to share a global belief between several compound hypotheses. Obviously, one expects the transform to promote the hypotheses the cardinality of which is the greatest... Roughly, it means that, if for the same book, it is possible to bet on the singleton hypothesis  $\{h_1\}$  or on the compound hypothesis  $\{h_1, h_2\}$ , then, this latter must be preferred (even if the chances for  $h_1$  are far more interesting than for  $h_2$ ). Practically, this intuitive behaviour looks perfectly accurate, and of course, the generalization proposed by Smets behaves so.

On the other hand, there are yet other situations, where it should be encouraged to bet on singleton hypotheses when possible, whereas it should remain allowed to bet on compound hypotheses when it is impossible to be more accurate. Hence, we depict a “progressive” decision process, where it is possible to remain slightly hesitant, and to manually tune the level between hesitation and bet. Let us imagine such a situation: the position of a robot is modelled by a state-machine, and its trajectory along a discrete time scale is modelled by a lattice. At each iteration of the discrete time, the sensors provide information to the robot, and these pieces of information are processed in the TBM framework: they are fused together (the credal level) and the state of the robot is inferred by a decision process (the pignistic level). At this point several stances are possible:

- the classical PT is used. Unfortunately, as the sensors are error-prone, the inferred state is not always the right one. Finally, the inferred trajectory is made of right and wrong states with respect to the ground-truth (Fig. 1). Of course, the TBM provides several tools to filter such trajectories [5,6,7], and, in spite of a relative computational cost, they are really efficient.
- Instead of betting on a single state at each iteration of the time, it is safer to bet on a compound hypothesis (i.e. on a group of several states, knowing that, the more numerous, the less chance to make a mistake). Unfortunately, the risk is now to face a situation where no real decision is made and the inferred trajectory is too imprecise (Fig. 2).
- The balance between these two extreme stances would be to automatically tune the level of hesitation in the bet: When the decision is difficult to make, a compound hypothesis is assessed to avoid a mistake, and otherwise, a singleton hypothesis is assessed, to remain accurate (Fig. 3).



**Fig. 1.** The space-time lattice: the horizontal axis represents the time iterations, and the vertical axis, the states. The real trajectory (ground truth) is represented by the black line, and the inferred states are presented by black dots linked by the grey line. The real and inferred trajectories differ, as few mistakes are made in the decision process.

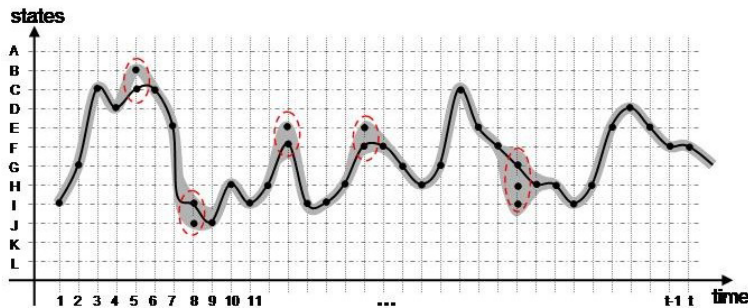


**Fig. 2.** In a similar manner to figure 1, the real trajectory (ground truth) is compared to the inferred one. As a matter of fact, no mistake is made on the inferred trajectory, but, as a drawback, it is really imprecise.

The first stance corresponds to classical decision making. The second and third stances both correspond to situations where it is possible to bet on compound hypotheses, but in a different manner. The second stance is rather classical from belief functions point of view, and several types of decision based on non-additive measures [8] achieve efficient results (such as [4,9]). Nevertheless, in spite of an adapted mathematical structure, they do not model the problem in a manner that corresponds to the kind of decision we expect in the third stance (as shown in 3.1). By now, the only way to perform a decision according to the third stance is to set an ad-hoc method. For instance, it is possible to consider compound hypotheses, and practise hypothesis testing, such as in classical statistical theory. With such a method, the size of the selected compound hypothesis is related to the p-value desired. In a similar way, but in a more subjective state of mind, it is also possible to associate a cost to each decision and to minimise the cost function. Finally, it is possible to simply assess a threshold  $T$  to the probability of each decision, to sort them in descending order, and to select the first  $n$  hypotheses so that their probabilities add up to a value superior to  $T$ . For all

these methods, we do not provide bibliographical links, as they are based on very basic scholar knowledge.

This paper aims at defining a decision process according to the third stance in the context of the TBM. In section 2, we briefly present the TBM. In section 3 we analyse related works and we focus on the Shapley value and the corresponding PT generalization. We show that minor modifications lead to the expected results. In section 4, we present our new method to generalize the PT, and give some interesting properties. Finally, section 5 illustrates it with real examples.



**Fig. 3.** In a similar manner to figure 1, the real trajectory (ground truth) is compared to the inferred one. A trade-off between risky bets (a singleton state is assessed) and imprecise decisions (circled by a dot line) allows limiting the number of mistake while remaining quite precise

## 2 Transferable Belief Model

In this section we rapidly cover the basis of the TBM [1] and of the belief function theory [10], in order to set the notations. We assume the reader to be familiar with belief functions.

Let  $\Omega$  be the set of  $N$  exclusive hypotheses  $\Omega = \{h_1, \dots, h_N\}$  for a variable  $X$ .  $\Omega$  is called the **frame of discernment**. Let  $2^\Omega$ , called the **powerset** of  $\Omega$ , be the set of all the subsets  $A$  of  $\Omega$ , including the empty set (it is the sigma-algebra of  $\Omega$ ):  $2^\Omega = \{A/A \subseteq \Omega\}$ .

A **belief function**, or a **basic belief assignment**(BBA)  $m(\cdot)$  is a set of scores defined on  $2^\Omega$  that adds up to 1:

$$m : 2^\Omega \rightarrow [0, 1]$$

$$A \mapsto m(A) \text{ with } \sum_{A \subseteq \Omega} m(A) = 1$$

A **focal element** is an element of the powerset to which a non-zero belief is assigned. We call as the **cardinality** of a focal element, noted  $|\cdot|$ , the number of elements of  $\Omega$  it contains. For sake of simplicity, we say that a hypothesis or a focal element is larger (or wider) than another when its cardinality is greater. Hence, a BBA represents a subjective belief in the propositions that correspond to the elements of  $2^\Omega$  and nothing wider or smaller.

The **conjunctive combination** is a  $N$ -ary symmetrical and associative operator that models the fusion of the pieces of information coming from  $N$  independent sources (it is the core of the **credal level**):

$$\begin{aligned} \circledast : \mathfrak{B}^\Omega \times \mathfrak{B}^\Omega \times \dots \times \mathfrak{B}^\Omega &\rightarrow \mathfrak{B}^\Omega \\ m_1 \circledast m_2 \circledast \dots \circledast m_N &\mapsto m_{\circledast} \end{aligned}$$

with,  $\mathfrak{B}^\Omega$  corresponding to the set of the BBA defined on  $\Omega$ , and

$$m_{\circledast}(A) = \sum_{\bigcap_{i=1}^N A_i = A} \left( \prod_{n=1}^N m_n(A_n) \right) \quad \forall A \subseteq 2^\Omega$$

The pignistic probability measure ( $\text{BetP}$ ) is defined by the use of the **pignistic transform** (in the **pignistic level**):

$$\text{BetP}(X = h) = \frac{1}{1 - m(\emptyset)} \sum_{h \in A, A \subset \Omega} \frac{m(A)}{|A|} \quad \forall h \in \Omega$$

Then, the pignistic probability distribution is computed:  $p(h) = \text{BetP}(X = h) \quad \forall h \in \Omega$ , or, in other words:  $p = \text{PT}(m)$ . Finally, the hypothesis of maximum pignistic probability  $\tilde{h}$  is selected:  $\tilde{h} = \text{argmax}_{h_i \in \Omega}(p(\cdot))$

### 3 The Shapley Value and the Pignistic Transform

#### 3.1 Related Work

Several generalizations/alternatives to the PT exist in the literature. When proposing such a work, authors do not have the same objective, which explains this manifold. In [11,12], the point is to find a conversion method between probabilistic models and evidential ones. Then, the PT is compared to the plausibility transform, and this later is assessed to be more adapted. In [13], the point is to face the computational complexity of BF by finding an adequate probabilistic approximation. In [16] the Generalized Pignistic Transformation is defined. In spite of its name, is not related to the TBM framework: it is the counterpart of the PT in a framework which is an attempt of generalization of the TBM. This framework, its potential applications and the way it generalizes the PT or propose alternative transforms have nothing in common with this work. In [14,15], several transforms are proposed as alternatives to derive a bet-like decision from a BF. Finally, in each of these works, the alternatives to the BFs (i.e. the alternative mathematical structures that support the information prior to the decision) are either out of our interests [16], either probabilistic. Consequently, as interesting these works remain, they are not in the scope of this paper, in the meaning that they do not propose any generalizations to partial bets, such as targeted here.

Nevertheless, there are several works in which alternative structures (i.e. neither BF, neither probabilities) are proposed and may fit our need here. These

structures belong to the general class of fuzzy measures [8] (also called capacities, or non-additive measures [19]). Unfortunately, these works aim at defining alternative structures which are computationally more efficient than BF; and the definition of a non-additive measure adapted to partial bet is not investigated. For instance, the works of Cuzzolin [17] prove that the space of the BFs defined on a dedicated frame is a simplex, and it provides a framework to analyse the various transforms of the DST from a geometrical point of view. For us, it brings new insights with respect to decision making as this geometrical work stresses the link between structures on which decision is classically made (e.g. a bayesian BF, a plausibility function), and the geometrical transforms that make the conversion amongst these structures (e.g. the PT, the Mbius transforms). In [18,19], Gravisch introduces  $k$ -order additive fuzzy measures, which can be seen as an intermediate type of structure between probabilities (1-order measures) and BF: it corresponds to BF for which the cardinality of the largest focal element is  $k$ . Once again, the main objective is to define more efficient structure for computational aspects.

These papers do not investigate the consequences of the use of these structures in decision making. To our knowledge, the only work in which decision making scenarios with compound hypotheses are considered was carried out by Shapley [4], and then re-investigated by Smets [2,3], in the context of the TBM. Hence, we mainly base our work on this latter.

### 3.2 The Shapley Value

In [3], Smets summarizes his work of [2] and explains how to derive the PT in case of non-singleton bets. As it is explained altogether with the assessment of the result, it concurs with the work of Shapley [4]:

$$\text{BetP}'(X = B) = \frac{1}{1 - m(\emptyset)} \sum_{A \subseteq \Omega} \frac{m(A) \cdot |A \cap B|}{|A|} \quad \forall B \subseteq \Omega$$

In this equation, the value associated to  $B$  is the sum of (1) all the hypotheses of strictly smaller cardinality which are nested in  $B$ , (2)  $m(B)$  itself (3) an "inherited" mass from wider hypotheses in which  $B$  is nested, and (4) all other hypotheses for which there is no inclusion relation with  $B$ , but the intersection of which with  $B$  is non-empty:

$$\begin{aligned} \text{BetP}'(X = B) = \frac{1}{1 - m(\emptyset)} \cdot & \left[ \sum_{A \subseteq B} m(A) + m(B) \right. \\ & + \sum_{B \subset A \subseteq \Omega} \frac{m(A) \cdot |B|}{|A|} \\ & \left. + \sum_{\text{other } A \subseteq \Omega} \frac{m(A) \cdot |A \cap B|}{|A|} \right] \quad \forall B \subseteq \Omega \end{aligned}$$

Of course, in case of  $B$  being a singleton hypothesis, it corresponds to the classical PT: The first and the fourth terms are zero-valued, and  $|B| = 1$ , so that:

$$\text{BetP}'(X = B) = \frac{1}{1 - m(\emptyset)} \cdot \left[ m(B) + \sum_{B \neq A, B \in A \subseteq \Omega} \frac{m(A)}{|A|} \right] = \text{BetP}(X = B) \quad \forall B \in \Omega$$

### 3.3 Application of Shapley Work to Partial Bets

Now, let us forget the original interest of this formula, and let us consider it through our own aim: Is it sensible to consider the Shapley value as a generalization of the PT which allows comparing hypotheses of different cardinality? Basically, the first term in the previous equation means, that the value associated to a compound hypothesis  $\{h_1, h_2\}$  is increased by the belief of all the hypotheses which are nested in it :  $\{h_1\}$  and  $\{h_2\}$ . Moreover, as all the considered hypotheses inherit belief of wider hypotheses in a manner proportional to their cardinality, it is impossible to assign a pignistic probability to  $\{h_1\}$  or  $\{h_2\}$  which is greater to the one assigned to  $\{h_1, h_2\}$ . As a consequence, larger hypotheses are always promoted in the decision making, which leads to situations such as the one illustrated in figure 2.

In addition, the fourth term is also problematic with respect to partial bets. Because of it, an important belief in a compound hypothesis  $\{h_1, h_2\}$  increases the Shapley value for another compound hypothesis  $\{h_1, h_3\}$  as their intersection is none empty. In our situation,  $\{h_1, h_2\}$  and  $\{h_1, h_3\}$  are different and exclusive choices for the decision. The value assess to a compound hypothesis must keep an evidential interpretation, as we deal with hypothesis of different cardinality simultaneously (as in the credal level). Then, it must not yet be understood as a probabilistic sigma-algebra, and we should stick to an interpretation similar to Shafer's concerning the belief assignment [10], reading that, *the BBA in a focal element models the belief that can be placed in it, and nothing smaller or larger.*

The transform leading to the Shapley value is really interesting as a natural generalization of the PT. Unfortunately, it does not lead to an acceptable solution when a partial bet is expected. On the other hand, as we root in the TBM, the decision process we aim at must also remain related to the PT. As a consequence, we propose to start from the Shapley value, and to modify it, so that it fulfils our requirements. The first natural step is to remove terms 1 and 4, as they appear to be problematic. On the contrary, the terms 2 (which represents the belief in the considered hypothesis) and 3 (which represents inherited beliefs) are perfectly natural, except for normalization considerations: As some of the redistributions of the belief have been discarded, it is natural to consider to renormalize their values so that the total mass is conservative.

### 3.4 Axiomatic Justification of the PT

By now, let us recall the axiomatic of the PT, in order to make sure that we respect it. In contradiction to what is often read, the PT is not justified by

the principle of insufficient reason [20], nor it is justified by the proof that it is impossible to built a Dutch book against the PT. As a matter of fact, Smets is rather explicit on these two points:

- An intuitive generalization of the principle of insufficient reason is a cue of the interest of the PT, but it does not justify it [21].
- In spite of a particular Dutch Book discarded in [3], the proof that it resists to all Diachronic Dutch Books in general is not given.

Then, the only justifications of the PT rely in five axioms (linearity, projectivity, efficiency, anonymity, false event). These axioms are not always accepted beyond the TBM interpretation of the Belief function theory, and consequently, the PT is also discussed by supporters of other interpretations [11,12]. Within the TBM, it is nonetheless the only decision process accepted. As this work roots in the TBM, **our single concern is to remain coherent with this framework and with these five axioms**, to which we add a sixth, the conservation principle.

#### 4 Generalization to Partial Bets

As explained in our introductive example, the point is to allow hesitation, but to control it, so that, when it is not necessary, no hesitation occurs. A very simple way to control the hesitation is that the decision maker defined a maximum amount of authorized hesitation. Thus, let  $\gamma \in \mathbb{N} \cap [1, |\Omega|]$  a threshold that models this amount. Let  $L_i$  the set of hypotheses of cardinality  $i$  ( $L$  stands for “level”). The decision is made within  $\Delta_\gamma = \{L_1, \dots, L_\gamma\}$ .

Then, our purpose is to define a probability measure  $\mathbb{B}$  on  $\Delta_\gamma$ , so that a decision can be made by selecting the element of  $\Delta_\gamma$  the value of which is the greatest. The corresponding probability space  $(\Delta_\gamma, \mathcal{F}, \mathbb{B})$ , where  $\mathcal{F}$  is the canonical sigma-algebra of  $\Delta_\gamma$ , must be derived from the measured space  $(\Omega, 2^\Omega)$  in a manner similar to the probability space  $(\Omega, 2^\Omega, \text{Bet}\mathbb{P})$ , i.e. by the definition of an appropriate transform. Intuitively,  $\mathbb{B}$  looks like a  $\gamma$ -additive BF [18], but as a matter of fact, its interpretation as such is problematic.

$\Delta_\gamma$  must be understood as a decision space in itself, in which a variable  $D$  (which stands for “Decision”) takes its value, and which is not related to  $2^\Omega$ , in which the variable  $X$  takes its value. This may appear as strange, as the elements of  $\Delta_\gamma$  corresponds to elements of  $2^\Omega$ , but from the decision point of view,  $\{h_1, h_2\}$  and  $\{h_1\}$  are two different elements of  $\Delta_\gamma$ , and they are decisions which are exclusive one another. On the contrary, for  $X$ ,  $\{h_1, h_2\}$  and  $\{h_1\}$  are nested. That is why  $\mathbb{B}$  is not a BF. As a consequence, in spite of a similar mathematical structure, it can not be interpreted as a  $\gamma$ -additive BF. In [18], Gravisch stresses that the interpretation of a non-additive measure is rather difficult, and the interpretation of  $\mathbb{B}$  perfectly illustrates this fact. Equivalently,  $\text{Bet}\mathbb{P}$ , the result of the PT, which has a structure equivalent to a bayseian BF, can not be interpreted as such [11,12]; otherwise its combination with a BF thanks to Dempster’s rule would be significant. As we consider the TBM as

the frame of this work, it is obvious that the interpretation of  $\mathbb{B}$  as a BF is as problematic as the interpretation of  $\text{BetP}$  as a BF.

Let  $A \notin \Delta_\gamma$ . As explained, the point is to “take” the belief  $m(A)$ , to “replace” it by a zero value, and to “redistribute” it to hypotheses within  $\Delta_\gamma$ . Let  $B$  such a hypothesis within  $\Delta_\gamma$ . In a way similar to the PT, the redistribution must be linear, that is, proportional to the size of the hypothesis that inherits it. Moreover, the redistribution must remain conservative, so that, making a decision on  $\Omega$  by selecting an element of  $\Delta_{\gamma_1}$ , then, making a more precise decision by selecting an element of  $\Delta_{\gamma_2}$  with  $\gamma_2 < \gamma_1$  is equivalent to directly make a decision on  $\Omega$  by selecting an element of  $\Delta_{\gamma_2}$ .

As a consequence, all the hypotheses within a level  $L_i$  must inherit the same amount of belief, and each level  $L_i$  must globally inherit a belief proportional to  $|L_i| \times i$ . Hence, we propose to share  $m(A)$  into  $N$  parts, so that, all the elements of  $L_i, \forall i \leq \gamma$  inherits  $i$  parts of  $m(A)$ .  $N$  depends on the number of hypotheses of  $D_\gamma$  which are nested in  $B$ . This number depends in turn of  $\gamma$ , and the size of  $A$ . An elementary enumeration leads to the following formula:

**Definition 1**

$$N(|A|, \gamma) = \sum_{k=1}^{\gamma} C_k^{|A|} \cdot k$$

where  $C_p^n = \frac{n!}{p!(n-p)!}$  is the number of combinations of  $p$  elements among  $n$ .

Now that the redistribution pattern is defined, let us derive the transform itself:

**Definition 2.** The probability measure  $\mathbb{B}_\gamma$  is derived from the BBA  $m(\cdot)$  by the following transform:

$$\mathbb{B}_\gamma(D = B) = \frac{1}{1 - m(\emptyset)} \cdot \left[ m(B) + \sum_{B \subset A \subseteq \Omega, A \notin \Delta_\gamma} \frac{m(A) \cdot |B|}{N(|A|, \gamma)} \right] \quad \forall B \in \Delta_\gamma$$

**Proposition 1.** The pignistic transform is a particular case of definition 2, as we have  $\mathbb{B}_1(\cdot) = \text{BetP}(\cdot)$ .

*Proof.* If  $\gamma = 1$ , several simplifications occur:  $\Delta_\gamma \equiv \Omega, X \equiv D, N(|A|, \gamma) = |A|$  and  $|B| = 1$ . As  $B$  is a singleton, let us note it  $h$ . One has  $\forall h \in \Omega$ :

$$\begin{aligned} \mathbb{B}_1(D = h) &= \frac{1}{1 - m(\emptyset)} \cdot \left[ m(h) + \sum_{h \in A \subseteq \Omega, |A| \neq 1} \frac{m(A)}{|A|} \right] \\ &= \frac{1}{1 - m(\emptyset)} \cdot \sum_{h \in A \subseteq \Omega} \frac{m(A)}{|A|} = \text{BetP}(D = h) = \text{BetP}(X = h) \quad \square \end{aligned}$$

Another interesting property is derived from the conservation principle:

**Proposition 2.** Making a decision by selecting  $\delta_1 \in \Delta_{\gamma_1}$  with the use of  $\mathbb{B}_{\gamma_1}$ , and then, making a decision on  $\delta_1$  by the use of  $\mathbb{B}_{\gamma_2}, \gamma_2 < \gamma_1$ , is equivalent to directly make a decision by the use of the probability measure  $\mathbb{B}_{\gamma_2}$ .

From an applicative point of view, this last result is really interesting, as it means, it is possible to make a decision on  $\Delta_{|\Omega|-1}$ , by redistributing the belief from  $L_{|\Omega|}$ , in order to discard a single element of  $\Omega$ , then to make a decision on  $\Delta_{|\Omega|-2}$  by redistributing only the belief from  $L_{|\Omega|-1}$ , and so on, until  $\Delta_1$ . This set of operations has a computational cost similar to the one necessary to make a decision over  $\Delta_1$ : the belief at each level  $L_i$  is redistributed one time to compute  $\mathbb{B}_1$  or  $\mathbb{B}_\gamma, \forall \gamma \in [1, |\Omega|]$ . Then, it is possible for the decision maker to rapidly analyse the capability of the decision process to focus on a compound hypothesis of restricted cardinality, prior to the definition of  $\gamma$ .

Now the transform is explicit, the removal of two of the four terms in the original Shapley value may appear as arbitrary: We mainly explain it from a “functional” point of view. On the other hand, here are strong evidences that a more mathematical construction is also achievable: (1) the  $\gamma$ -additive structure, with  $\gamma = 1$  being equivalent to the PT and with  $\gamma = |\Omega|$  being equivalent to the original BF, and (2) our formula has strong similarities with the orthogonal projection of a BF on the probability simplex [17]. Hence, identifying the geometrical transform that justify it is an interesting future works to focus on.

## 5 Applications to American Sign Language Recognition

In this section, we briefly summarize a previous work of ours [22], in which the transform has been used for gesture recognition: We proposed to recognize an American Sign Language gesture performed in front of a video camera, among a set 19 possible gestures. For any gesture  $G^i$ , a dedicate Hidden Markov Model  $HMM^i$  is trained. For any new occurrence  $G^j$  to recognize, the system computes the probability of the observed gesture to be the observation sequence produced by each  $HMM^i$ . A classical method is to recognize the new gesture as an occurrence of the gesture  $G^*$  which  $HMM^*$  produces the highest likelihood.

Amongst the 19 gestures, few pairs of them are so closed to each others than the system does not discriminate. Consequently, the overall accuracy for the recognition task is reasonably good (75.88% on 228 items), but several mistakes occur between the similar pairs or triplets. This is why, in this article, we have first proposed to set a decision method which allows to produce a single decision when it is possible and to produce an incomplete decision otherwise, in order to complete it in a second step. The use of the  $\mathbb{B}_2$  and  $\mathbb{B}_3$  provides far better result than  $\mathbb{B}_1$  or equivalently  $\text{Bet}\mathbb{P}$ . On the 228 items in the test set, there are 189 examples for which a complete decision is made. For this singleton decision, the accuracy is 79.37%: the fewer remaining singletons are less error-prone. For the other examples the decision is imprecise. If we consider the decision as a right one when one of the elements of the compound hypothesis is the good one, then, the overall accuracy is 82.02%. From the applicative point of view, it shows that the concept of such a decision process is accurate as it allows focusing the imprecision of a decision only when it is necessary.

In a second step, we fuse the information of the manual gestures with additional non-manual gestures (face/shoulders motions, facial expression, which

are very important in ASL) in order to help do discriminate among a cluster of similar gestures. These non-manual gestures are completely inefficient to discriminate if they are used in the first step altogether with the manual ones, as their variability is hidden by variability of the manual feature. Hence, the progressive nature of the decision process is helpful for a hierarchical data fusion.

Finally, when compared to classical Bayesian methods, it appears that our complete system is both more accurate and more robust: For instance, 31.1% of the mistakes are avoided with respect to a situation where the second step is systematically used: In such a use, the second step put back into question some good decisions of the first step (see [22] for a comprehensive evaluation).

## 6 Conclusion

In this article, we have presented a generalisation of the pignistic transform that differs from Smets' one, as it does not corresponds to the Shapley value. It provides an alternative when it is necessary to control the trade-of between hesitation and bet in decision making. Moreover, the classical pignistic transform from Smets appears to be a particular case of this generalisation. From a theoretical point of view, the main change with respect to Shapley's work relies in (1) the manner the belief in too large focal elements is normalized prior to its redistribution, and (2) the restriction to focal elements of cardinality  $\leq \gamma$ , as with  $k$ -additive belief functions. From a practical point of view, its application to Sign Language recognition stresses its interest on real problems. Three directions are considered for future works: (1) application to other real problems in pattern recognition, (2) its application to credal time-state models [5], and (3) the geometrical explanation of the transform in the probability simplex [17].

## References

1. Smets, P., Kennes, R.: The transferable belief model. *Art. Int.* 66(2), 191–234 (1994)
2. Smets, P.: Constructing the pignistic probability function in a context of uncertainty. *Uncertainty in Artificial Intelligence* 5, 29–39 (1990)
3. Smets, P.: No Dutch book can be built against the TBM even though update is not obtained by bayes rule of conditioning. In: *Workshop on probabilistic expert systems*, Societa Italiana di Statistica, Roma, pp. 181–204 (1993)
4. Shapley, L.: A Value for  $n$ -person Games. *Contributions to the Theory of Games*. In: Kuhn, H.W., Tucker, A.W. (eds.) *Annals of Mathematical Studies*, vol. 2(28), pp. 307–317. Princeton University Press, Princeton (1953)
5. Ramasso, E., Rombaut, M., Pellerin, D.: Forward-Backward-Viterbi procedures in the Transferable Belief Model for state sequence analysis using belief functions, ECSQARU, Hammamet, Tunisia (2007)
6. Xu, H., Smets, P.: Reasoning in Evidential Networks with Conditional Belief Functions. *International Journal of Approximate Reasoning* 14, 155–185 (1996)
7. Ristic, B., Smets, P.: Kalman filters for tracking and classification and the transferable belief model. In: *FUSION 2004*, pp. 4–46 (2004)

8. Dennenberg, D.: *Non-Additive Measure and Integral*. Kluwer Academic Publishers, Dordrecht (1994)
9. Dubois, D., Prade, H.: *Possibility theory: an approach to computerized processing of uncertainty*. Plenum Press (1988)
10. Shafer, G.: *A Mathematical Theory of Evidence*. Princeton University Press, Princeton (1976)
11. Cobb, B., Shenoy, P.: On the plausibility transformation method for translating belief function models to probability models. *Int. J. of Approximate Reasoning* (2005)
12. Cobb, B., Shenoy, P.: A Comparison of Methods for Transforming Belief Functions Models to Probability Models. In: Nielsen, T.D., Zhang, N.L. (eds.) *ECSQARU 2003*. LNCS (LNAI), vol. 2711, pp. 255–266. Springer, Heidelberg (2003)
13. Voorbraak, F.: A computationally efficient approximation of Dempster-Shafer theory. *International Journal on Man-Machine Studies* 30, 525–536 (1989)
14. Daniel, M.: Probabilistic Transformations of Belief Functions. In: Godo, L. (ed.) *ECSQARU 2005*. LNCS, vol. 3571, pp. 539–551. Springer, Heidelberg (2005)
15. Sudano, J.: Pignistic Probability Transforms for Mixes of Low- and High- Probability Events. In: *Int. Conf. on Information Fusion*, Montreal, Canada (2001)
16. Dezert, J., Smarandache, F., Daniel, M.: The Generalized Pignistic Transformation. In: *7th International Conference on Information Fusion* Stockholm, Sweden (2004)
17. Cuzzolin, F.: Two new Bayesian approximations of belief functions based on convex geometry. *IEEE Trans. on Systems, Man, and Cybernetics - B* 37(4), 993–1008 (2007)
18. Grabisch, M.: K-order additive discrete fuzzy measures and their representation. *Fuzzy sets and systems* 92, 167–189 (1997)
19. Miranda, P., Grabisch, M., Gil, P.: Dominance of capacities by k-additive belief functions: *EJOR* 175, 912–930 (2006)
20. Keynes, J.: *Fundamental Ideas. A Treatise on Probability*. Macmillan, Basingstoke (1921)
21. Smets, P.: Decision Making in the TBM: the Necessity of the Pignistic Transformation. *Int. J. Approximate Reasoning* 38, 133–147 (2005)
22. Aran, O., Burger, T., Caplier, A., Akarun, L.: A Belief-Based Sequential Fusion Approach for Fusing Manual and Non-Manual Signs. *Pattern Recognition* (2009)