A Survey on "Binary Message" LDPC decoder

Emmanuel Boutillon*, Chris Winstead°

*Université de Bretagne Sud
°Utah State University

November the 4th, 2014

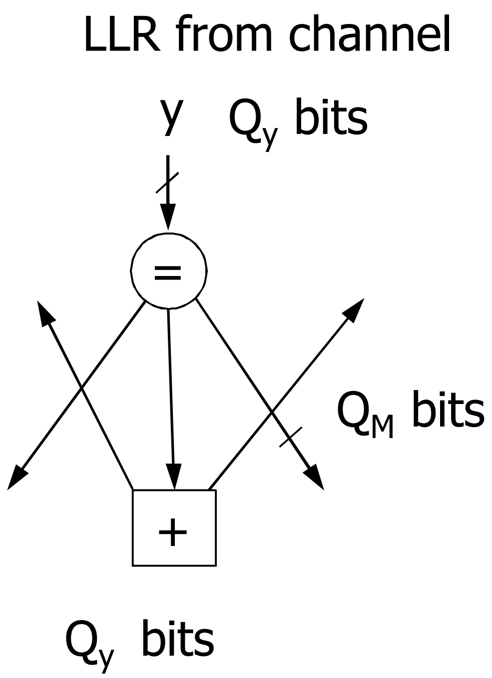
Outline

- Classifications of BM LDPC decoder
- State of the art
- Recent results
- Conclusions

Outline

- Classifications of BM LDPC decoder
- State of the art
- Recent results
- Conclusions

Binary message LDPC decoder

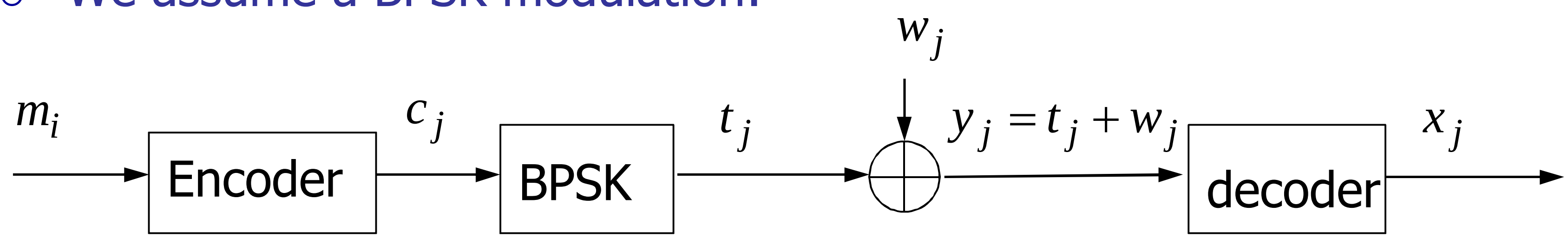


	HARD DECISION $Q_y = 1$	SOFT DECISION $Q_y \geq 1$
$Q_M = 1$	Gallager A,B	Binary message
$Q_M \geq 1$	FAID	BP, Min-Sum

Performance ⟷ Complexity

Notations for BM LDPC decoder

- We assume a BPSK modulation:



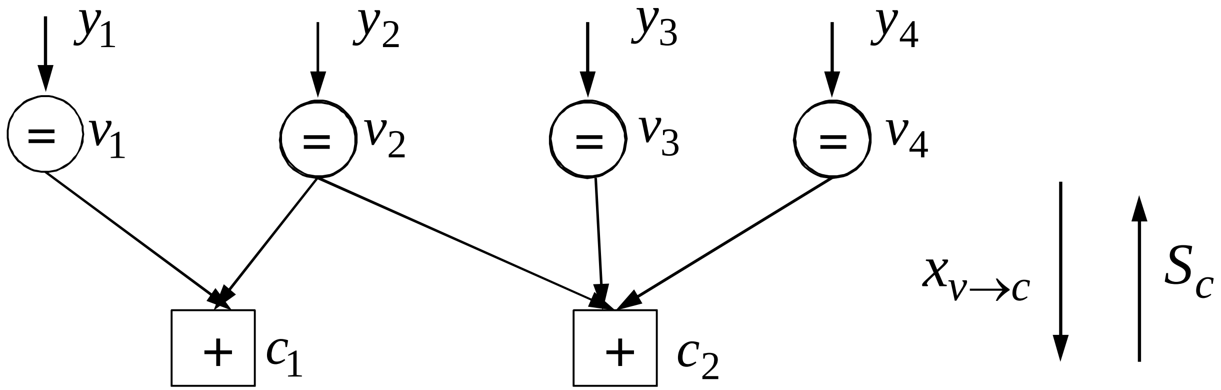
  - Bit $c = 0$ is associated to $t = +1$
  - Bit $c = 1$ is associated to $t = -1$
  - x_j is the estimated decoded value of t_j.

- The first estimate of x is the sign of y: $x^{(0)} = \text{sign}(y)$.

LDPC representation

Variable node

Check node



$C(v)$ set of check nodes connected to variable *v*.

$V(c)$ set of variable node connected to check node *c*.

$x_{v \to c}$ Binary message (-1 or 1) from variable node *v* to check node *c*

$S_c = \prod_{v' \in V(c)} x_{v' \to c}$ syndrome of check node *c* (1 if OK, -1 if NOK)

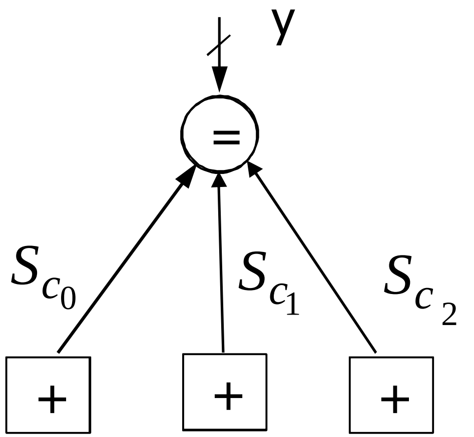
Note: since message are binary: $x_{c \to v} = x_{v \to c} S_c = \prod_{v' \in V(c)/v} x_{v' \to c}$

Memoryless – State variable

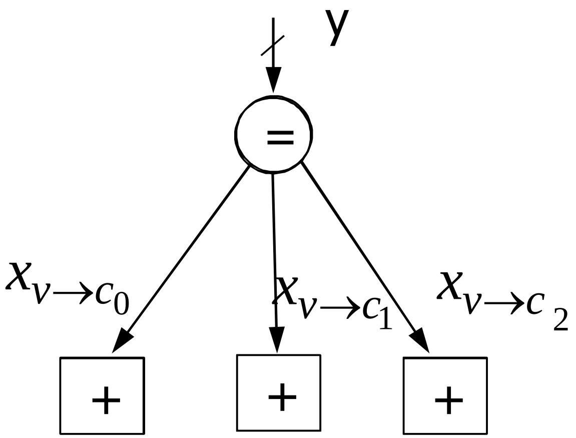
- In a memoryless node, the output messages depend only on the current input messages.
- In a State-variable node, the output message depend also from an internal state *State* (memory effect).
- For example, Self Corrected Min-Sum is an algorithm where the check node posses an internal variable (the sign of the messages of the previous iteration).

Extrinsic/Broadcast messages

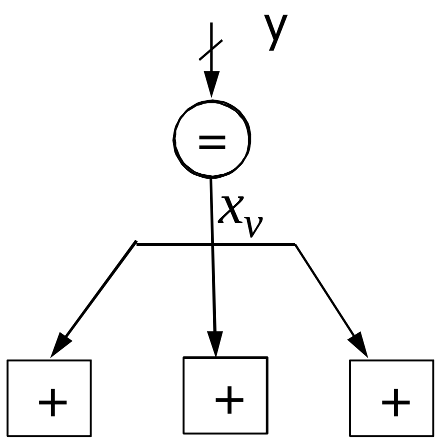
Check->variable



Extrinsic messages



Broadcast message



$$x_{v \to c_0} = f(y, S_{c_1}, S_{c_2}, State)$$
$$x_{v \to c_1} = f(y, S_{c_0}, S_{c_2}, State)$$
$$x_{v \to c_2} = f(y, S_{c_0}, S_{c_1}, State)$$

$$x_v = f(y, S_{c_0}, S_{c_1}, S_{c_2}, State)$$

Note: check node can send syndrome (broadcast) or $x_{c \to v}$ messages.
Both are equivalent (implementation detail).

Deterministic/ Non-Deterministic

- Deterministic: the decoded output is only function of the received message.
- Non-Deterministic: some "randomness" is introduced in the decoding process: the decoded output become a random process.

Classification of BM–LDPC decoder

Density evolution
is feasable

	Memoryless		State variable	
Extrinsic				
Broadcast				
	Non-D	Deterministic	Deterministic	Non-D

- We can construct a grid to classify the algorithms
  => may give some ideas.

Outline

- Classifications of BM LDPC decoder
- State of the art
- Recent results
- Conclusions

Stochastic decoder

	Memoryless		State variable	
Extrinsic		BP →	→	STOCH.
Broadcast				
	Non-D	Deterministic		Non-D

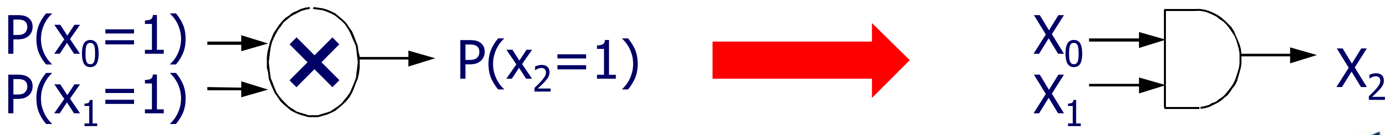
Stochastic decoder

- Based on the Believe Propagation algorithm with probability encoded by a random binary stream.

- Principle: P(x=1) the probability of a random variable x to be 1 is represented by a random binary stream, where the probability to have one is exactly P(x=1).

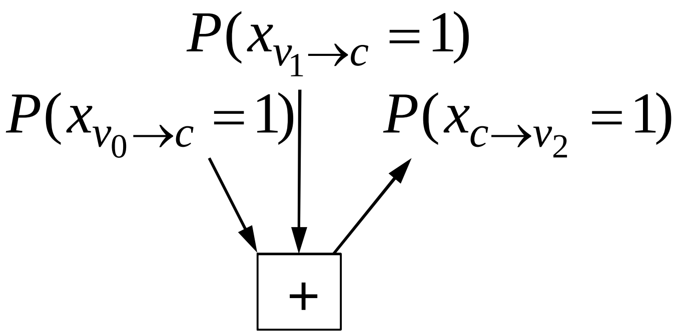
- For example X="001000110010000001001000000001001000…"
  is an alternative (an redundant) representation of P(x=1)=0.2.

- Arithmetic is trivial with this representation.



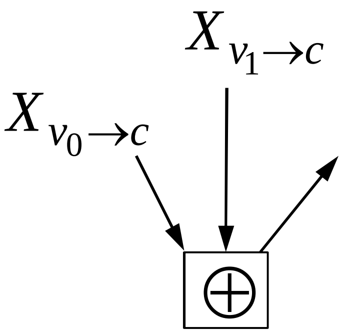
Check node stochastic architecture

- Check node equation in probability domain (BP)



$$P(x_{c \to v_2} = 1) = P(x_{v_0 \to c} = 1)P(x_{v_1 \to c} = 0) + P(x_{v_0 \to c} = 0)P(x_{v_1 \to c} = 1)$$

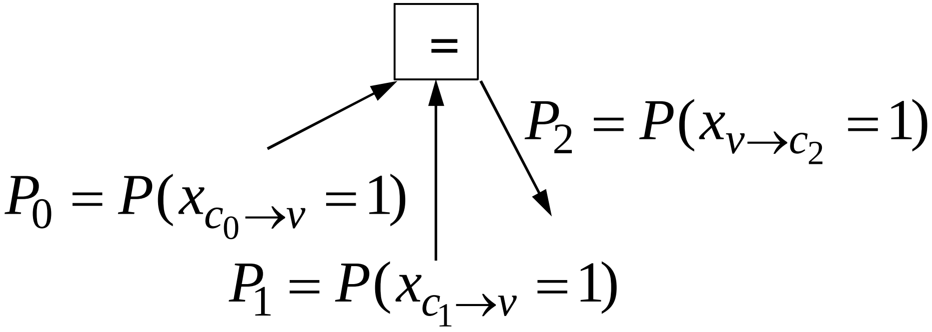
- Check node computation in stochastic domain: a simple XOR.



$$X_{c \to v_2} = X_{v_0 \to c} \oplus X_{v_1 \to c}$$

Variable node stochastic architecture

- Variable node equation in probability domain (BP)



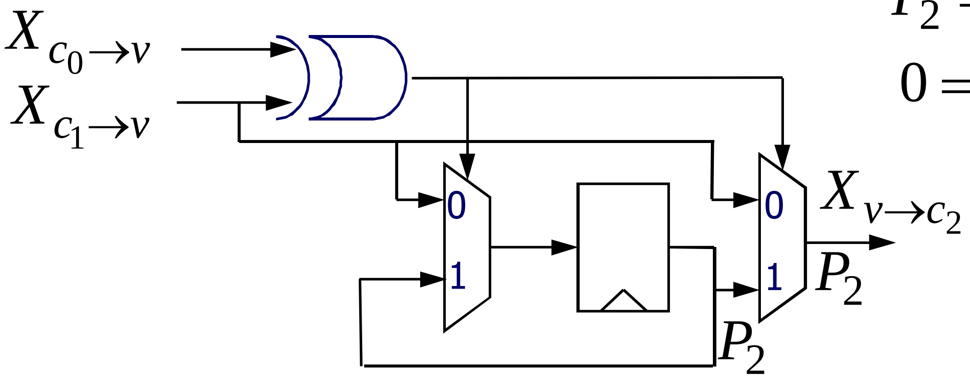
$$P_2 = \frac{P_0 P_1}{(1-P_0)(1-P_1) + P_0 P_1}$$

- Variable node computation in stochastic domain:

$$P_2 = P_0 P_1 + ((1-P_0)P_1 + P_0(1-P_1))P_2$$

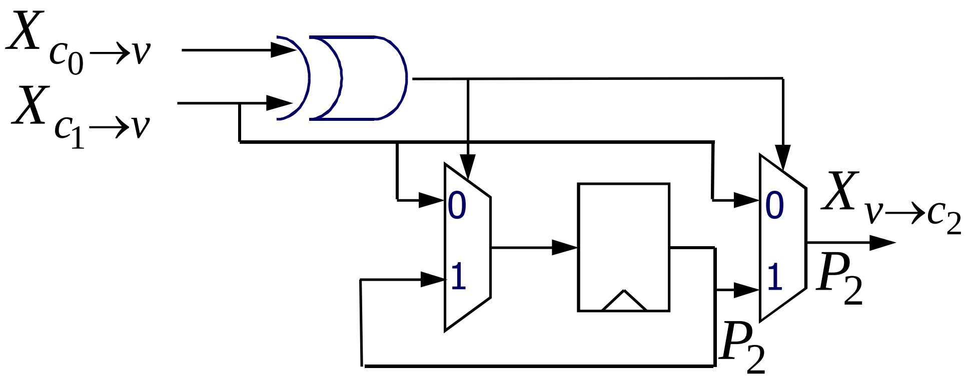
$$P_2 = P_0 P_1 + (1 - P_0 P_1 - (1-P_0)(1-P_1))P_2$$

$$0 = P_0 P_1 - (P_0 P_1 + (1-P_0)(1-P_1))P_2$$

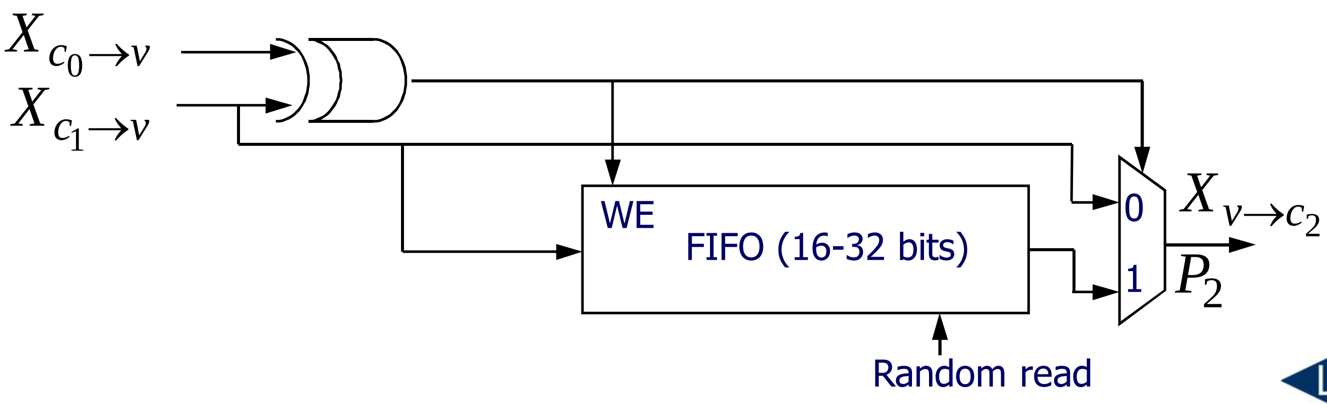


Variable node stochastic architecture

- To avoid deadlock cycle in the graph



- Is replaced by (edge memory)



References on Stochastic LDPC dec.

[1] Rapley, A.; Gaudet, V.; Winstead, C., "On the simulation of stochastic iterative decoder architectures," *Canadian Conference on ECE, 2005.* , vol., no., pp.1868,1871, 1-4 May 2005

[2] Tehrani, S.S.; Gross, W.J.; Mannor, S., "Stochastic decoding of LDPC codes," *Communications Letters, IEEE* , vol.10, no.10, pp.716,718, Oct. 2006

[3] Djordjevic, I.B et al., "LDPC codes and stochastic decoding for beyond 100 Gb/s optical transmission," *Optical Communication, 2008. ECOC 2008. 34th European Conference on* , vol., no., pp.1,2, 21-25 Sept. 2008

[4] Tehrani, S.S. et al., "Majority-Based Tracking Forecast Memories for Stochastic LDPC Decoding," *Signal Processing, IEEE Transactions on* , vol.58, no.9, pp.4883,4896, Sept. 2010

[6] Onizawa, N. et al., "Clockless Stochastic Decoding of Low-Density Parity-Check Codes," *Signal Processing Systems (SiPS), 2012 IEEE Workshop on* , vol., no., pp.143,148, 17-19 Oct. 2012

[7] Leduc-Primeau, Francois; Hemati, Saied; Mannor, Shie; Gross, Warren J., "Relaxed Half-Stochastic Belief Propagation," *Communications, IEEE Transactions on* , vol.61, no.5, pp.1648,1659, May 2013

[8] Lee, X.-R. et al., "A 7.92 Gb/s 437.2 mW Stochastic LDPC Decoder Chip for IEEE 802.15.3c Applications," IEEE Trans. On *Circuits and Systems I: Regular Papers,*, vol.PP, no.99, pp.1,10, 2014.

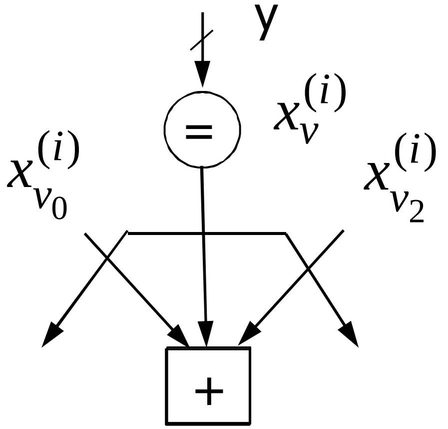
Lab-STICC cnrs

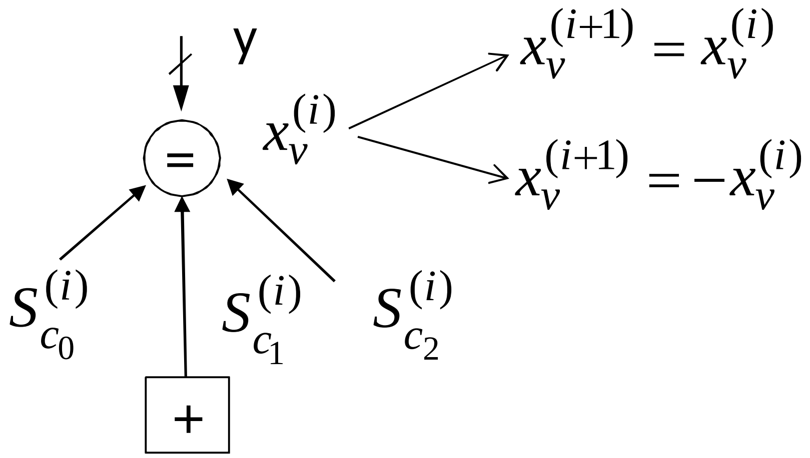
Bit flipping algorithm (BFA)

	Memoryless		State variable	
Extrinsic				STOCH.
Broadcast			BPA	
	Non-D	Deterministic		Non-D

Principle of Bit–Flipping algorithm

- First step: initialize local decision $x^{(0)}$ with sign(y).
- While there are still syndromes equal to -1, or while not reaching the maximum number of iteration it_{max} do
  - 1) Variable nodes broadcast their values to the check nodes
  - 2) Check node broadcast their syndrome values. Then, variable nodes decide to flip (or not) the local decision.
- Output the estimated value x.





- BFA differs by the decision rules

Lab-STICC CNRS

Gradient Bit Flipping Algorithm

- Maximum Likelihood decoding $$X = \arg\max\left\{\sum_{j=1}^{N} x_j \cdot y_j, x \in C\right\}$$
- If X belongs to the code C, then, all syndromes are +1, thus:

$$X = \arg\max\left\{\sum_{j=1}^{N} x_j \cdot y_j + \sum_{i=1}^{M} S_i, x \in C\right\}$$

- Decoding method: when x is not a codeword, try to flip bits of x to maximize objective function O.

$$O = \sum_{j=1}^{N} x_j \cdot y_j + \sum_{i=1}^{M} S_i$$

Gradient Bit Flipping Algorithm

- The dependence of the objective function on variable x_v is:

$$\frac{dO}{dx_v} = x_v \cdot y_v + \sum_{c' \in C(v)} S_{c'}$$

- If $E_v = \frac{dO}{dx_v} < 0$ then, it worth flipping bit x_j to increase O.

$$E_v^{(i)} = x_v^{(i)} y + \sum_{c' \in C(v)} S_{c'}^{(i)} < 0$$

$$\downarrow \quad x_v^{(i+1)} = -x_v^{(i)}$$

$$E_v^{(i+1)} = x_v^{(i+1)} y + \sum_{c' \in C(v)} S_{c'}^{(i+1)} > 0$$

$$S_v^{(i)} = \prod_{v' \in V(c)} x_{v'}^{(i)}$$

$$\downarrow \quad x_v^{(i+1)} = -x_v^{(i)}$$

$$S_v^{(i+1)} = \prod_{v' \in V(c)} x_{v'}^{(i+1)} = -S_v^{(i)}$$

Gradient Bit Flipping Algorithm

- Several update rules
  - Single bit flipping: at each iteration, flip the bit with the smallest score E_v.
  - Multiple bit flipping: at each iteration, flip the bit if
    Ev < threshold.

- Possibility to adapt the threshold with the iterations

- In terms of implementation, GBDA gives very low complexity hardware.
  - Possibility of parallel implementation with tenth of Gbit/s throughput.
- Drawback: GBDA is easily stuck in "trapping sets", giving medium performance.

References on BFA

[1] Miladinovic, N.; Fossorier, M.P.C., "Improved bit flipping decoding of low-density parity check codes," ISIT *2002*, vol., no., pp.229, 2002

[2] Miladinovic, N.; Fossorier, M.P.C., "Improved bit-flipping decoding of low-density parity-check codes," *Information Theory, IEEE Transactions on* , vol.51, no.4, pp.1594,1606, April 2005

[3] Xiaofu Wu; Chunming Zhao; Xiaohu You, "Parallel Weighted Bit-Flipping Decoding," *Communications Letters, IEEE* , vol.11, no.8, pp.671,673, August 2007

[4] Xiaofu Wu; Cong Ling; Ming Jiang; Enyang Xu; Chunming Zhao; Xiaohu You, "New insights into weighted bit-flipping decoding," *IEEE Trans. On Communication*, vol.57, no.8, pp.2177,2180, Aug. 2009

[5] Junho Cho; Wonyong Sung, "Adaptive Threshold Technique for Bit-Flipping Decoding of Low-Density Parity-Check Codes," *Communications Letters, IEEE* , vol.14, no.9, pp.857,859, September 2010

[6] Catala-Perez, Joan-Marc et al., "Reliability-based iterative decoding algorithm for LDPC codes with low variable-node degree," *Communications Letters, IEEE* , vol.PP, no.99, pp.1,1, 2014

Differential Decoding with Binary Message Passing

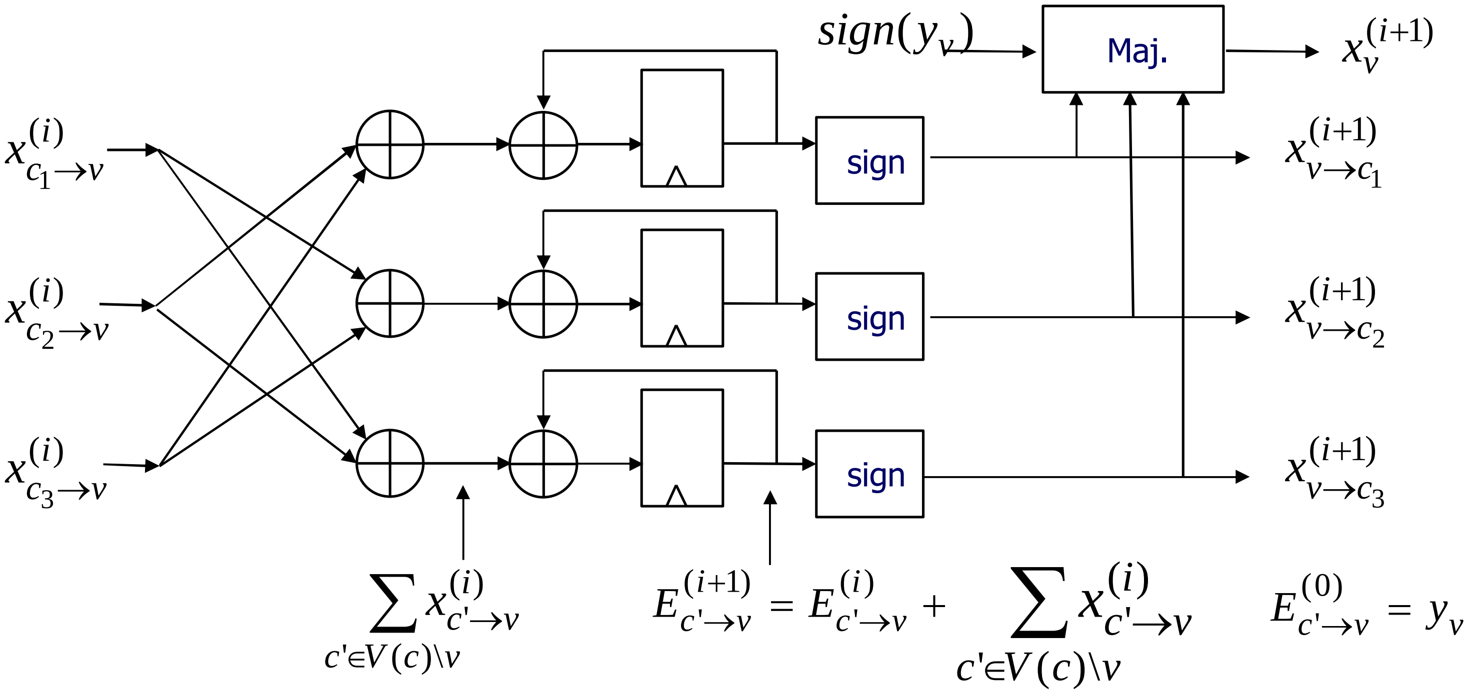
	Memoryless		State variable	
Extrinsic			DD-BMP	STOCH.
Broadcast			BPA	
	Non-D	Deterministic		Non-D

[1] Janulewicz, E.; Banihashemi, A.H., "Performance Analysis of Iterative Decoding Algorithms with Memory over Memoryless Channels," IEEE. Trans. On *Com.*, vol.60, no.12, pp.3556,3566, December 2012

[2] Cushon, K.; Hemati, S.; Leroux, C.; Mannor, S.; Gross, W.J., "High-Throughput Energy-Efficient LDPC Decoders Using Differential Binary Message Passing," *Signal Processing, IEEE Transactions on* , vol.62, no.3, pp.619,631, Feb.1, 2014

Differential Decoding with Binary Message Passing

- Use extrinsic principle ; track the long term tendency.



[1] Mobini, N.; Banihashemi, A.H.; Hemati, S., "A Differential Binary Message-Passing LDPC Decoder," *Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE* , vol., no., pp.1561,1565, 26-30 Nov. 2007

[2] Cushon, K.; Hemati, S.; Leroux, C.; Mannor, S.; Gross, W.J., "High-Throughput Energy-Efficient LDPC Decoders Using Differential Binary Message Passing," *Signal Processing, IEEE Transactions on* , vol.62, no.3, pp.619,631, Feb.1, 2014.

Outline

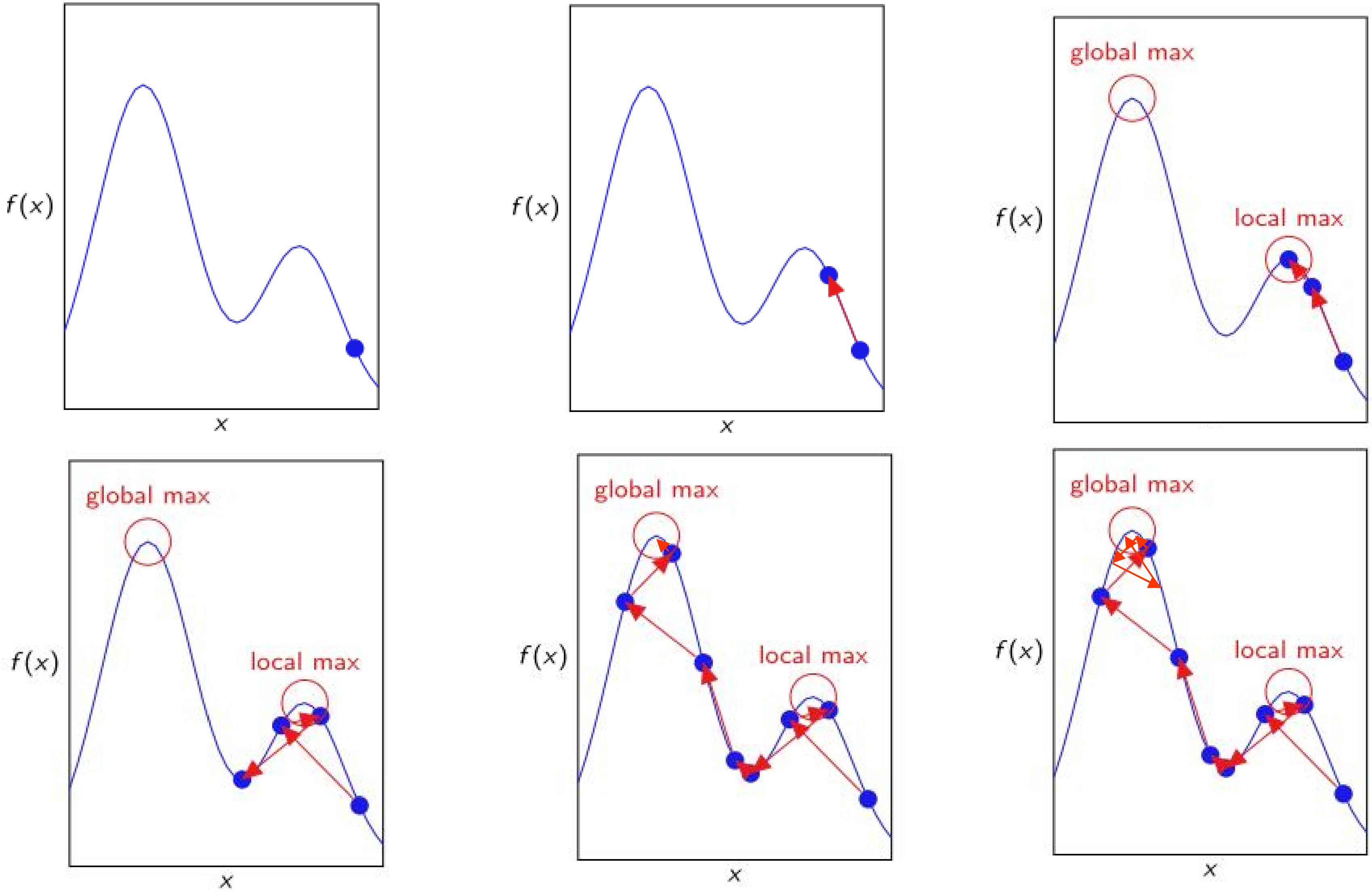
- Classifications of BM LDPC decoder
- State of the art
- Recent results
- Conclusions

Noisy Gradient Decent Bit Flipping Decoder

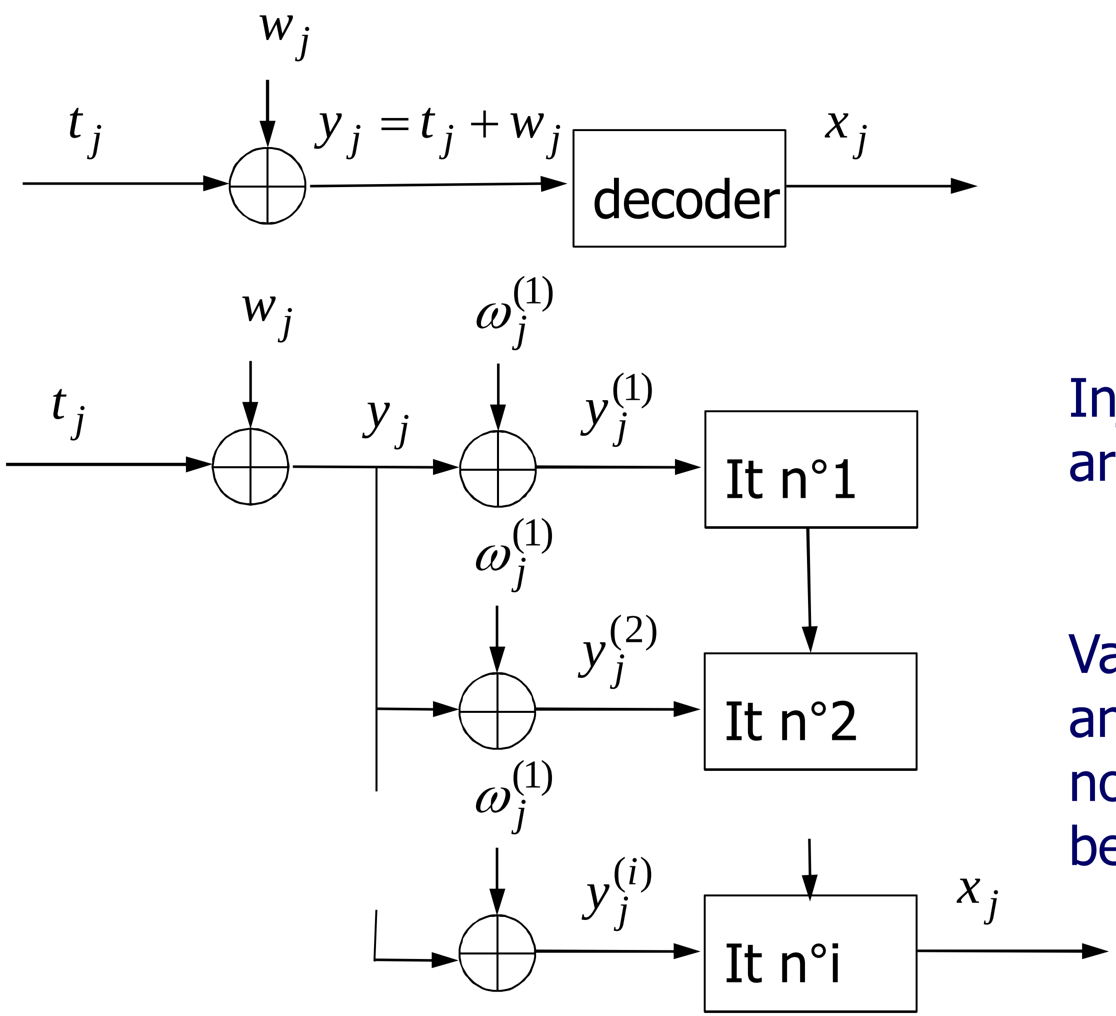
	Memoryless		State variable	
Extrinsic			DD-BMP	STOCH.
Broadcast			BFA →	NGDBFD
	Non-D	Deterministic		Non-D

[1] Sundararajan, G.; Winstead, C.; Boutillon, E., "Noisy Gradient Descent Bit-Flip Decoding for LDPC Codes," *Communications, IEEE Transactions on* , vol.62, no.10, pp.3385,3400, Oct. 2014

GBFA with noise to escape local minima.



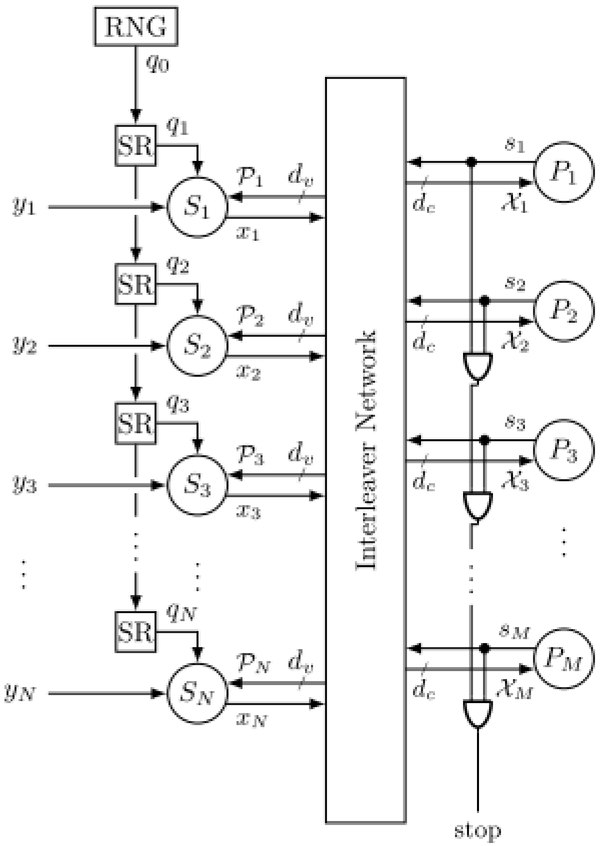
Optimal injection of noise

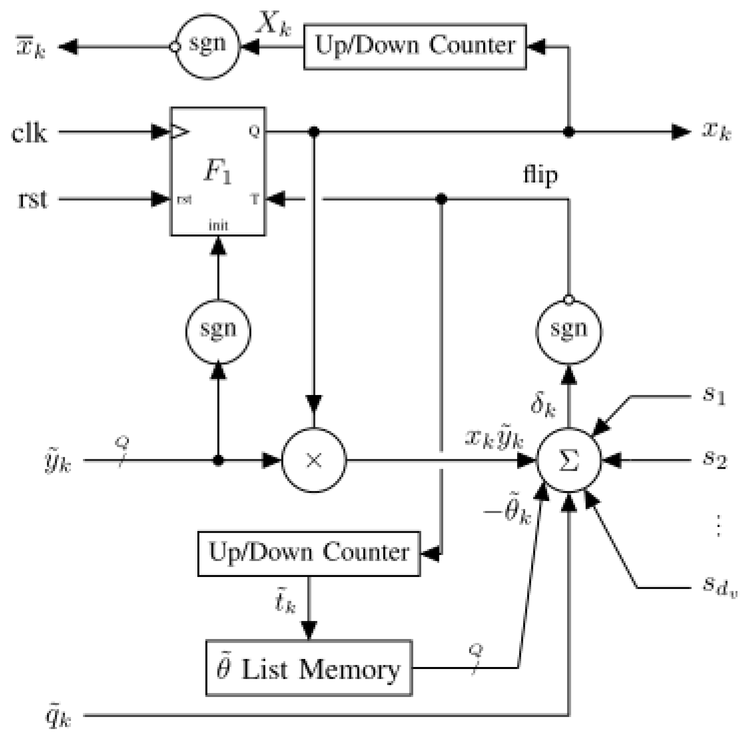


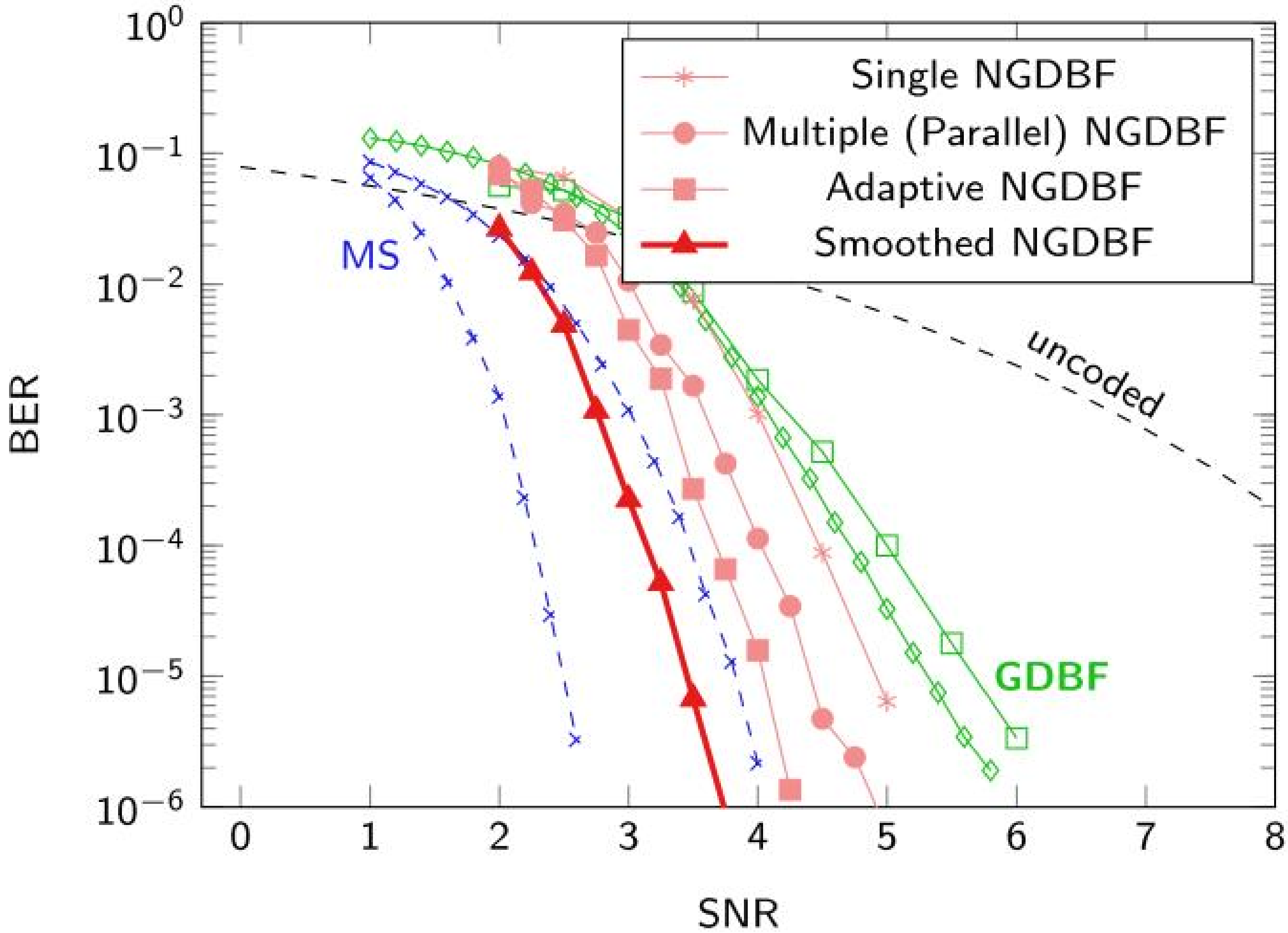
Injected noise samples are independant

Variance of channel noise and variance of «injected» noise are equal to obtain best performance.

Very efficient hardware implementation

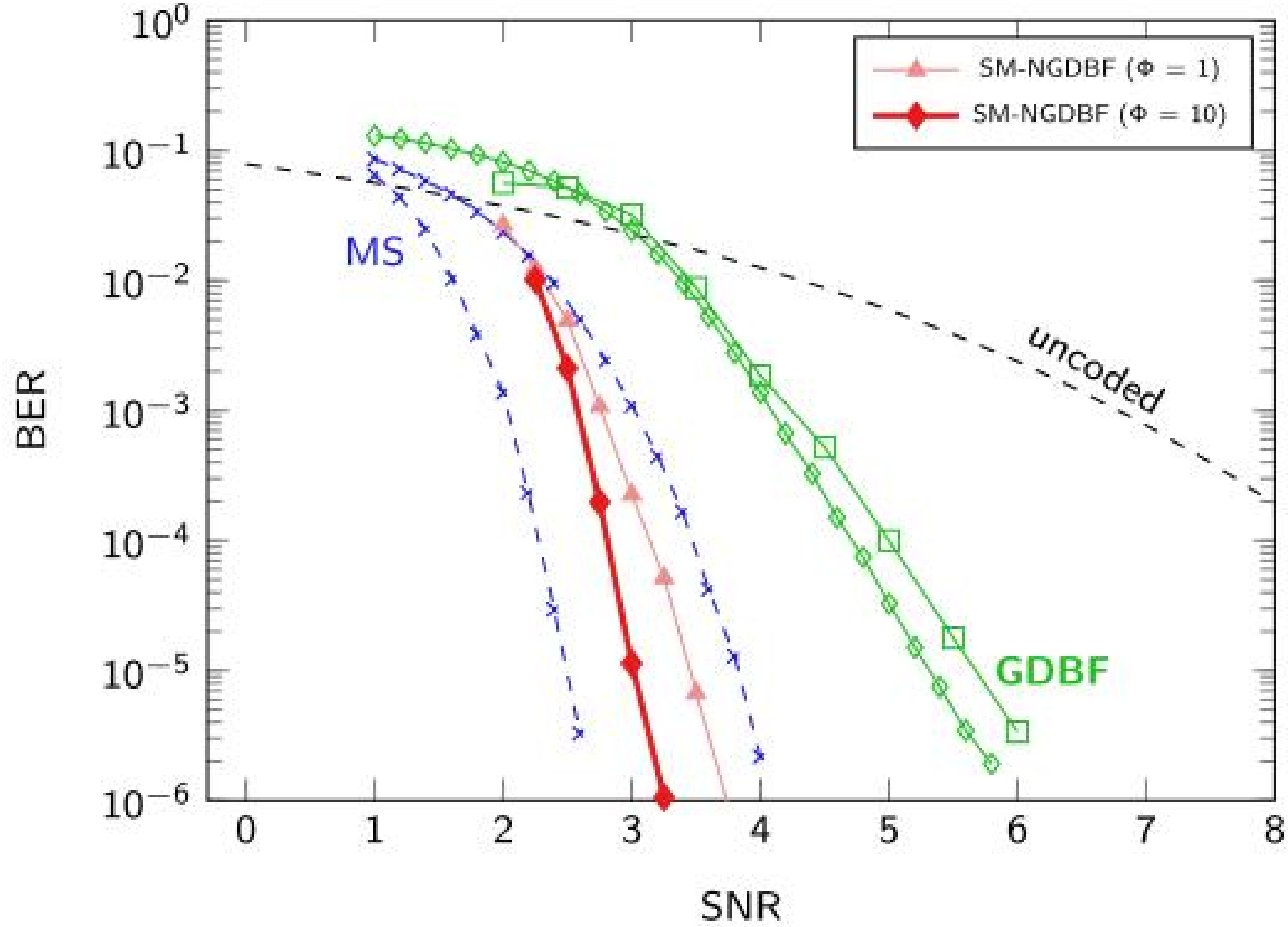





Single NGDBF
Multiple (Parallel) NGDBF
Adaptive NGDBF
Smoothed NGDBF
MS
uncoded
GDBF
BER
SNR
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
1
2
3
4
5
6
7
8

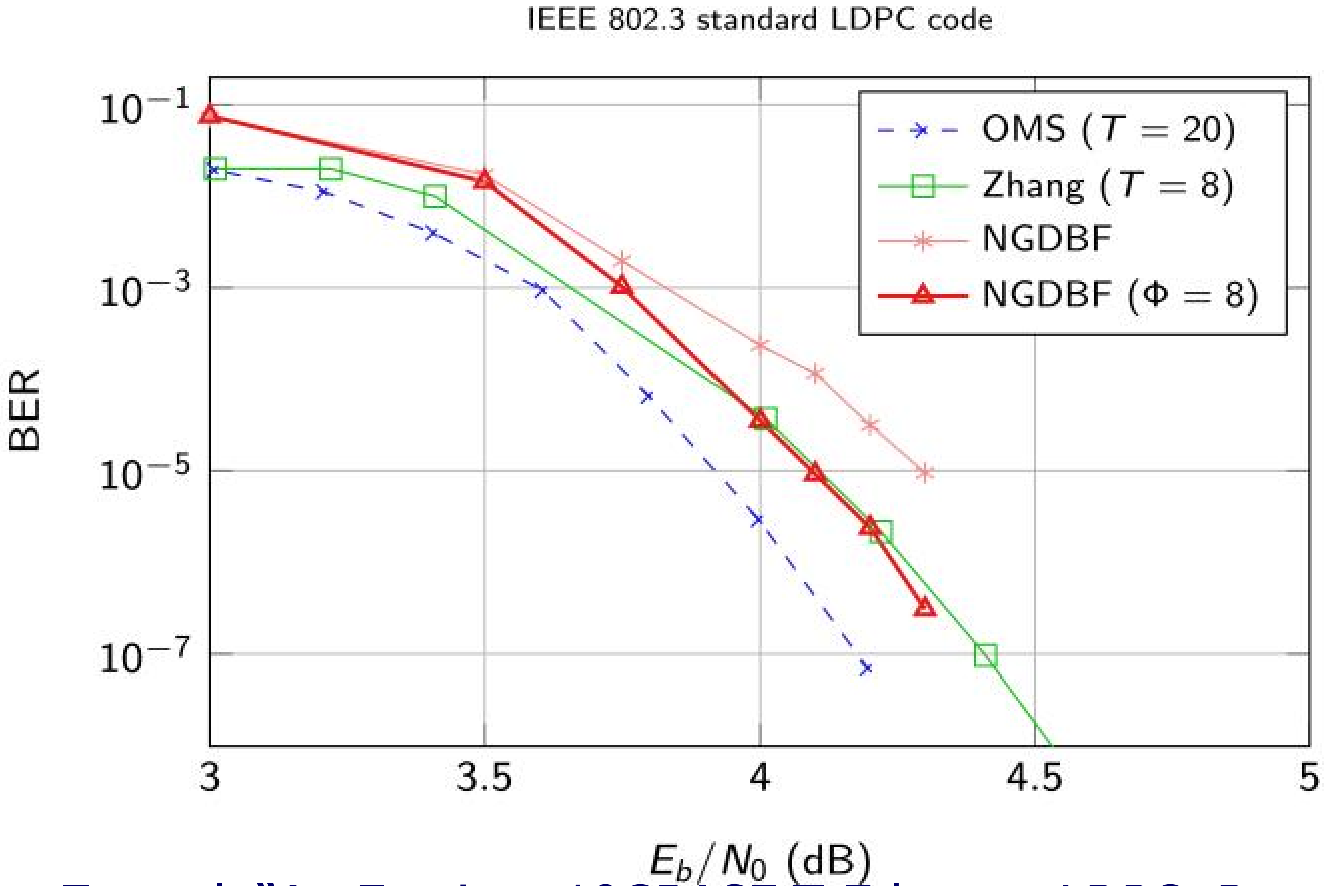
Repeated decoding

NGDBF is non-deterministic: improve performance by repeatedly decoding failed frames until success (up to a maximum of Φ phases).



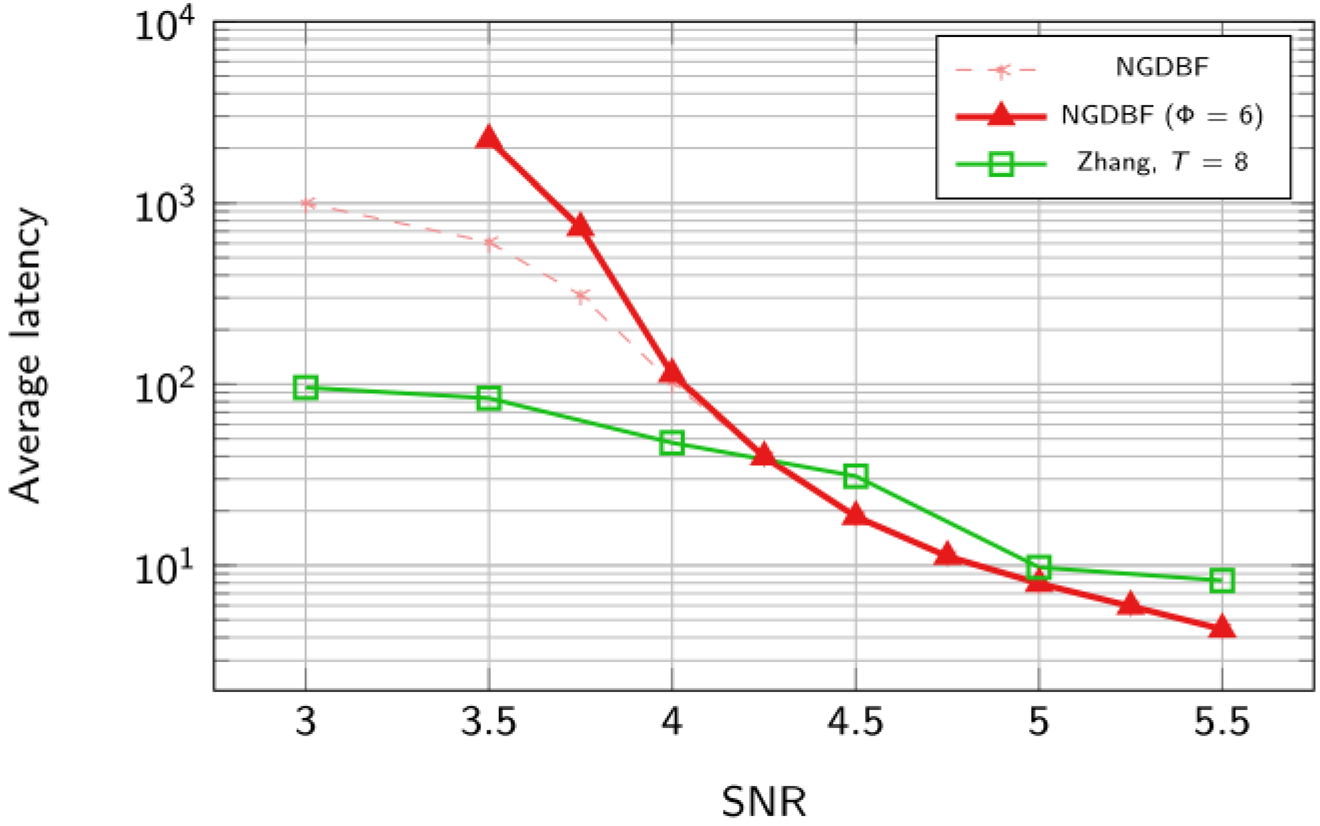
TICC

Application to 10GBaseT standard



[1] Zhang, Z. et al. "An Efficient 10GBASE-T Ethernet LDPC Decoder Design With Low Error Floors". In: IEEE J. Solid-State Circ. 45, pp. 843–855.

Comparison in average clock cycles



For low BER, NGDBF becomes more competitive than OMS

Locally Maximum-Likelihood Binary Message

	Memoryless		State variable	
Extrinsic		LMLBM	DD-BMP	STOCH.
Broadcast			BFA	NGDBFD
	Non-D	Deterministic		Non-D

[1] Winstead, C.; Boutillon, E., "Decoding LDPC Codes with Locally Maximum-Likelihood Binary Messages," *Communications Letters, IEEE* , vol.PP, no.99, pp.1,1

Principle of Density Evolution of LMLBM

Knowing the quantization rule and the variance of the noise, we can compute the probabilities $P(y_v / x_v = +1)$ and $P(y_v / x_v = -1)$

Knowing the input error probability $P_{v \to c}^{(0)}$ of first message and the check node degree, we can compute $P_{c \to v}^{(0)}$

From $P_{c \to v}^{(0)}$ we can compute $P(S_{c \to v}^{(0)} = a / x_v = +1)$ and $P(S_{c \to v}^{(0)} = a / x_v = -1)$

With $S_{c \to v}^{(0)} = \sum_{v' \in V(c) \backslash c} x_{c' \to v}$ and $a \in \{-d_c, d_c - 2, \ldots, d_c\}$

Thus, knowing the observed a and y values, the Local Maximum Likelihood decision is:

$$x_{v \to c}^{(1)} = \arg\max\{P(a / x) + P(y / x), x \in \{-1, +1\}\}$$

We can deduce $P_{v \to c}^{(1)}$ … and iterate the process.

Density evolution result

TABLE I
DENSITY EVOLUTION THRESHOLDS FOR REGULAR ENSEMBLES.

d_v	d_c	R	$\mathbf{SNR}^*_{\mathrm{LMLBM}}$	$\mathrm{SNR}^*_{\mathrm{BP}}$	$\Delta\mathrm{SNR}$
4	62	0.9356	**4.79**	4.25	0.54
5	10	0.5	**3.15**	2.05	1.1
4	8	0.5	**2.87**	1.62	1.25
4	6	0.333	**3.29**	1.67	1.6
3	6	0.5	**2.94**	1.11	1.83
3	5	0.4	**3.04**	0.97	2.07
3	4	0.25	**3.74**	1	2.74

Simulation results

N=4000, (4,8), it_{max} =30
(+10 Gallager A iterations),

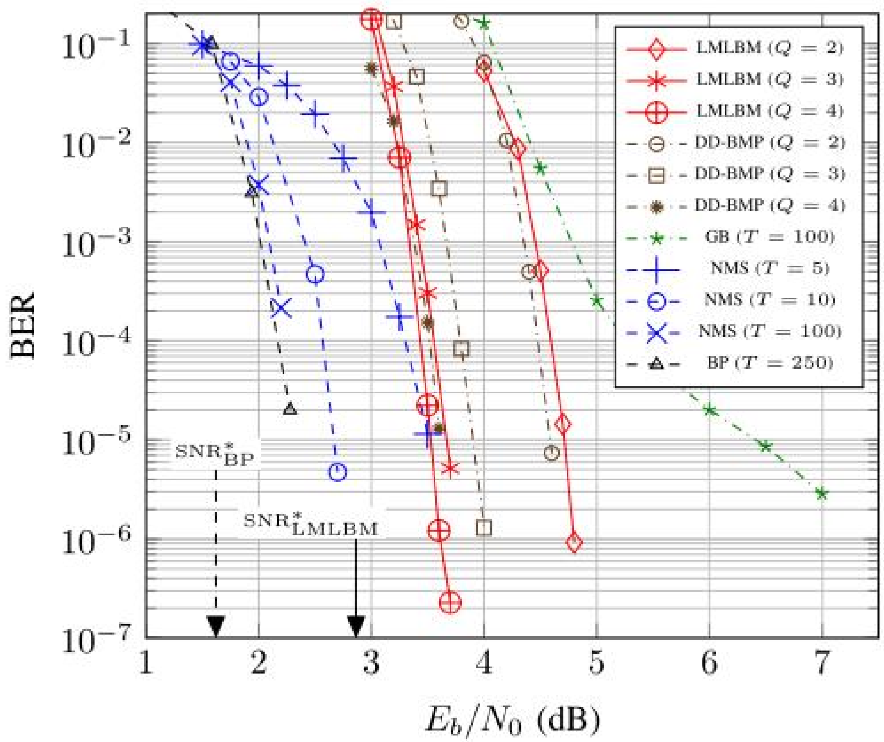


Fig. 2. Simulation results for a (4, 8) rate-1/2 regular code of length $n = 4000$ bits, identified as code 4000.2000.4.244 in MacKay's encyclopedia. For $Q = 2$, 3 and 4, LMLBM was simulated with $T = 30$, $\gamma = 0.55$, $K = 3$ and $Y_{\max} = 1.1$, using a Gallager-A stage with a maximum of 10 iterations.

N=4376, (4,62), it_{max} =50

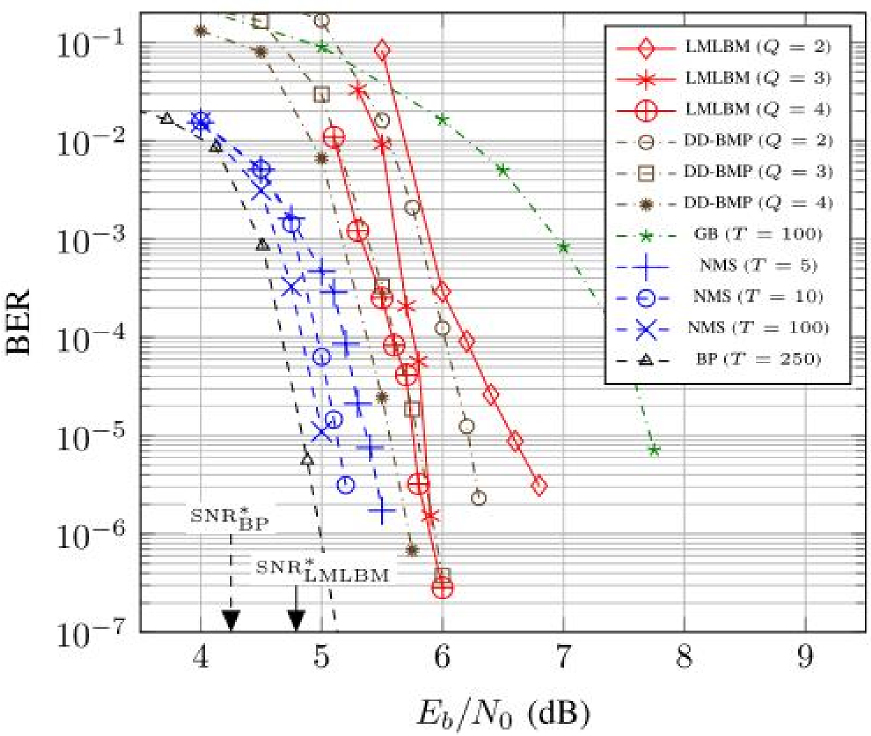


Fig. 3. Simulation results for the (4, 62) rate-0.9356 regular code of length $n = 4376$ bits, identified as code 4376.282.4.9598 in MacKay's encyclopedia. For $Q = 4$, LMLBM was simulated with $T = 100$, $Y_{\max} = 1.1$, $\gamma = 0.05$ and $K = 9$. For $Q = 3$, $T = 50$, $\gamma = 0.11$, $K = 6$ and $Y_{\max} = 0.8$. For $Q = 2$, $T = 50$, $\gamma = 0.01$, $K = 6$ and $Y_{\max} = 0.9$. No Gallager-A stage.

Natural future extension

Add some « noise » in LMLBM, generalizing the idea of [1]

	Memoryless		State variable	
Extrinsic	NLMLBM ←	LMLBM	DD-BMP	STOCH.
Broadcast			BFA	NGDBFD
	Non-D	Deterministic		Non-D

[1] Miladinovic N., Fossorier M., "Improved Bit-Flipping Decoding of Low-Density Parity-Check Codes", IEEE Trans. On Inf. Theory, vol. 51, n. 4, APRIL 2005

Outline

- Classifications of BM LDPC decoder
- State of the art
- Recent results
- Conclusions

Conclusion

Far from exhaustive survey of the state of the art…

There is still « terra incognita » in the table, possible mixt of algorithm

	Memoryless		State variable	
Extrinsic	?	LMLBM	DD-BMP	STOCH.
Broadcast	?	?	BFA	NGDBFD
	Non-D	Deterministic		Non-D

Already some hardware for several tenth of Giga bit decoders.

Questions ?