Sign-Preserving algorithm or "forget the zero"

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Outline

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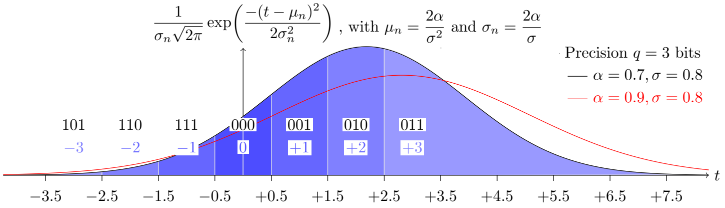
Classical OMS decoder: quantification of channel LLR

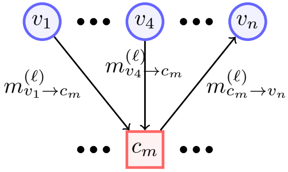
Quantified intrinsic information I_n is obtained from channel LLR y_n as $I_n = \mathcal{Q}(\alpha \times y_n)$, with $y_n \in \mathbb{R}$, α a scaling factor and $\mathcal{Q}$ a quantification rules from $\mathbb{R}$ to $\mathcal{A}_C = \{-3, -2, -1, 0, +1, +2, +3\}$ defined as $\mathcal{Q}(a) = \mathcal{S}_3(\lfloor a + 0.5 \rfloor)$, with $\mathcal{S}_3$ the saturation function toward [-3, 3].



Among the 8 levels of a 3 bits representation, only 7 are used.

- **Update rule at a CNU**

$$m^{(\ell)}_{c_m \to v_n} = \left(\prod_{v \in \mathcal{V}(c_m) \setminus \{v_n\}} \text{sign}\left(m^{(\ell)}_{v \to c_m} \right) \right) \times \max\left\{ \min_{v \in \mathcal{V}(c_m) \setminus \{v_n\}} \left| m^{(\ell)}_{v \to c_m} \right| - 1, 0 \right\}.$$

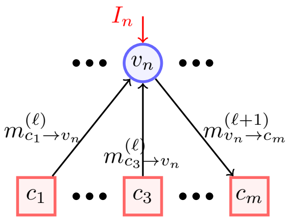


- Note that for low precision $q = 3$ (same for $q = 4$), the offset applied in CNUs only gives us the possibility to use 5 values ($m^{(\ell)}_{c_m \to v_n} \in \{-2, -1, 0, +1, +2\}$) instead of the 7 values of $\mathcal{A}_C$.

- **Update rule at a VNU**

$$m^{(\ell+1),U}_{v_n \to c_m} = I_n + \sum_{c \in \mathcal{V}(v_n) \setminus \{c_m\}} m^{(\ell)}_{c \to v_n}.$$

exponent U indicates unsaturated message.



$$m^{(\ell+1)}_{v_n \to c_m} = \mathcal{S}_3 \left(m^{(\ell+1),U}_{v_n \to c_m} \right).$$

$$m^{(\ell+1)}_{v_n \to c_m} \in \{-3, -2, -1, 0, +1, +2, +3\}.$$

Low precision Offset Min-Sum Based Decoders

From the analysis of OMS-based decoders with $q = 3$

(i) the quantized LLRs $I_n \in \mathcal{A}_C = \{-3, -2, -1, 0, +1, +2, +3\}$,
(ii) v-to-c messages $m^{(\ell+1)}_{v_n \to c_m} \in \mathcal{A}_C = \{-3, -2, -1, 0, +1, +2, +3\}$, and
(iii) c-to-v messages $m^{(\ell)}_{c_m \to v_n} \in \mathcal{A}_C \backslash \{-3, +3\} = \{-2, -1, 0, +1, +2\}$.

The OMS-based decoders are suboptimal

It can be clearly noted that all combinations that can be obtained from $q = 3$ bits is not used.

How to use the 8 quantization levels for precision $q = 3$

We define a new decoder called Sign-Preserving decoder where:

- The quantization of LLRs has to be changed.
- CNU has to be changed.
- VNU has to be changed.

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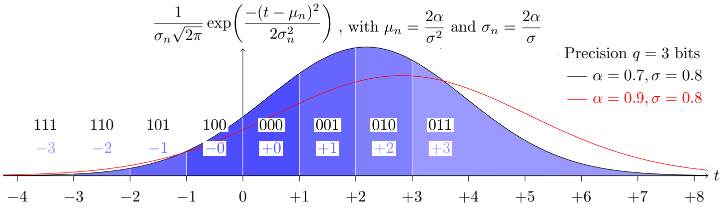
Quantization used for Sign-Preserving Min-Sum Decoders

- In order to use the 8 levels of $q = 3$ bits, we define the message alphabet as $\mathcal{A}_S = \{-3, -2, -1, -0, +0, +1, +2, +3\}$. Using the sign-and-magnitude representation we have $100_2 = -0$, $000_2 = +0$, etc.
- A new quantification process $\mathcal{Q}^*$ is used: $I_n = \mathcal{Q}^* (\alpha \times y_n) \in \mathcal{A}_S$ where $\mathcal{Q}^* (a) = (\text{sign}(a), \mathcal{S}_3 (\lceil \alpha \times |a| \rceil - 1))$

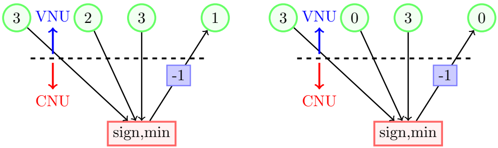


The *channel gain factor* α represents a degree of freedom in the decoder definition.

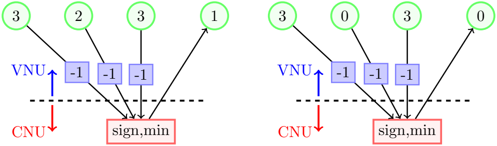
CNU of Sign-Preserving Min-Sum Decoders

The update rule at a CNU can be written in two equivalent ways.

Classical method: $\left|m^{(\ell)}_{c_m \to v_n}\right| = \max\left\{ \min_{v \in \mathcal{V}(c_m)\backslash\{v_n\}} \left|m^{(\ell)}_{v \to c_m}\right| - 1, 0 \right\}.$



Equivalent method: $\left|m^{(\ell)}_{c_m \to v_n}\right| = \min_{v \in \mathcal{V}(c_m)\backslash\{v_n\}} \left(\max\left(\left|m^{(\ell)}_{v \to c_m}\right| - 1, 0 \right) \right).$



Therefore, we can move the offset from CNs to VNs.

VNU of Sign-Preserving Min-Sum Decoders

Moving the offset from CNs to VNs, the update rule at a VNU of OMS-based decoders can be rewritten as

$$m^{(\ell+1),U}_{v_n \to c_m} = I_n + \sum_{c \in \mathcal{V}(v_n) \setminus \{c_m\}} m^{(\ell)}_{c \to v_n}.$$

The offset is subtracted before saturation to allow -3 and 3 values in the variable to check message.

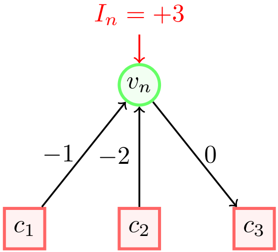
$$m^{(\ell+1)}_{v_n \to c_m} = \text{sign}\left(m^{(\ell+1),U}_{v_n \to c_m}\right) \times \mathcal{S}_3\left(\max\left(\left|m^{(\ell+1),U}_{v_n \to c_m}\right| - 1, 0\right)\right)$$

Problem of the update rule at VNUs

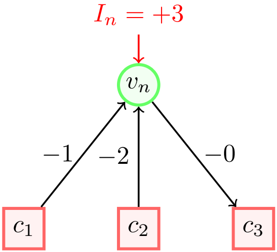
The v-to-c message $m^{(\ell+1)}_{v_n \to c_m}$ can be zero, zero does not have any information about the bit value, this means that the VNU can erase the bit value.

VNU of Sign-Preserving Min-Sum Decoders

Classical VNU:



Modified VNU:



Idea to avoid the zero value.

- **Modified VNU:** we propose to use half the sum of the signs of the c-to-v messages and the sign of the quantized LLR
  Example: $a = (+\mathbf{3}-\mathbf{1}-\mathbf{2}) + (+\mathbf{1}-\mathbf{1}-\mathbf{1})/2 = -\mathbf{0.5}$, hence $m^{(\ell),U}_{v_n \to c_3} = (\text{sign}(a), \lfloor |a| \rfloor) = -\mathbf{0}$ and $m^{(\ell)}_{v_n \to c_3} = -\mathbf{0}$.

We define the Sign-Preserving Min-Sum decoders as decoders that always preserve the sign of the messages [1].

Update rules for SP-MS Decoders

Let us denote by $\mu_{v_n \to c_m}^{(\ell)}$ the *sign-preserving factor* of the message $m_{v_n \to c_m}^{(\ell+1)}$, defined as

$$\mu_{v_n \to c_m}^{(\ell)} = \xi \times \text{sign}(I_n) + \sum_{c \in \mathcal{V}(v_n) \backslash \{c_m\}} \text{sign}\left(m_{c \to v_n}^{(\ell)}\right),$$

where $\xi = \begin{cases} 0, & d_v = 2, \\ 1, & d_v \in \{3, 5, 7, ...\}, \\ 2, & d_v \in \{4, 6, 8, ...\}. \end{cases}$

By construction $\mu_{v_n \to c_m}^{(\ell)}$ take its value between $\{-1, 1\}$ for $d_v = 2$, $\{-d_v, -d_v + 2, \ldots, -1, 1, \ldots, d_v\}$ for d_v odd and $\{-d_v - 1, -d_v + 1, \ldots, -1, 1, \ldots, d_v + 1\}$ for d_v even.

Update rules for SP-MS Decoders

- Let us redefined $m^{(\ell+1),U}_{v_n \to c_m}$ as

$$m^{(\ell+1),U}_{v_n \to c_m} = \frac{\mu^{(\ell)}_{v_n \to c_m}}{2} + I_n + \sum_{c \in \mathcal{V}(v_n) \setminus \{c_m\}} m^{(\ell)}_{c \to v_n}$$

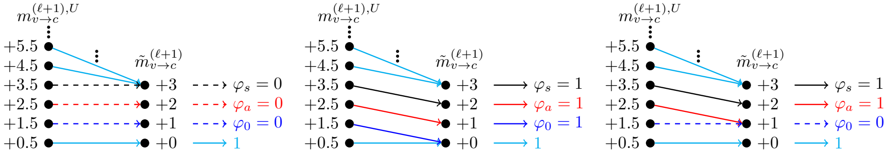
- $m^{(\ell+1),U}_{v_n \to c_m}$ takes its value in the set $\mathcal{A}_U = \{\dots, -1.5, -0.5, +0.5, +1.5, \dots\}$.
- and we obtain a message $m^{(\ell+1)}_{v_n \to c_m}$ that has always a defined sign as

$$m^{(\ell+1)}_{v_n \to c_m} = \left(\text{sign}\left(m^{(\ell+1),U}_{v_n \to c_m}\right), \mathcal{S}_3\left(\max\left(\left\lfloor \left| m^{(\ell+1),U}_{v_n \to c_m} \right| \right\rfloor - 1, 0\right)\right)\right).$$

Outline

Optimization of Sign-Preserving Min-Sum Decoders

Thanks to density evolution, we can assess the optimal saturation rules at variable node level. For example, for $q_{ch} = 3$ (channel quantization), $q = 3$ (message quantization), $(d_v, d_c) = (4, 8)$ we obtained:



(a) SP-MS. (b) SP-OMS. (c) Optimized SP-MS.

MS Decoder	OMS decoders	Optimized SP-MS	DE gain	SNR gain
$\delta_{db} = 2.736$	$\delta_{db} = 2.322$	$\delta_{db} = 1.982$	**0.3399**	**0.32**

SNR gains in the waterfall correspond to what was predicted with DE.

SP-MS decoders for (5,20)-regular LDPC codes

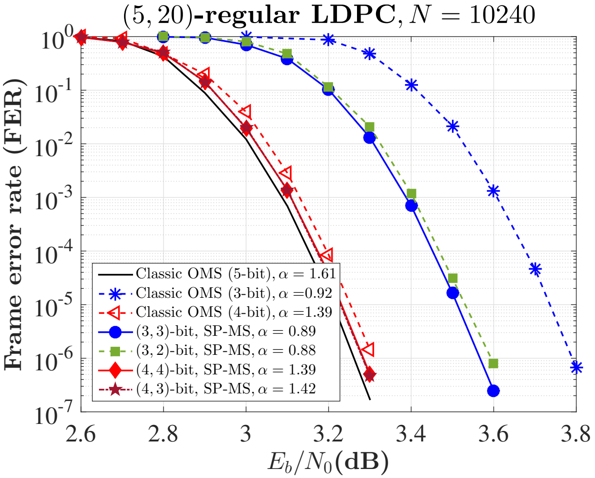


Figure: FER performance of SP-MS decoders for precision $q \in \{2, 3, 4\}$.

SP-MS decoders for (6,32)-regular LDPC codes

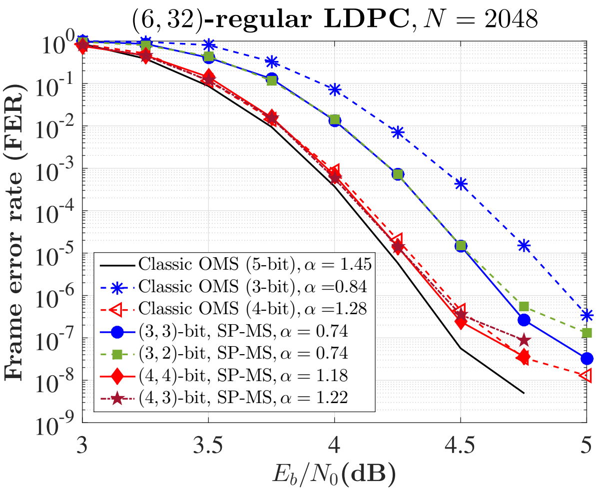


Figure: FER performance of SP-MS decoders for precision $q \in \{2, 3, 4\}$.

SP-MS decoders for (4,64)-regular LDPC codes

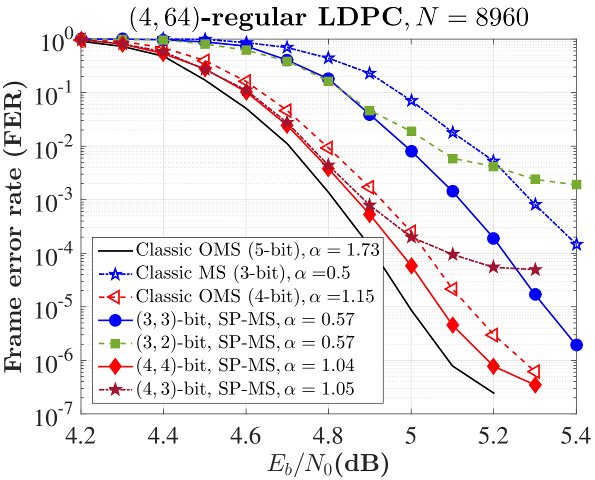


Figure: FER performance of SP-MS decoders for precision $q \in \{2, 3, 4\}$.

SP-MS decoders for (3,18)-regular LDPC codes

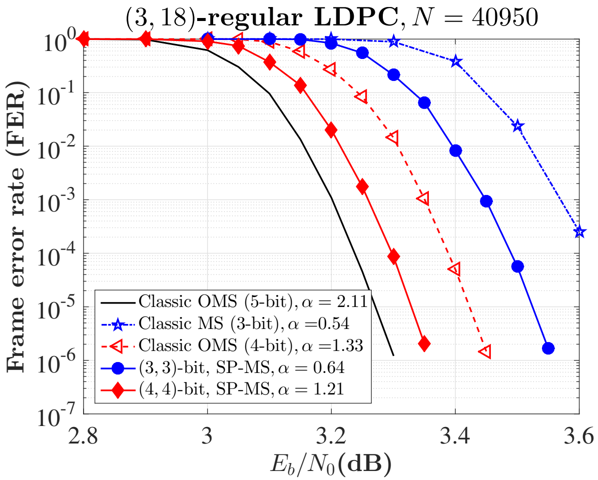


Figure: FER performance of SP-MS decoders for precision $q \in \{3, 4\}$.

Qualitative result obtained by SP-MS algorithm

For $(q_{ch}, q) = (4, 4)$, SP-MS gives small gain compared to MS or OMS

For $(q_{ch}, q) = (4, 3)$, SP-MS is almost equivalent to SP-MS with $(q_{ch}, q) = (4, 4)$!

For $(q_{ch}, q) = (3, 3)$, SP-MS gives 0.2 up to 0.4 dB of gain compared to MS or OMS.

For $(q_{ch}, q) = (3, 2)$, SP-MS is almost equivalent to SP-MS with $(q_{ch}, q) = (3, 3)$!.

Outline

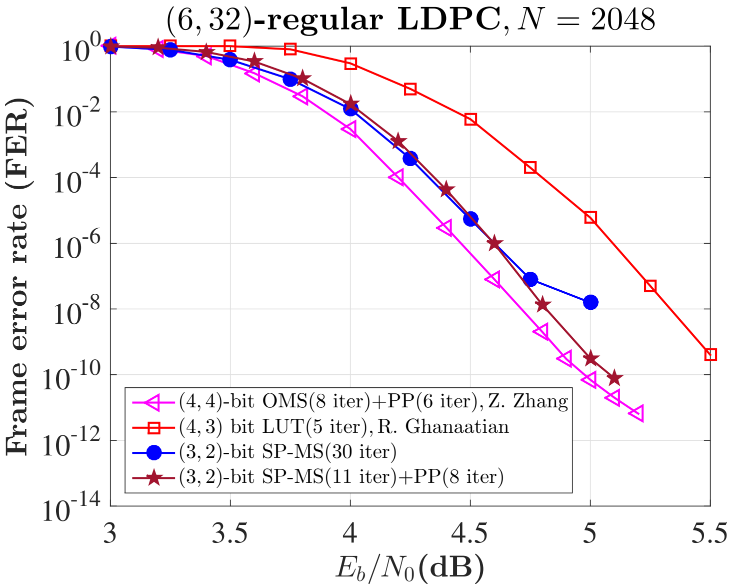
Synthesis results for regular LDPC Codes

IEEE 802.3 LDPC code: Rate 0.8143 $(2048, 1723)$ with regular $(6, 32)$ degree distribution, quantification (3,3).

	This work	Ghanaatian,2018,[2]	Zhang,2010,[3]
Technology	28nm FDSOI	28nm FDSOI	65nm CMOS
Decoder	SP-MS	finite-alphabet	OMS
Quantization	3 bits	3 bits	4 bits
Iterations	9+6	5	8+6
Architecture	full-parallel	unrolled full-parallel	partial-parallel
E_b/N_0 @ FER= 10^{-10}	5.03 dB	5.51 dB	4.98 dB
Frequency	500 MHz†	862 MHz	700 $\xrightarrow{28nm}$ 1000 MHz
Core area	2.56 mm^{2†}	16.2 mm^2	5.05 $\xrightarrow{28nm}$ 1.77 mm^2
Throughput	68.3 Gbit/s†	588 Gbit/s	13.3 $\xrightarrow{28nm}$ 19 Gbit/s
Hardware efficiency	26.7 Gbit/s/mm^{2†}	36.3 Gbit/s/mm^2	2.63 $\xrightarrow{28nm}$ 10.7 Gbit/s/mm^2
Throughput (4.5 dB)	256 Gbit/s/mm^{2†}	588 Gbit/s/mm^2	33.3 $\xrightarrow{28nm}$ 48.2 Gbit/s/mm^2
Hardware efficiency (4.5 dB)	100 Gbit/s/mm^{2†}	36.3 Gbit/s/mm^2	6.59 $\xrightarrow{28nm}$ 26.91 Gbit/s/mm^2

† Preliminary results.

Emulation on FPGA for the IEEE 802.3 ETHERNET code



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Conclusions

We believe that NGDBF and SP-MS can reach 200 Gbit/s/mm2 on 28 nm technology with very good FER.

Could be very interesting to assess the algorithm performance for a convolutional LDPC code used in optical fiber.

Tera bit/s decoding throughput with ultra low FER should be feasible in an ASIC with low energy per bit.

Thank you for listening!

Questions?

References I

[1] F. Cochachin, E. Boutillon, and D. Declercq, "Optimization of sign-preserving noise-aided min-sum decoders with density evolution", in *2018 IEEE 10th International Symposium on Turbo Codes Iterative Information Processing (ISTC)*, Dec. 2018, pp. 1–5. DOI: 10.1109/ISTC.2018.8625309.

[2] R. Ghanaatian, A. Balatsoukas-Stimming, T. C. Müller, M. Meidlinger, G. Matz, A. Teman, and A. Burg, "A 588-gb/s ldpc decoder based on finite-alphabet message passing", *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol. 26, no. 2, pp. 329–340, Feb 2018, ISSN: 1063-8210. DOI: 10.1109/TVLSI.2017.2766925.

[3] Z. Zhang, V. Anantharam, M. J. Wainwright, and B. Nikolic, "An efficient 10gbase-t ethernet ldpc decoder design with low error floors", *IEEE Journal of Solid-State Circuits*, vol. 45, no. 4, pp. 843–855, April 2010, ISSN: 0018-9200. DOI: 10.1109/JSSC.2010.2042255.