Good LDPC decoders with exchanged messages of size 1, 2 and 3 bits.

Workshop on Coding for Future Optical Communications (Munich, 27 February 2018)

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Outline

- Syndrome Bit Flipping:

  BSC channel, no extrinsic, (Bit Flipping Algorithm).

- Noisy Gradient Bit Flipping Decoder:

  AWGN Channel, no extrinsic (Bit Flipping Algorithm).

- Sign-Preserving algorithm.

  AWGN Channel, message passing algorithm

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Syndrome Bit Flipping

For LDPC code IEEE802.3 (size 2048, rate 0.84, $d_v = 6$, $d_c = 32$):

20 errors among 2048 bits generate in average 60 unsatisfied parities among 384.

$$\binom{2048}{20}=10^{48} \quad \text{but} \quad \binom{384}{60}=10^{70}!$$

Syndrome can be viewed as a compressed representation of the error pattern:

It should be possible to consider only the syndrome to suppress the error floor.

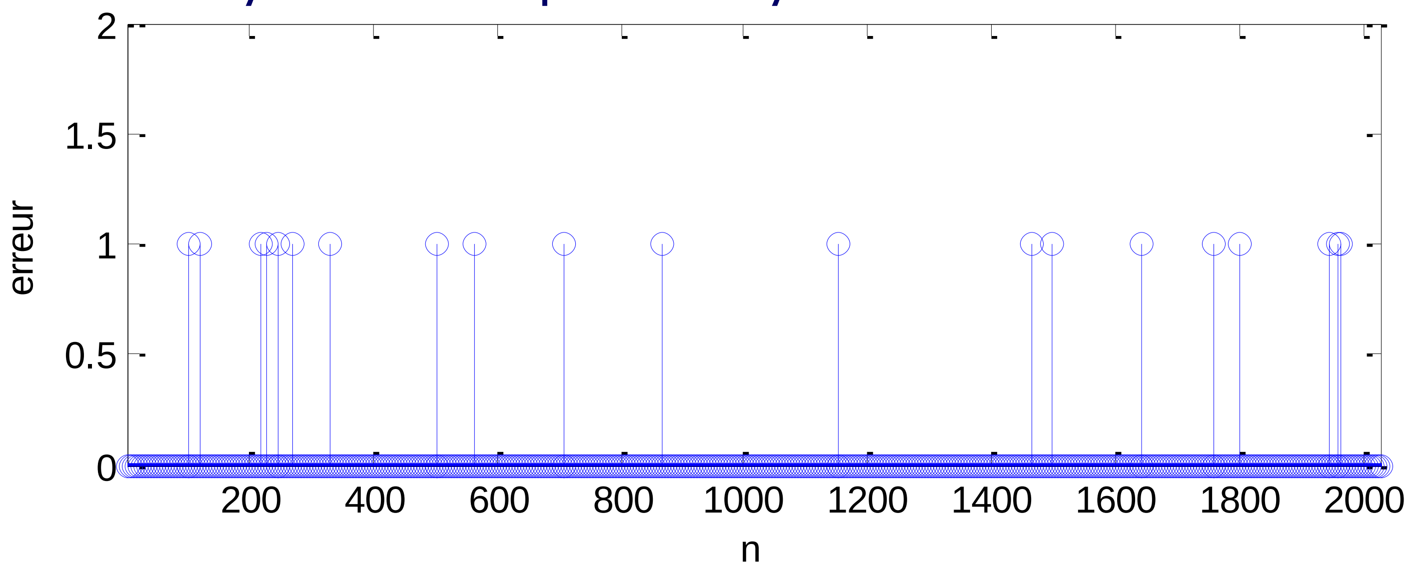
Old idea [1]: used the number of unsatisfied check connected to a variable to flip its state.

[1] Raul Benet, Adriaan De Lind Van Wijngaarden, Ralf Dohmen, Thomas Richardson, Rudiger Urbanke, "Iterative decoding of low-density parity-check (LDPC) codes", Patent EP1643653A1, 5 april 2006.

Example of SBF on a real case

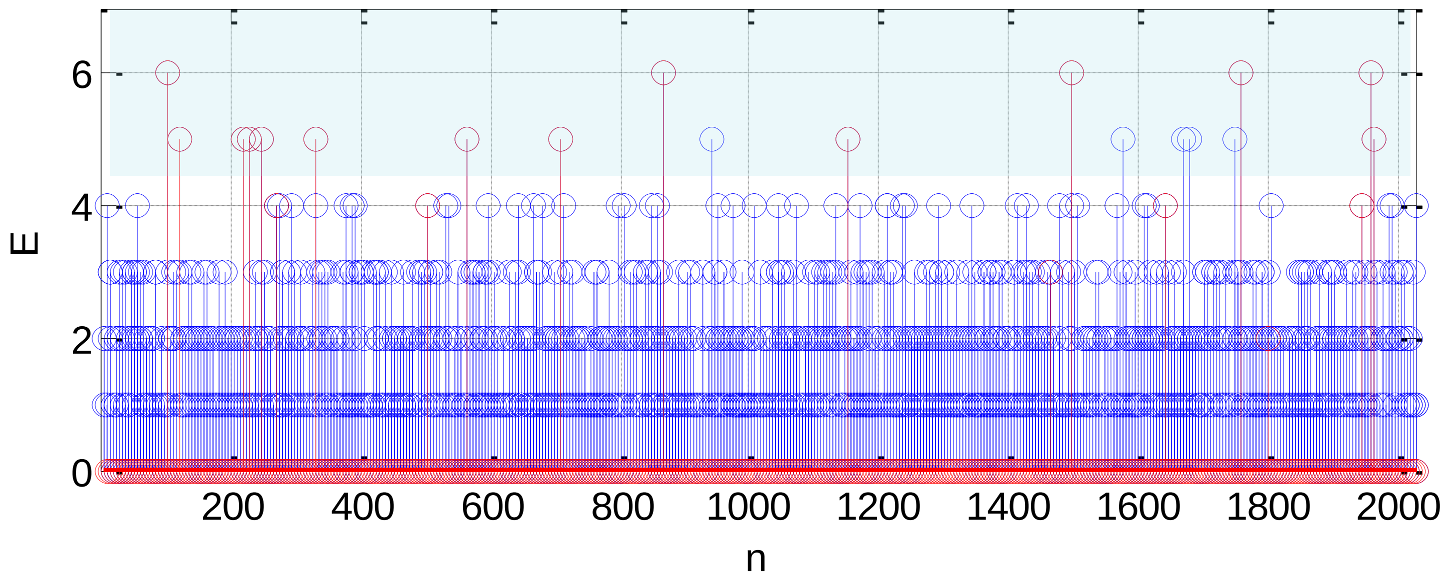
First decoder failed: the final state x doesn't respect H.x = 0.

x = c + err => S = H.x mod 2 = H(c+err) mod 2 = 0 + H.err mod 2
=> syndrom S depends only of the error vector err.



20 errors, 92 checks non fulilled (sum(S) = 92)

Value of E at the first iteration



Flip bits with E > 4 => 19 bits flipped :

14 that was in error

5 that was correct

Only 20 – 14 + 5 = 11 errors after the first iteration.

Sequence θ	=		4	4	4	3	3
nb_err	=	20	11	6	5	3	0
nb_synd	=	92	44	22	18	14	0

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Impact of threshold sequence

Threshold sequence [4 4 4 3 3] =>

nb_err = 20 11 6 5 3 0

nb_synd = 92 44 22 18 14 0

Threshold sequence [4 4 4 4 4 3 3]) =>

nb_err = 20 11 6 5 5 5 3 0

nb_synd = 92 44 22 18 18 18 14 0

Threshold sequence [3 3 3 3] =>

nb_err = 20 54 157 542 941

nb_synd = 92 118 172 206 202

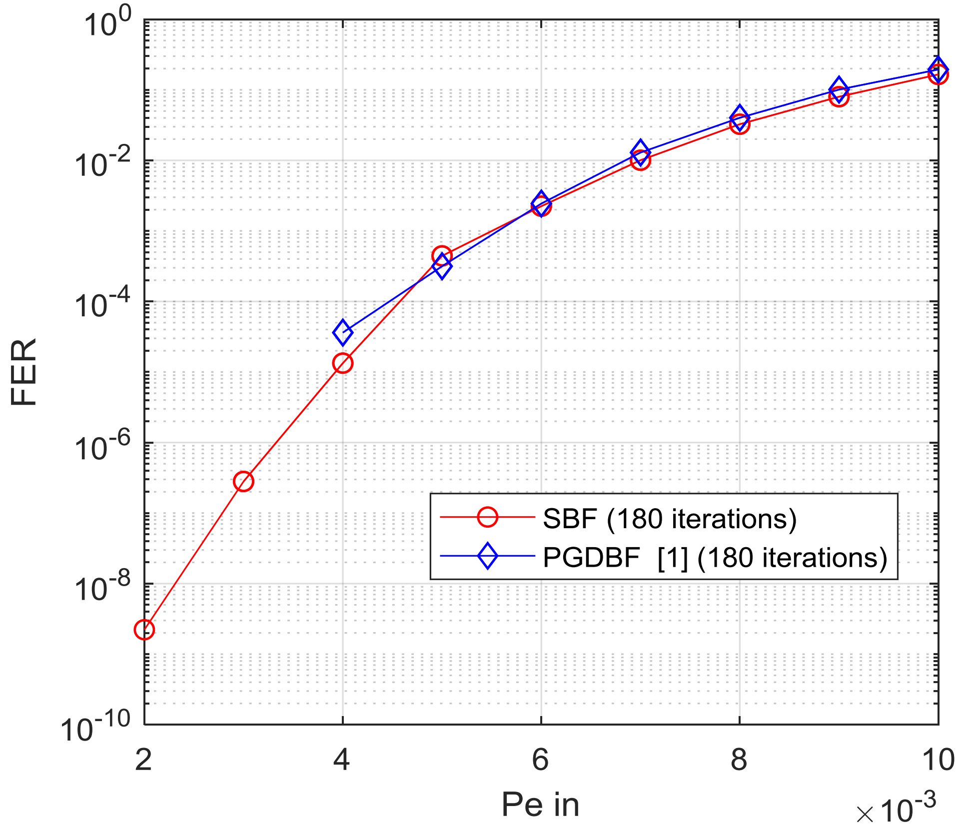
NOTE: there is error patterns solved by [3 3 3 3] but not by [4 4 4 3 3] !

A threshold sequence is « a decoding key ».

Each key can resolve a subset of error patterns.

SBF alone in BSC for IEEE 802.3

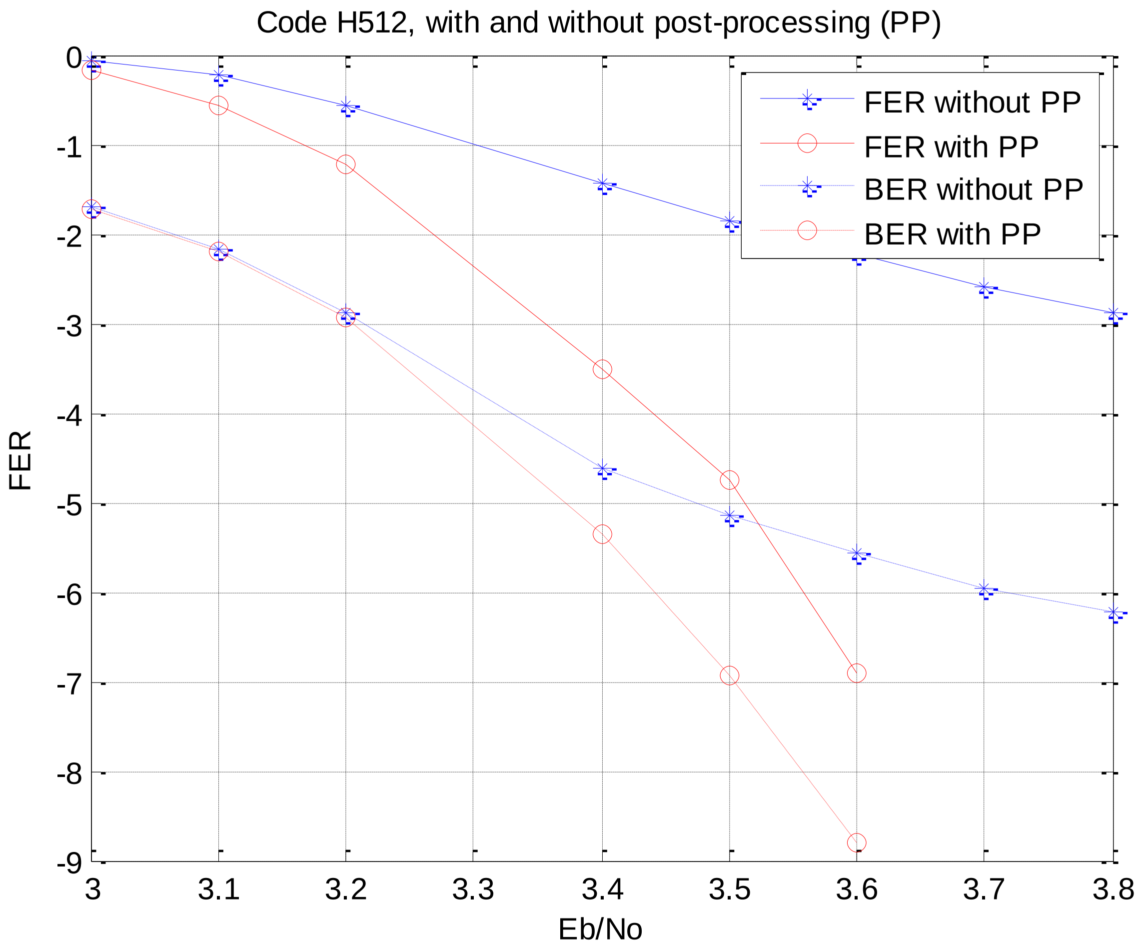
$(d_v,d_c)=(6,32)$, N=2048



SBF alone can outperform PGDBF at low FER

[1] K. Le et al. "A novel high-throughput, low-complexity bit-flipping decoder for LDPC codes,"

Example of SBF Post-Processing
d_v = 5, d_c = 20, N =10240



Initial algorithm that converge fast toward low weigth error pattern.

Then post-processing with SBF to clear remaining error Patern.
(only 11 iterations of SBF in this example).

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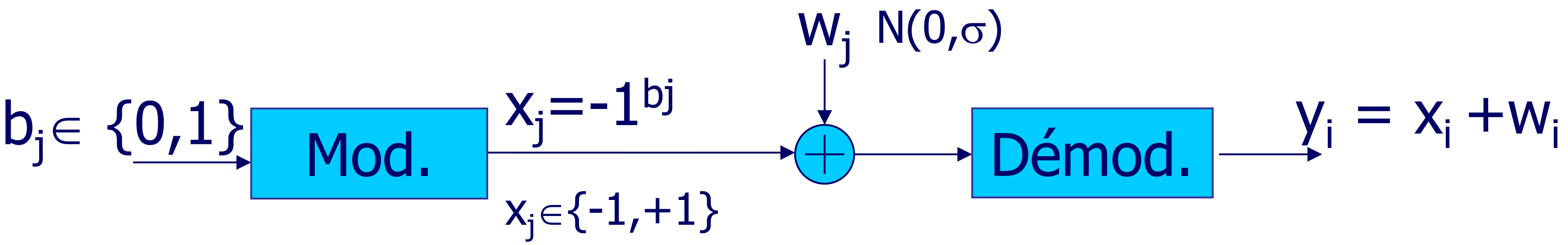
Optimum decoding

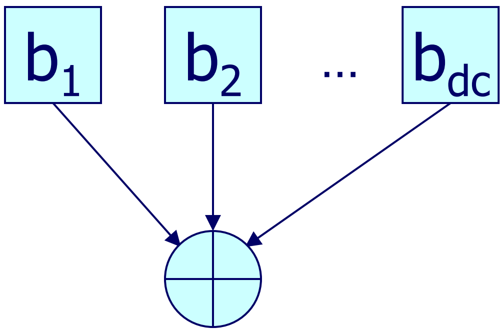
Bit of a codeword are transmitted in a noisy channel.



Decision with channel value: $x_i = \text{sgn}(y_i)$

ML decoder (optimal decoder): $\hat{x} = \arg\max_{x \in C} \left\{ \sum_{i=1}^{n} x_i y_i \right\}$

Check with sign (BPSK 0=>1, 1=>-1)



$$b_1 \oplus b_2 \oplus \ldots \oplus b_{dc} = 0$$

=> Even number of bit 1

If there is no error, then, for all $j = 1 \ldots m$ $\quad S_j = \prod_{k \in N(j)} x_k = +1$

$$\hat{x} = \arg\max_{x \in C} \left\{ \sum_{i=1}^{n} x_i y_i \right\} \quad <=> \quad \hat{x} = \arg\max_{x \in C} \left\{ \sum_{i=1}^{n} x_i y_i + \sum_{j=1}^{m} S_j \right\}$$

Try to maximise the energy function $E(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i y_i + \sum_{j=1}^{m} S_j$

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Gradient bit flipping algorithm

Let assume that x is not a codeword

Question: do we increase energy function flipping bit x_l?

If x_l flips to $\overline{x_l}$ then

$$E_{old} \Rightarrow E_{new} = E_{old} + (\bar{x}_l - x_l)\frac{dE}{dx_l} = E_{old} - 2x_l\frac{dE}{dx_l}$$

Thus, energy increases only if $E_l = x_l \frac{dE}{dx_l} < 0$.

with $E_l = x_l \frac{dE}{dx_l} = x_l y_l + \sum_{j \in N(l)} S_j$

Parallel update rule: flip the bits l verifying that $E_l < \theta$, with θ a negative threshold.

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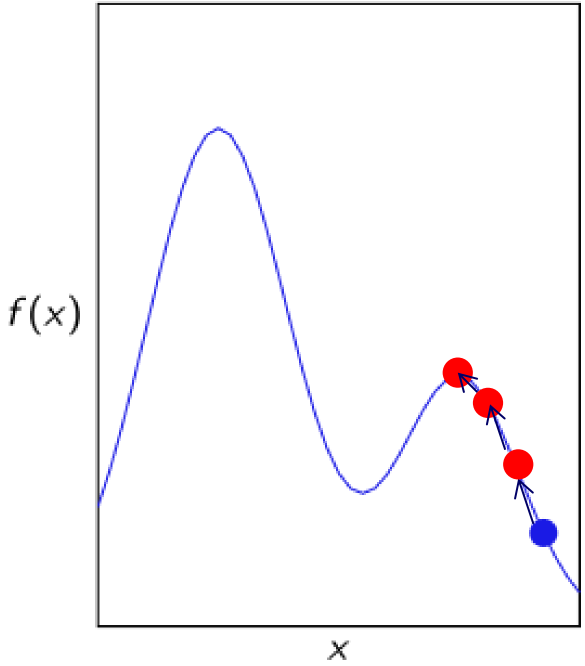
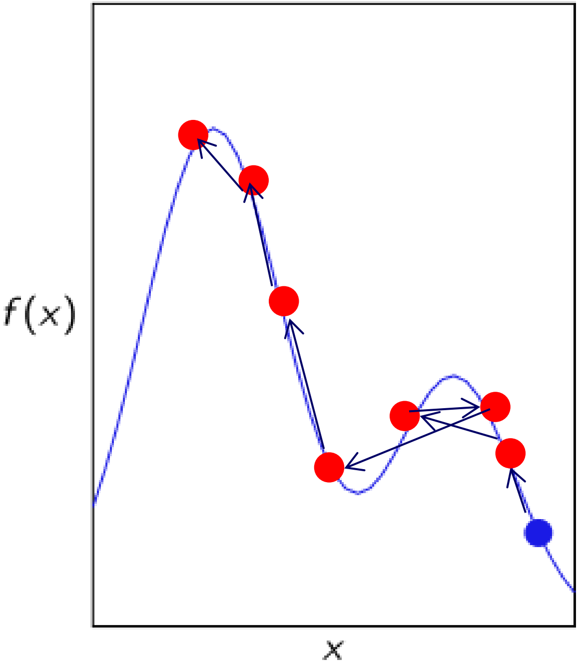
Problem with Gradient bit flipping algorithm





Problem: algorithm stucks in a local maxima.

$$E_i = x_i y_i + \sum_{j \in M(i)}^{m} S_j$$

Solution to escape the local maxima: add perturbation.

$$E_i = x_i y_i + \sum_{j \in M(i)}^{m} S_j + q_i$$

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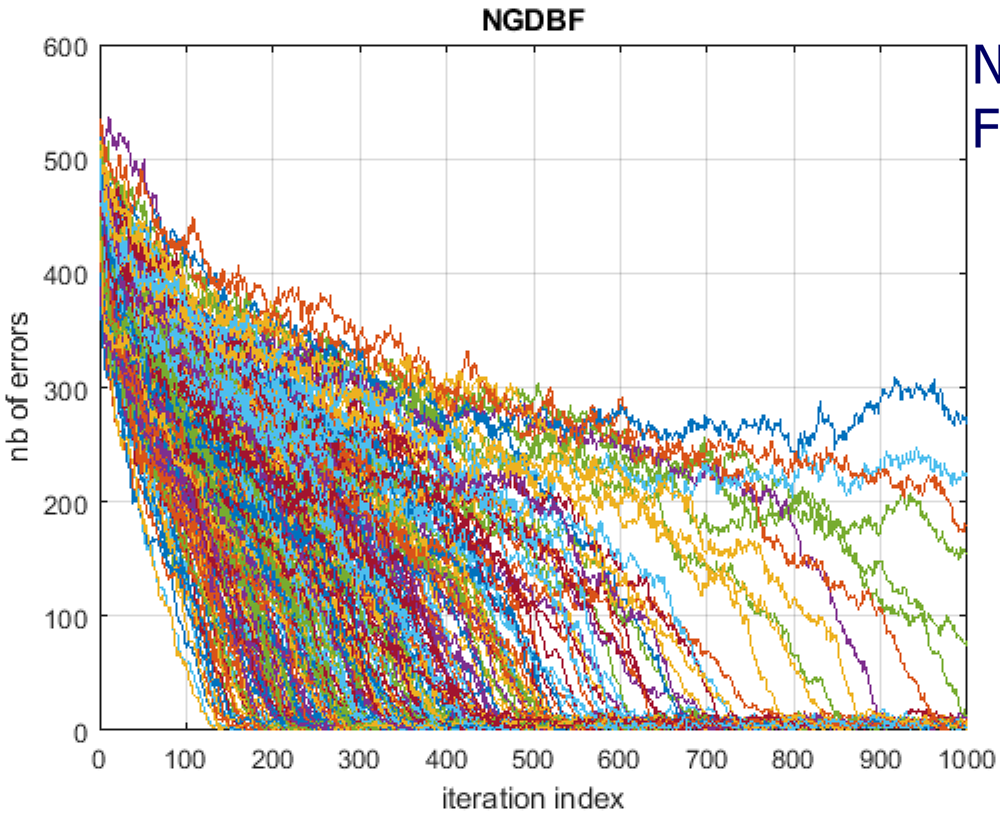
NGDBF alone



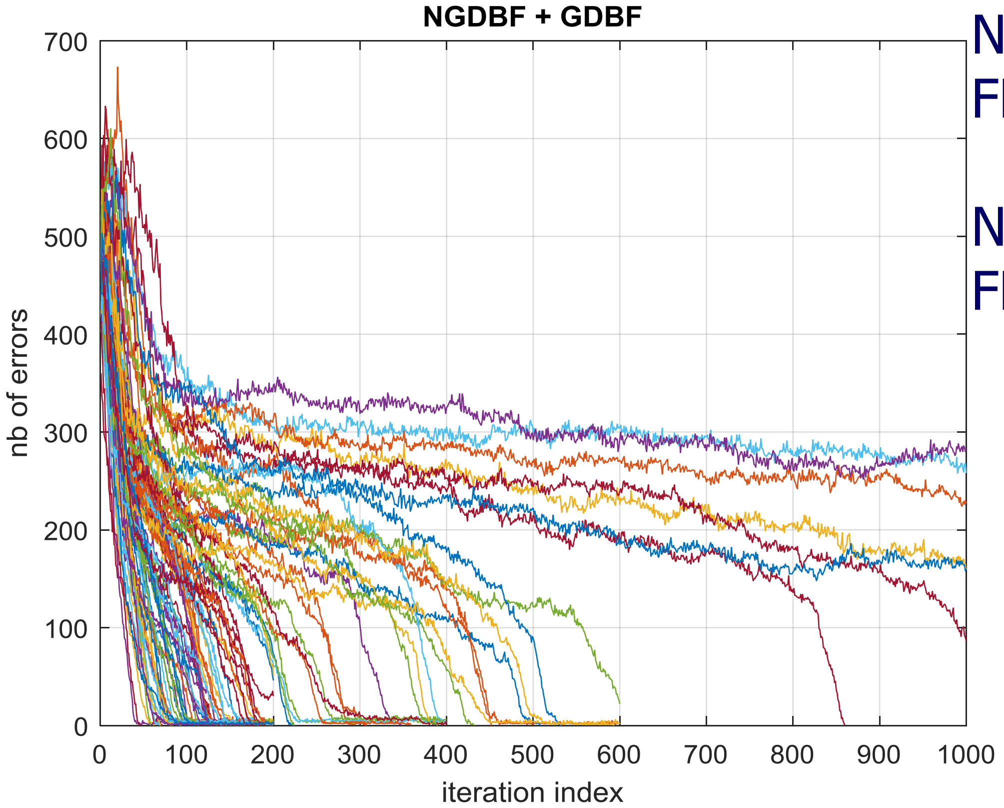
NGDBF + SBF
FER = 0.0278

(dv=5,dc = 20),
N = 10240,
Eb/N0 = 3.2 dB

Performance reference

Mixte NGDBF and GDBF

(dv=5,dc = 20),
N = 10240,
Eb/N0 = 3.2 dB



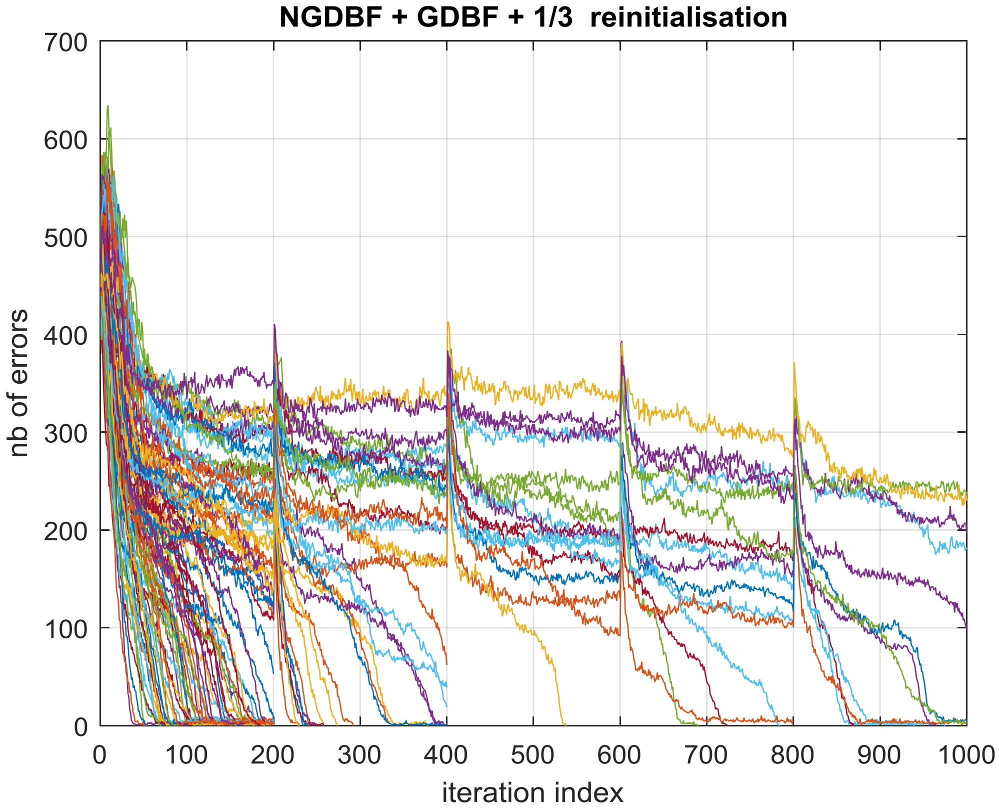
NGDBF + SBF
FER = 0.0278

NGDBF/GDBF + SBF
FER = 0.0585

Faster convergence, but FER degradation

Every 200 cycles, 1/3 of states take their initial value.

(dv=5,dc = 20),
N = 10240,
Eb/N0 = 3.2 dB



NGDBF
FER = 0.0278

NGDBF + GDBF
FER = 0.0585

NGDBF + GDBF
+ partial restart
FER = 0.0188

Faster convergence, better performance.

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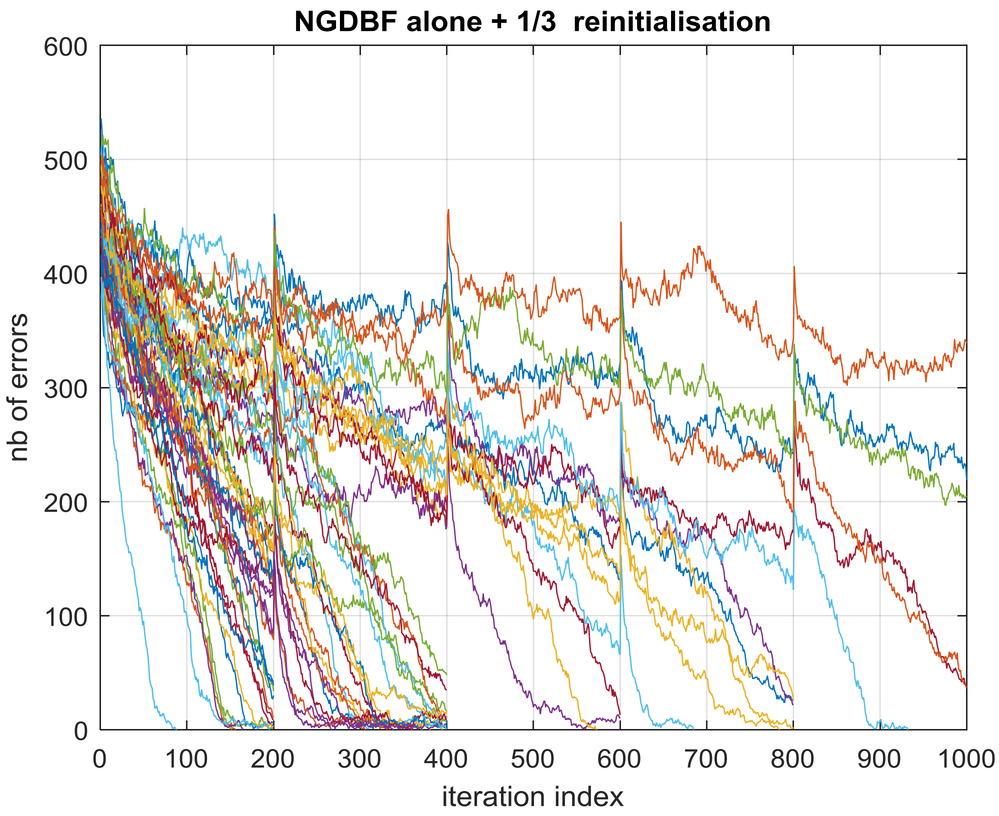
Réinitialisation partielle + NGDBF simple.

(dv=5,dc = 20),
N = 10240,
Eb/N0 = 3.2 dB



NGDBF
FER = 0.0278

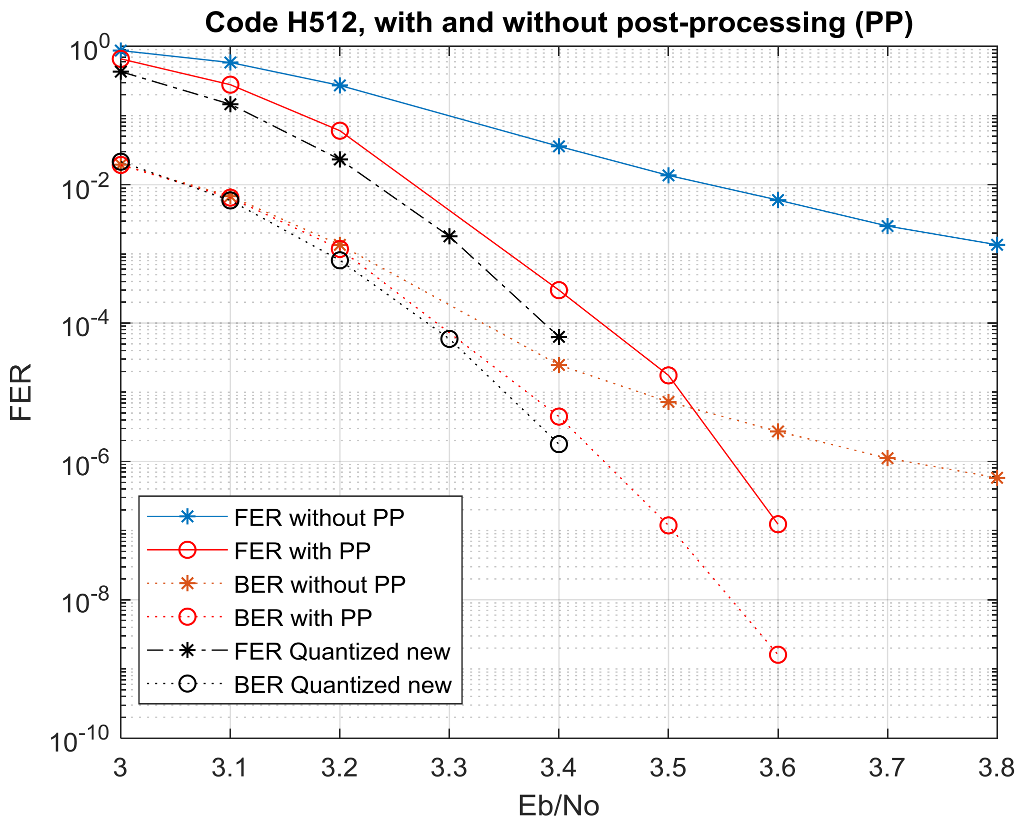
NGDBF + GDBF
FER = 0.0585

NGDBF + GDBF
+ partial restart
FER = 0.0188

NGDBF alone
+ Partial restart
FER = 0.0333

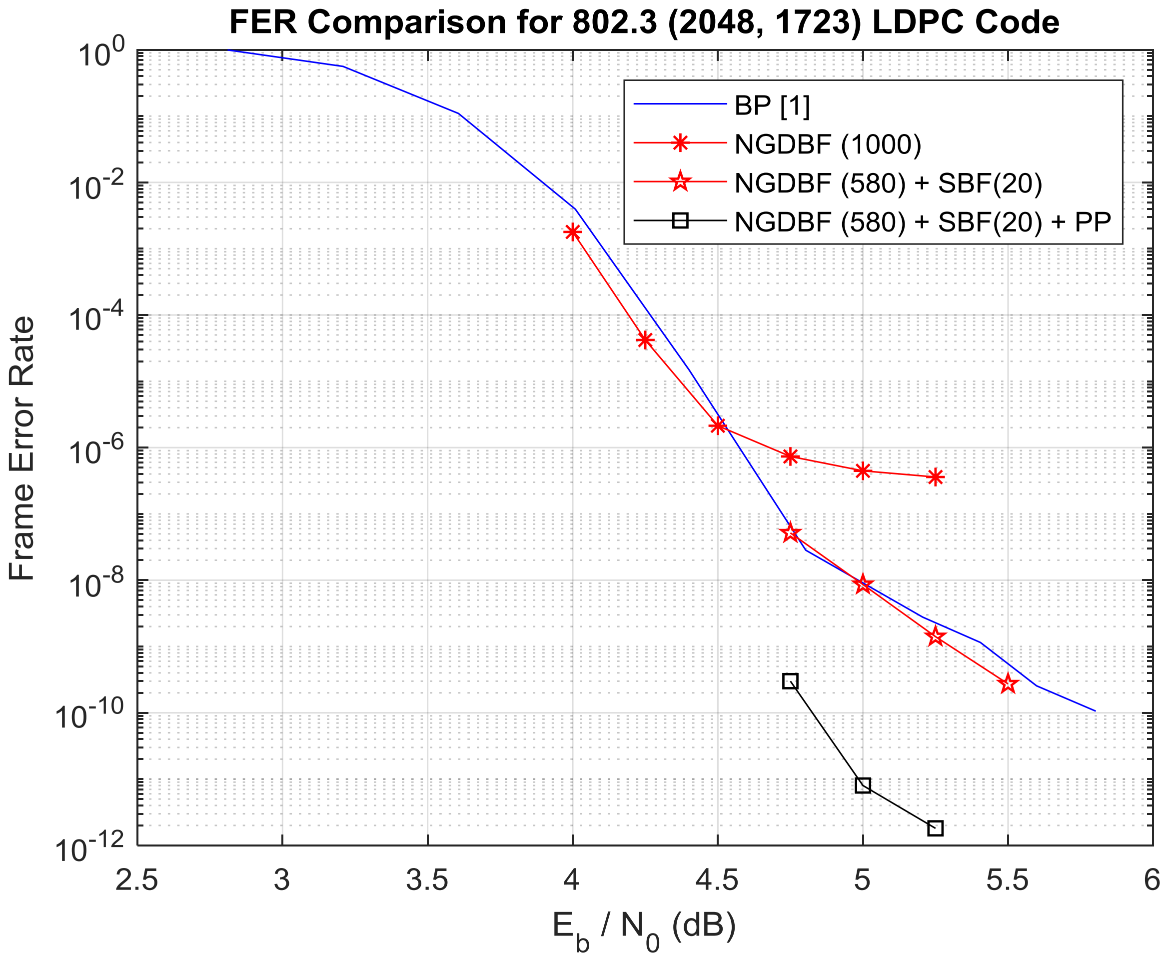
Lower performance because of lower convergence

Nouveau NGDBF
$d_v = 5$, $d_c = 20$, N =10240



1000 iterations
+ SBF

NGDBF + SBF for IEEE 802.3 $(d_v,d_c)=(6,32)$, N=2048



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Oups!

Let’s us shift toward slides generated by latex…