

The Benefits of Being Erratic:

Correcting Errors with Noise-Enhanced Gradient Algorithms

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- ▶ We interpret noise as a form of **algorithm diversity**.

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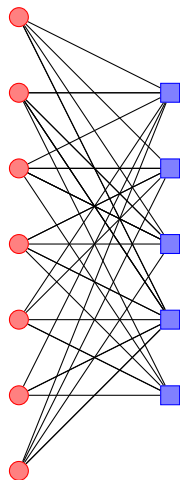
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 - ▶ Parity-only bit flipping
 - ▶ Threshold sequences tailored to dominant trapping sets.

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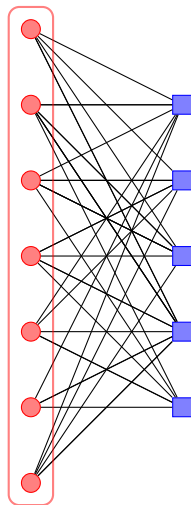


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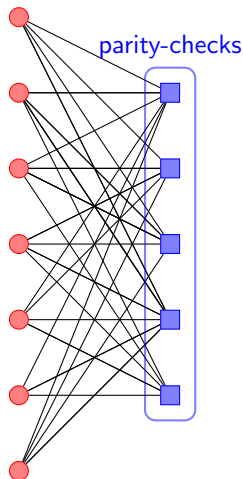


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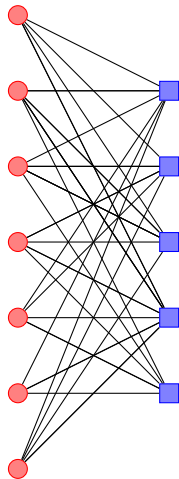
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The graph's **parity-checks** constrain even parity among adjacent bits.



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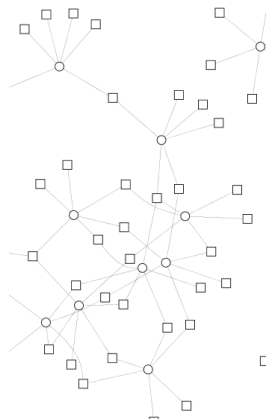
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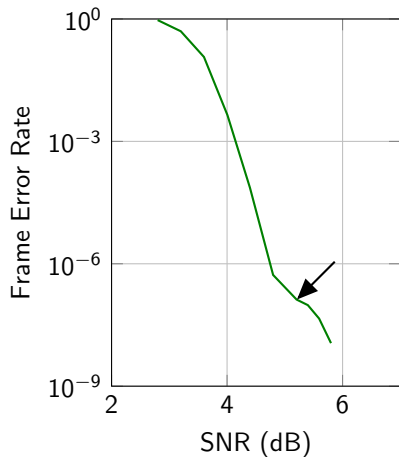
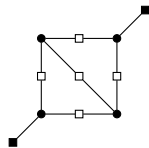
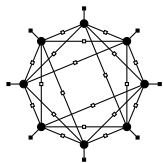
This problem is usually modeled as a **Bayesian Belief Network**, using the **Belief Propagation (BP)** algorithm. Probability calculations can be expensive, and **cycles in the network** distort the calculations, leading to failures.



Trapping Sets

Trapping sets or Absorbing Sets are repeated cyclic patterns in the code graph.

They induce **error floors** in LDPC decoders based on BP.



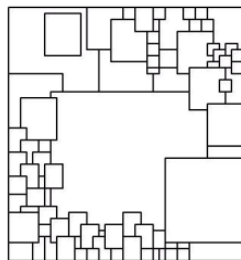
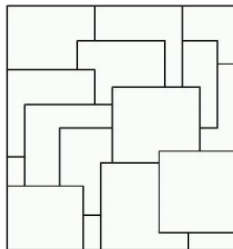
FAIDs & Decoder Diversity

There are many approaches for dealing with absorbing sets.

The general framework of **Finite Alphabet Iterative Decoders (FAID)** describes a class of decoders by LUTs rather than explicit Bayesian calculations^a.

Any particular FAID has small error coverage, but **decoder diversity** guarantees total coverage by alternating between different LUTs.

^aPlanjery et al. 2013; Declercq et al. 2013.



Single-Bit Algorithms

Several classes of low-complexity algorithms have been studied. Some notable algorithms:

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- ▶ **Bit-Flipping** – Heuristics *loosely based* on BP or gradient descent.

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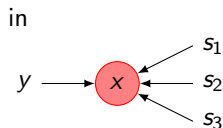
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Parallel Bit-Flipping Decoders

A binary alternative, simpler than BP or Stochastic decoders.

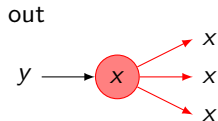
Inputs

channel sample y_i .
hypothesis $x_i \in \{-1, +1\}$.
parity checks $s_j \in \{-1, +1\}$



Operations

Reliability metric: Δ_i (unique to algorithm)
Threshold update: flip x_i if $\Delta_i \leq \theta$



Then transmit x_i to adjacent nodes.

Example Algorithms

Impr. Diff. Binary (IDB):⁴ $\Delta_i^{(t+1)} = \Delta_i^{(t)} + w \sum_{j \in \mathcal{M}_i \setminus \mathcal{J}} s_{i,j} - d x_i^{(t)}$

Grad. Desc. BF (GDBF):⁵ $\Delta_i^{(t+1)} = x_i y_i + \sum_j s_j$

Noisy GDBF:⁶ $\Delta_i^{(t+1)} = x_i y_i + w \sum_j s_j + q$

Probabilistic GDBF:⁷ If $\Delta_i < \theta$, flip with probability p .

Traditional rule of thumb: higher complexity = better performance.

⁴Cushon et al. 2014.

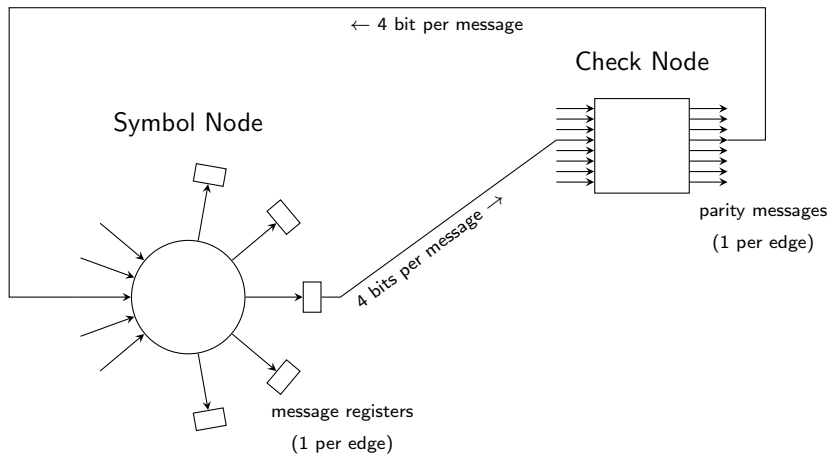
⁵Wadayama et al. 2008.

⁶Sundararajan, Winstead, and Boutillon 2014.

⁷Rasheed, Ivanis, and Vasic 2014.

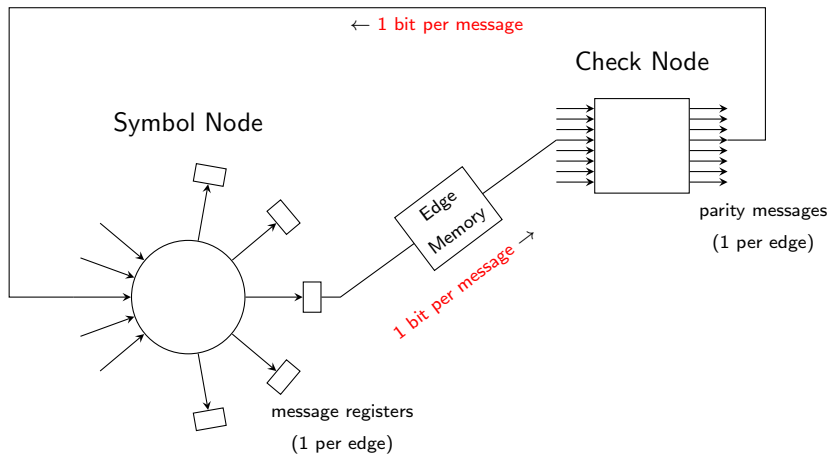
Motivation: Reduced Complexity

Standard BP-based or FAID algorithm:



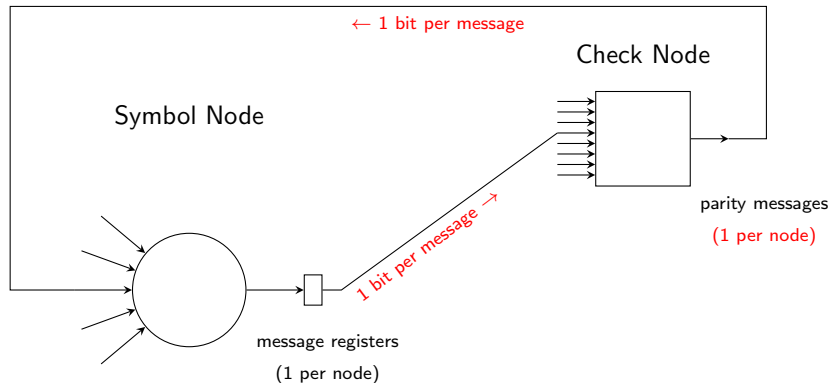
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Stochastic algorithm (successive relaxation):



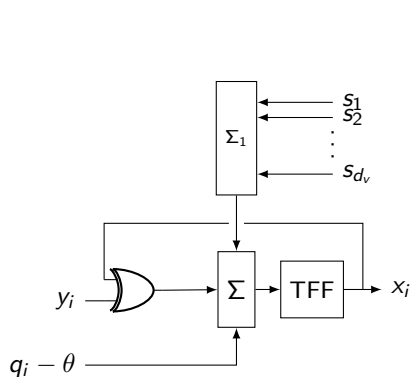
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Bit-flipping algorithm:

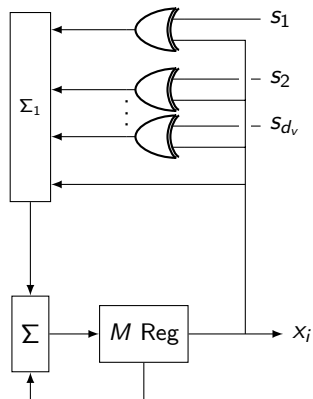


Example Node Designs

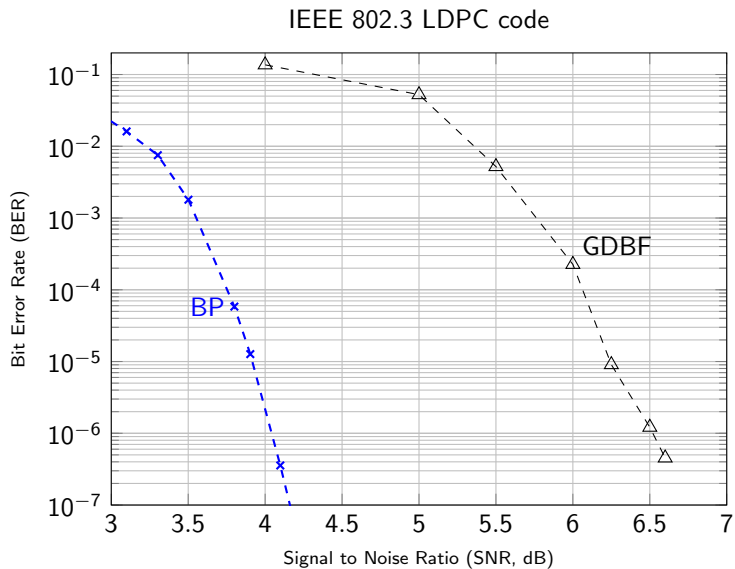
NGDBF



IDB

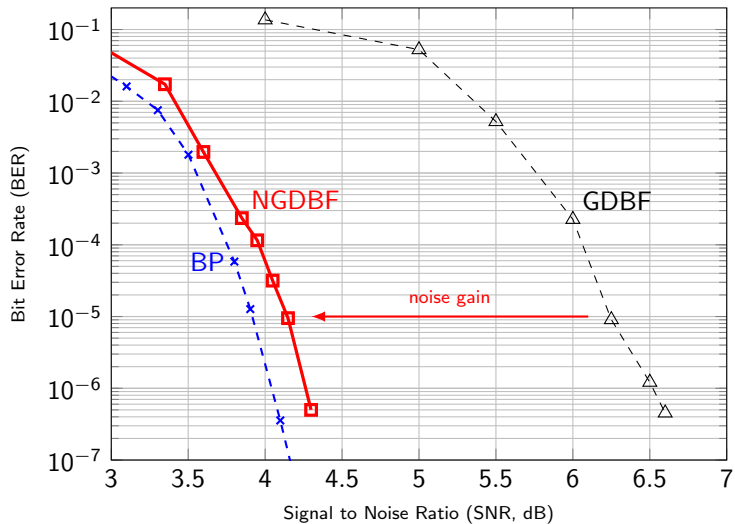


Performance: Noise Enhancement



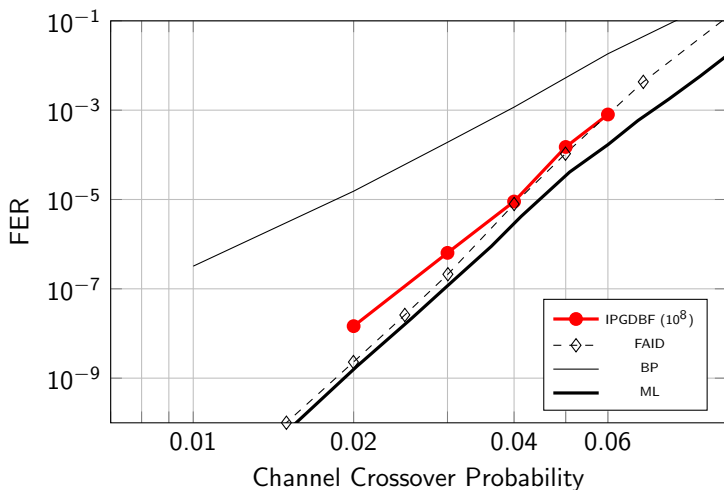
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IEEE 802.3 LDPC code



PGDBF FER

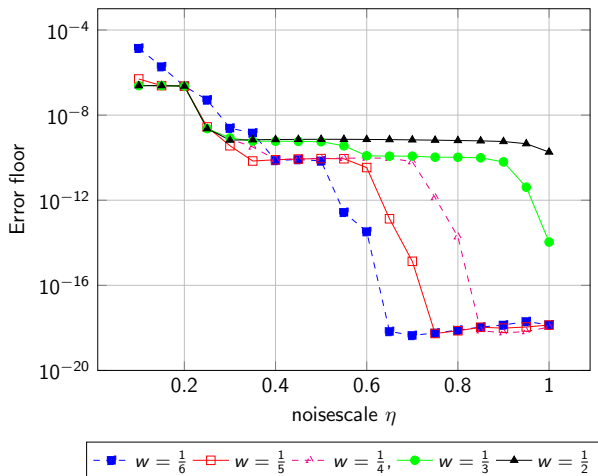
FAID and PGDBF approach Maximum Likelihood (ML) limit⁸



⁸Declercq et al. 2016.

We know that noise works...

Treating each trapping set as a Markov process, we can predict the error rate and optimize algorithm parameters:



What is the bigger picture? *Why* does non-determinism enhance these algorithms?

Parrondo's "paradox"

Parrondo's paradox considers noise-perturbed particles on various sloped surfaces. These are "failing" strategies:⁹

Parrondo's "paradox"

But by alternating between losing strategies at the right times, and with the right amount of noise energy, it's possible to "win" with high probability:

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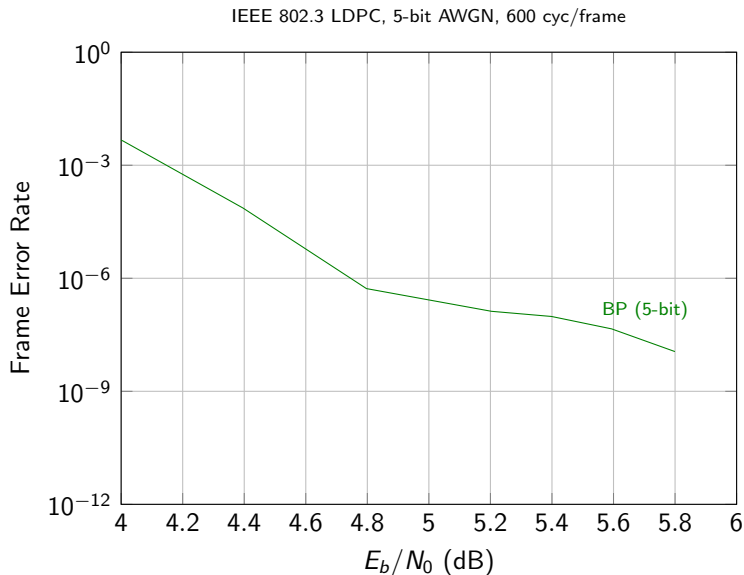
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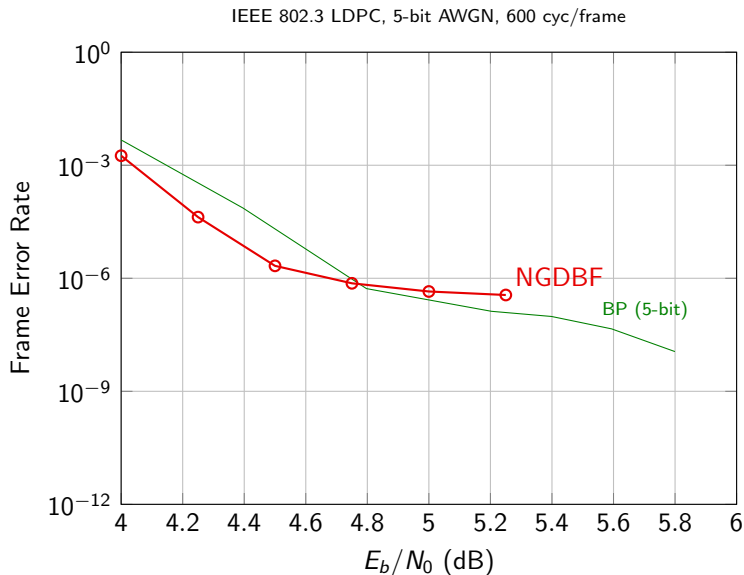
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- ▶ Noise is important to the demonstration only as another source of diversity – it could be replaced by additional strategies.
- ▶ Bit-flipping algorithms are “losers,” and noise turns them into “winners” . . .
- ▶ But maybe there are other ways to achieve this diversity.

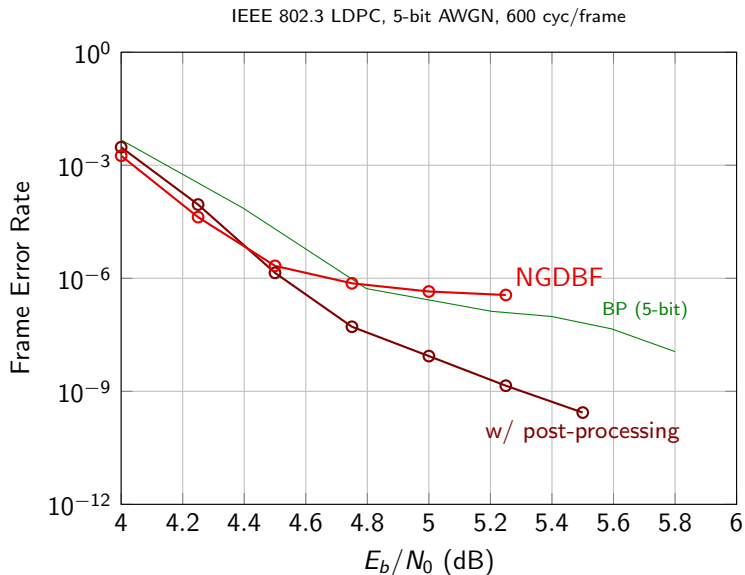
Lowering the Error Floor



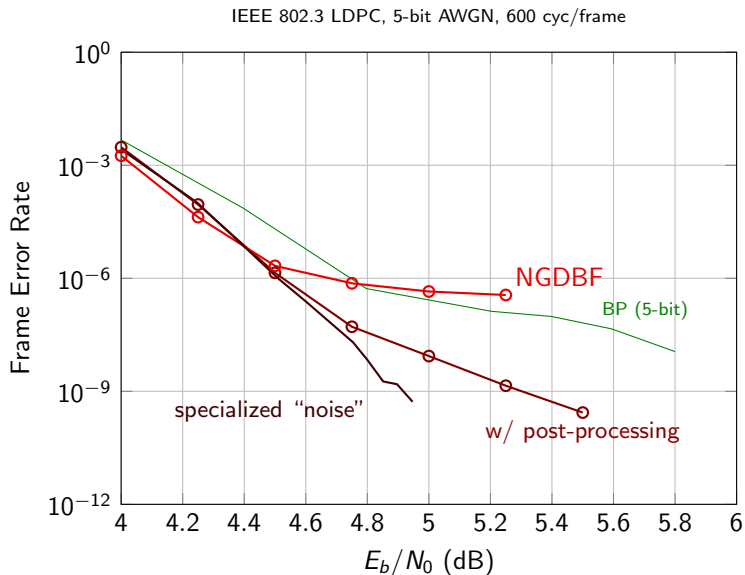
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- ▶ We now have a basic theory to explain and optimize noise enhancement.
- ▶ Algorithm diversity remains an open frontier.

Acknowledgements






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