Noise-Aided Gradient Descent Bit-Flipping Decoders approaching Maximum Likelihood Decoding

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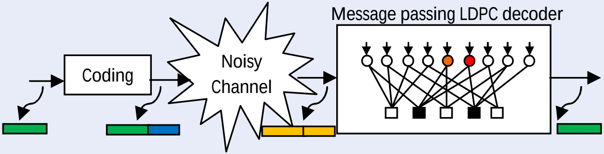
Outline

Contents

Context & Objectives

Low Density Parity Check (LDPC) Decoders

- Applications: Wired, Wireless communication, Storage...



2 types of LDPC decoding algorithms:

- **Soft information algorithms :** Sum-Product, Min-Sum..., powerful error correction capacity but high complexity.
- **Hard-decision algorithms :** Bit Flipping (BF), Gradient Descent Bit Flipping (GDBF), Gallager-B, etc. low complexity, usually weak in error correction.
- **Towards a new type of decoder :** low complexity + noise perturbation ⇔ powerful error correction,

Context and Objectives

A new noisy decoding framework

- In [Varshney 2011], [Vasic 2007], [Yazdi 2013], authors focused on trying to compensate faulty hardware effects, by making the iterative decoders robust and fault-tolerant,
- Inspired by the robustness of iterative decoders, a new direction of research has been identified:

Additionnal noise could help error correction instead of being an enemy

- the Additional noise could come from the circuit design, or through deliberate noise injection.

Injected Noise to break the attraction of Trapping Sets (TS)

- Failures of iterative decoders are mainly due to TS : fixed points or loopy attractors,
- Not only can it help in the error floor, but also in the waterfall,

[Varshney 2011] L. Varshney, "Performance of LDPC codes under faulty iterative decoding", *IEEE Trans. on Info. Theory*, 2011
[Vasic 2007] B. Vasic et al., "An information theoretical framework for analysis and design of nanoscale fault-tolerant memories based on LDPC codes", *IEEE Trans. on Circuits and Systems I*, 2007
[Yazdi 2013] S. Yazdi et al., "Gallager-B decoder on noisy hardware", *IEEE Trans. on Commun.*, 2013

Very simple decoder + Noise ⇔ Powerful Decoder

Injected Noise in the decoder can help to combat the channel errors

- First demonstration of this phenomenon on the Gradient-Descent Bit-Flipping decoder:
  - Probabilistic Gradient Descent Bit-Flipping (PGDBF) for the BSC, [Rasheed 2014]
  - Noisy Gradient Descent Bit-Flipping (NGDBF) for the BI-AWGN, [Sundararajan 2014]
- Could be generalized to other hard-decision decoders (Gal-B for example), [Ivanis 2015]

Issues raised about noise injection

- **Where ?** VNU ? CNU ? Memories ? Only localized computing units ?
- **When ?** From the first iteration ? After a given number of iterations ? When a decoding failure is detected ?
- **How ?** Which Noise model ? Values of the parameters ? Hardware realization ?

[Rasheed 2014] O. Rasheed et al., "Fault-tolerant probabilistic gradient-descent bit flipping decoder", *IEEE Communications Letters*, 2014

[Sundararajan 2014] Sundararajan et al., "Noisy gradient descent bit-flip decoding for LDPC codes", *IEEE Transactions on Communications*, 2014

[Ivanis 2015] P. Ivanis et al., "MUDRI: A fault-tolerant decoding algorithm", *ICC*, 2015

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Gradient Descent Bit-Flipping Decoder

Concept of Gradient Descent Bit Flipping (GDBF) Algorithm [Wadayama 2010]

- Iterative propagation of binary information between 2 groups of processing units:
  1. **Check Nodes Units** (CNU): compute parity check equations (XOR operations),

$$c_m^{(l)} = \bigoplus_{v_n \in \mathcal{N}(c_m)} v_n^{(l)}$$

  2. **Variable Nodes Units** (VNU): the VN value is flipped if the number of violated CN neighbors is too large.
- Requires the computation of an Energy/Inversion Function in order to select the bit-flips.

Energy function for the BSC channel (high energy = low reliability)

$$E_{v_n}^{(l)} = L(y_n)^{(l)} + \sum_{c_m \in \mathcal{N}(v_n)} c_m^{(l)} \qquad L(y_n)^{(l)} = y_n \oplus v_n^{(l)}$$

Energy function for the AWGN channel (low energy = low reliability)

$$E_{v_n}^{(l)} = L(y_n)^{(l)} + w \sum_{c_m \in \mathcal{N}(v_n)} (1 - 2\, c_m^{(l)}) \qquad L(y_n)^{(l)} = (1 - 2\, v_n^{(l)}) \log \left(\frac{prob(y_n | v_n = 0)}{prob(y_n | v_n = 1)} \right)$$

Flipping Sets

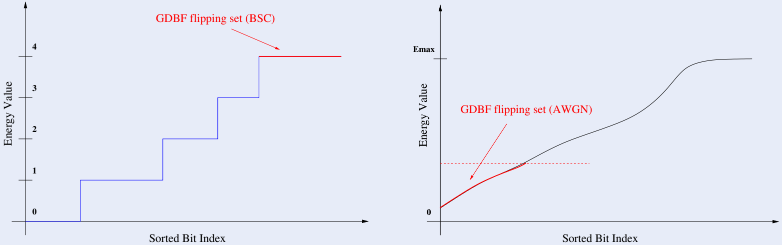
Flipping sets contain indices of bits to be flipped

$$\mathcal{F}_{\text{BSC}}^{(l)} = \left\{ n \in \{1, N\} \,;\, E_{v_n}^{(l)} = b^{(l)} \right\} \qquad \text{[BSC]}$$

$$\mathcal{F}_{\text{AWGN}}^{(l)} = \left\{ n \in \{1, N\} \,;\, E_{v_n}^{(l)} \leq \theta \right\} \qquad \text{[AWGN]}$$

- $b^{(l)} = \max_{1 \leq n \leq N} (E_{v_n}^{(l)})$ is the maximum energy (BSC), and θ is a pre-determined threshold.

Graphical representation of flipping sets



Noisy Flipping Sets

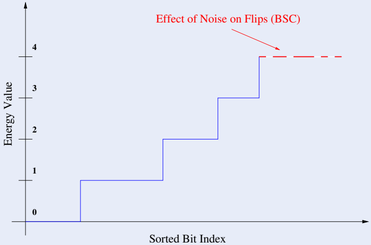
PGDBF : random variable $\epsilon_n^{(l)} \sim \mathcal{U}_{[0,1]}$ main parameter p_0

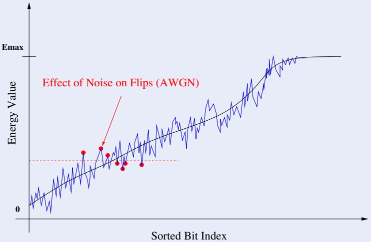
$$\widetilde{\mathcal{F}}_{\text{PGDBF}}^{(l)} = \left\{ n \in \mathcal{F}_{\text{BSC}}^{(l)} \; ; \; \epsilon_n^{(l)} < p_0 \right\}$$ [PGDBF]

NGDBF : random variable $\epsilon_n^{(l)} \sim \mathcal{N}(0, \sigma_p)$ main parameter σ_p

$$\widetilde{\mathcal{F}}_{\text{NGDBF}}^{(l)} \left\{ n \in [1, N] \; ; \; E_{v_n}^{(l)} \leq \theta + \epsilon_n^{(l)} \right\}$$ [NGDBF]

Graphical representation of noisy flipping sets





Unified presentation of GDBF and Noisy GDBF decoders

GDBF and NA-GDBF Algorithms : [iteration l]

[Step 1] Compute CNs values

$c_m^{(l)}, \forall m = 1, \dots, M,$

[Step 2] Compute Energy functions at VNs

$E_{v_n}^{(l)}, \forall n = 1, \dots, N$

[Step 3] Compute the flipping sets

$\mathcal{F}_{\text{BSC}}^{(l)}$ or $\mathcal{F}_{\text{AWGN}}^{(l)}$ for the deterministic GDBF

$\mathcal{F}_{\text{PGDBF}}^{(l)}$ or $\mathcal{F}_{\text{NGDBF}}^{(l)}$ for the noise-aided GDBF

[Step 4] Bit flipping

$$\forall n \in \mathcal{F}^{(l)} \quad v_n^{(l+1)} = \overline{v_n^{(l)}}$$

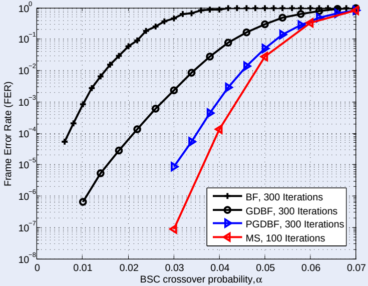
$$\forall n \notin \mathcal{F}^{(l)} \quad v_n^{(l+1)} = v_n^{(l)}$$

Complexity and Performance of Noise Injection

Deterministic GDBF and MS versus Probabilistic GDBF

- Regular quasi-cyclic LDPC code $(d_v, d_c) = (3, 6)$, $N = 1296$ bits,

Noise-aided simple decoders can approach performance of powerful decoders with a very small hardware overhead ($\simeq$ 5%)



dv3R050N1296	Area and Throughput θ Comparison							
	Code length	AREA (μm^2)	f_{max} (MHz)	N_c	$FER = 1e-5$		$\alpha = 0.02$	
					It_{ave}	θ (Gbit/s)	It_{ave}	θ (Gbit/s)
GDBF	1296	87810	222	1	2.00 (@$\alpha = 0.005$)	144.00	2.95 ($FER = 3e^{-4}$)	97.63
PGDBF	1296	92645	232	1	3.50 (@$\alpha = 0.012$)	86.11	2.88 ($FER = 5e^{-6}$)	104.65
Min-Sum	1296	950000	111	6	1.94 (@$\alpha = 0.025$)	12.36	1.15 ($FER = 1e^{-7}$)	20.85

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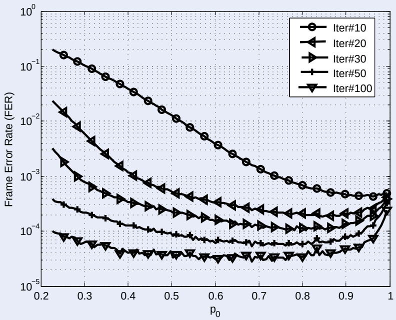
Statistical Analysis in the Waterfall : PGDBF

Analyze and quantify the amount of noise that should be introduced

- **Objective :** optimize the amount of noise that maximizes the coding gains
- Through Monte-Carlo simulations in the waterfall and the error-floor.

Binary Symmetric Channel - Waterfall Region - Tanner Code ($M = 93$, $N = 155$)



- **Conclusion 1:** for the first decoding iterations random noise does not help,
- **Conclusion 2:** a wide range of $p_0 \in [0.5; 0.9]$ values achieve the same coding gain.

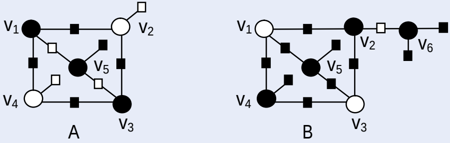
Statistical Analysis in the Error Floor : PGDBF

Errors located on Trapping Sets

- In the error floor region, the dominant uncorrectable error configurations are concentrated on Trapping Sets
- Trapping Sets $TS(a, b)$ are defined as a small set of a VNs for which the neighboring CNs contains exactly b odd degree CNs
- $TS(5, 3)$ is the smallest trapping set for regular $d_v = 3$ LDPC codes with girth $g = 8$.

Smallest Error Events not correctable by the GDBF

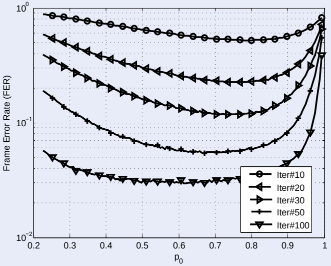
- weight-3 error patterns which does not satisfy 5 parity-checks,
- weight-4 error patterns which does not satisfy 10 parity-checks,



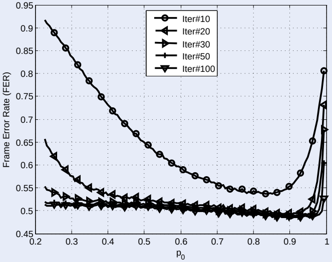
Statistical Analysis in the Error Floor : PGDBF

Frame error rate with fixed input errors

- for each Monte-Carlo round, only the random noise $\epsilon_n^{(l)}$ differ, the channel errors are kept the same.



3 bits error pattern



4 bits error pattern

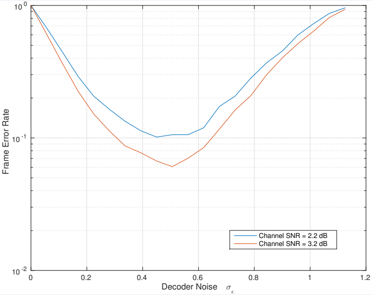
- **Conclusion 1:** the random noise is useful in the first iterations,
- **Conclusion 2:** the same wide range of $p_0 \in [0.5; 0.9]$ values achieve the maximum coding gain.

Statistical Analysis in the Error Floor : NGDBF

Errors located on Trapping Sets

- **Step 1 :** apply a bias on the channel samples associated with a TS,
- **Step 2 :** select only frames for which the GDBF fails on the selected TS,
- **Step 3 :** restart decoding of the same frame with the NGDBF,
- **Step 4 :** compute the residual FER on the selected frames.

TS(5, 3) on the Tanner (155, 64) code



- **Conclusion :** the choice of optimum σ_p is narrower than for the PGDBF.

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Trapping sets of noiseless GDBF decoders

Assumptions/Definitions

- We analyze errors located on a trapping set that is isolated from the rest of the graph,
- Shortest example for $d_v = 3$, $g = 8$ LDPC codes: the TS(5, 3) trapping set,
- Definition of a TS state : $S = (v_1, v_2, v_3, v_4, v_5)_2$
- *Examples :* correct state $S_0 = (0, 0, 0, 0, 0)_2$ - error state $S_{21} = (1, 0, 1, 0, 1)_2$.

A trapping set for the GDBF decoder : oscillating behavior between S_{21} and S_{26}.

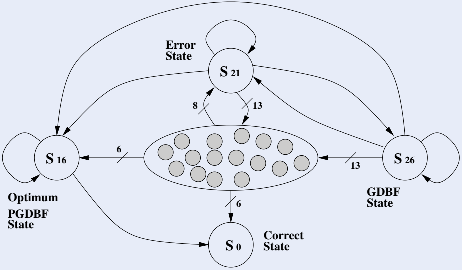
- black circles are in error,
- circled-dashed circles have maximum energy,

State space of the PGDBF decoder

Some observations starting with error state $S_{21} = (1,0,1,0,1)_2$

- Deterministic GDBF oscillates indefinitely between S_{21} and S_{26},
- Only 20 out of the 32 possible states are achievable
- S_0 is one of the achievable states,

State space of the PGDBF on the TS(5,3)

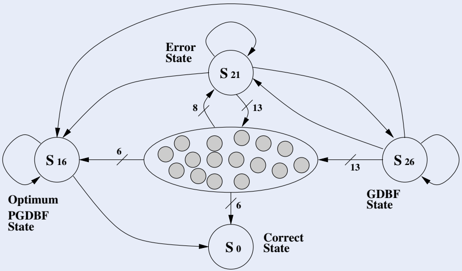


PGDBF probability of escaping a TS

Optimization of the parameter p_0

- The shortest path between the error and the correct state is : $S_{21} \rightarrow S_{16} \rightarrow S_0$,
- This is the only path with 2 transitions, its probability of occurence is $p_0^3(1-p_0)^2$,
- Optimization strategy : maximize the probability of the shortest path $\rightarrow$ $p_0 = 0.6$

State space of the PGDBF on the TS(5,3)



NGDBF probability of escaping a TS

Assumptions/Definitions

- All 32 states are achievable,
- let S and S' be two consecutive states, and $T(S,S')$ the indices of bits in which S and S' differ,
- Under the isolation assumption, only the state nodes $(v_1, v_2, v_3, v_4, v_5)$ have negative LLRs,

Transition Probabilities

- The transition probabilities between states S and S'

$$\Lambda(S,S') = \prod_{n \in T(S,S')} F(E_{v_n}, \theta, \sigma_p) \prod_{n \notin T(S,S')} (1 - F(E_{v_n}, \theta, \sigma_p)),$$

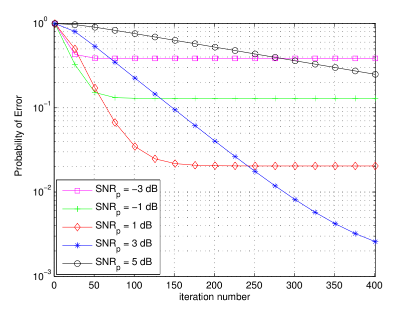
- with

$$F(E_{v_n}, \theta, \sigma_p) = \frac{1}{\sqrt{2\pi}\sigma_p} \int_{-\infty}^{\theta - E_{v_n}} e^{-\frac{x^2}{2\sigma_p^2}} dx,$$

- using $\Lambda(S,S')$ and the Perron-Frobenius theorem, we can compute the probability of escaping a TS, at each iteration.

NGDBF probability of escaping a TS

Probability of TS error for the NGDBF



Optimization of σ_p

- Trade-off between convergence speed and TS correction probability,
- Optimization of σ_p depends on how the TS are effectively isolated, and on the required latency.

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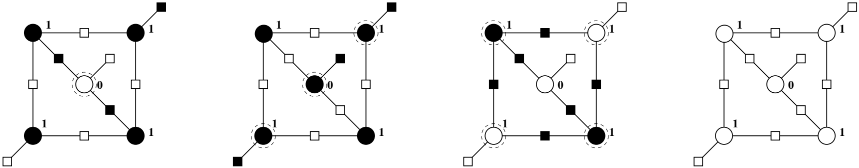
Advantage of modifying the PGDBF threshold

PGDBF with maximum threshold $E_{v_n}^{(l)} = b^{(l)}$: oscillation

PGDBF with decreased threshold $E_{v_n}^{(l)} \geq b^{(l)} - 1$: convergence



Decoder-Dynamic Shift PGDBF

- Instead of decreasing the threshold randomly, we let the threshold variations follow the dynamics of the decoder,
- Our proposed modification is to use the value of the maximum energy at the previous iteration $b^{(l-1)}$.

DDS-PGDBF : [iteration l]

[Step 1] Compute CNs values

$c_m^{(l)}, \forall m = 1, \ldots, M,$

[Step 2] Compute energy values at VNs

$E_{v_n}^{(l)}, \forall n = 1, \ldots, N,$

[Step 3] Compute the modified flipping set with threshold value from the previous iteration

$$\widetilde{\mathcal{F}}^{(l)}_{\text{DDS-PGDBF}} = \left\{ n \in \{1, N\} \,;\, E_{v_n}^{(l)} \geq b^{(l-1)} \text{ and } \epsilon_n^{(l)} < p_0 \right\}$$

[Step 4] Bit flipping

$$\forall n \in \widetilde{\mathcal{F}}^{(l)}_{\text{DDS-PGDBF}} \quad v_n^{(l+1)} = \overline{v_n^{(l)}}$$

$$\forall n \notin \widetilde{\mathcal{F}}^{(l)}_{\text{DDS-PGDBF}} \quad v_n^{(l+1)} = v_n^{(l)}$$

[Step 5] Update energy values at VNs

$$E_{v_n}^{(l)} = E_{v_n}^{(l)} - y_n \oplus v_n^{(l)} + y_n \oplus v_n^{(l+1)}.$$

[Step 6] Compute new maximum energy

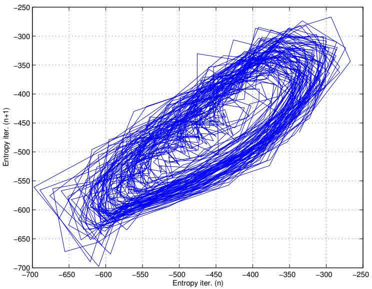
$$b^{(l)} = \max_{1 \leq n \leq N} \left(E_{v_n}^{(l)}\right)$$

Decoding Dynamics

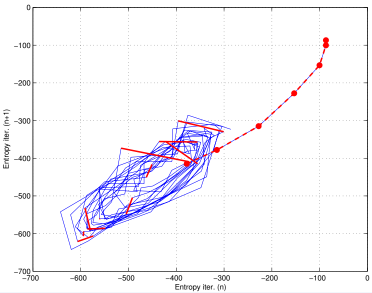
Behavior of the DDS-PGDBF decoder

- We plot the state space of the average binary entropy for the VN values,
- Errors are mainly located on a collection of uncorrectable TS,
- Using the previous threshold allows big jumps in the state space of the decoder,

PGDBF decoder ($p_0 = 0.6$)

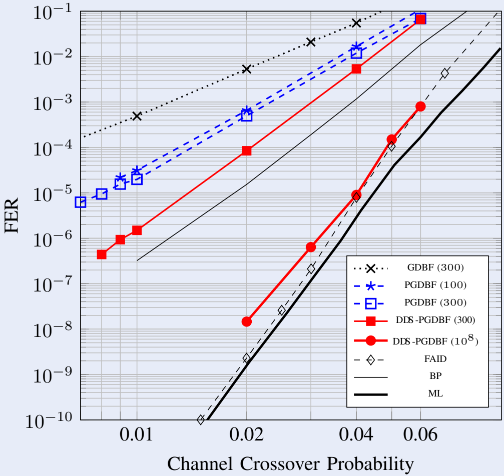


DDS-PGDBF decoder ($p_0 = 0.6$)



Performance of DDS-PGDBF

With a very large number of iterations, DDS-PGDBF approaches MLD



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Conclusion

Noise-Aided Iterative Decoders

- Injected Noise turn **weak decoders into powerful decoders**,
- Extra hardware complexity is negligible,
- The coding gains come at the cost of a larger number of iterations.

Optimization of Noise Statistics

- for the PGDBF on the BSC and for the NGDBF on the AWGN channel,
- using Monte-Carlo simulations and a semi-analytical analysis on isolated TS,
- the NGDBF gains require a precise optimization of σ_p.

PGDBF Decoders with Dynamic Shifts

- we proposed a modified PGDBF algorithm, with low complexity, and the ability to escape strong TS attraction,
- the DDS-PGDBF greatly improves the PGDBF performance, and approaches MLD using a very large number of iterations.