Reduced Complexity Decoding Algorithms for Low-Density Parity Check Codes

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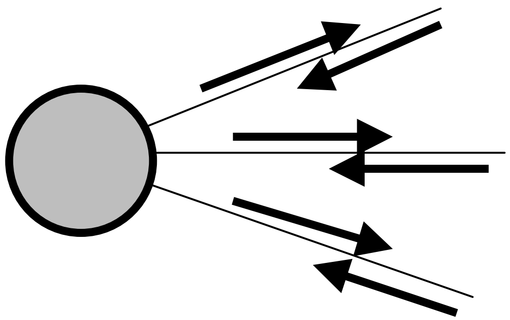
Outline

- Review of existing decoding algorithms for LDPC codes: BP, BP-based, APP, APP-based.
- Improvements to simplified BP algorithms.
  - Normalized BP-based algorithm,
  - Offset BP-based algorithm.
- A simple criterion to determine decoder parameters.
- Optimizing decoder parameters by density evolution.
- Implementation issues on the LDPC decoding algorithms.

Belief Propagation (BP) Algorithm

- BP algorithm is an iterative soft decision decoding algorithm [Gallager-IRE62, MacKay-IT99] .
- Each bit/check node is a processor, receiving messages from neighbor nodes, and sending back messages after processing.

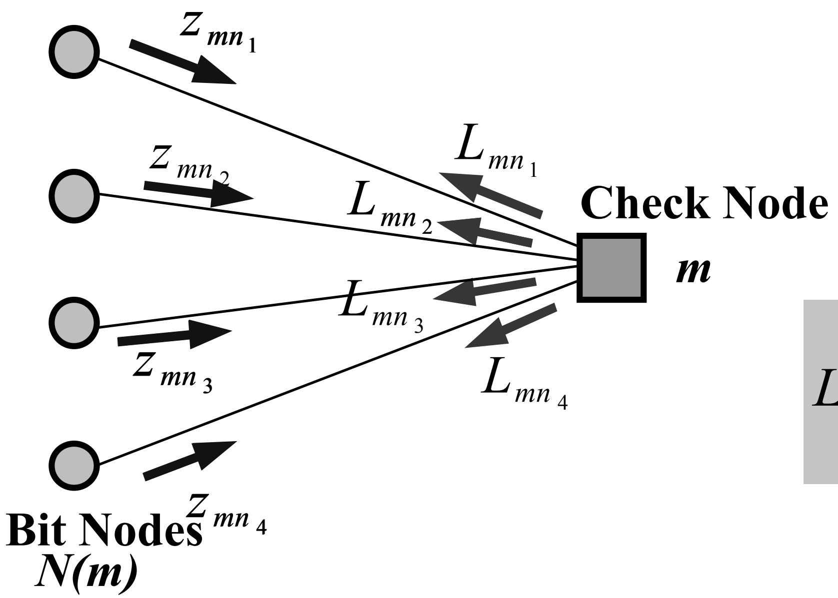


- Messages can be probabilities, and more conveniently, log-likelihood ratios (LLR's) for binary LDPC codes.

Processing in check nodes:

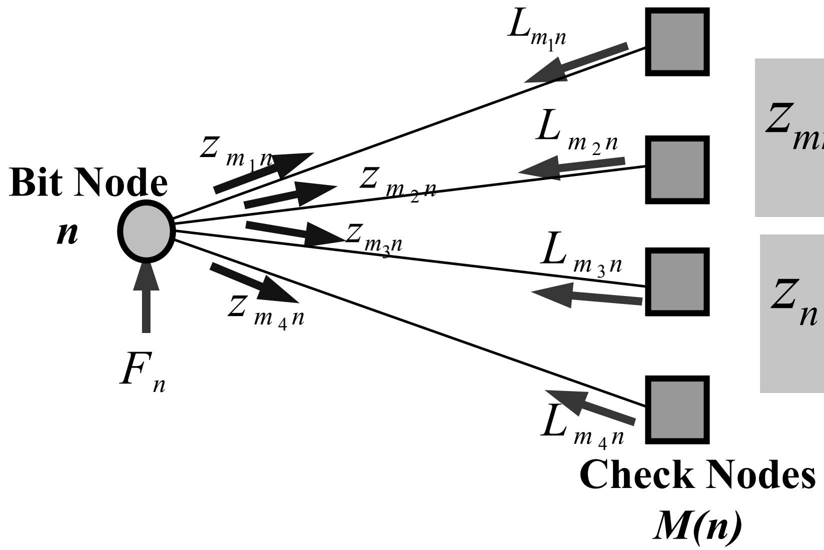
Principles:

incoming messages + constraints ⇒ outgoing messages



$$L_{mn} = 2\tanh^{-1}\left(\prod_{n' \in N(m)\backslash n} \tanh\left(z_{mn'}/2\right)\right)$$

Processing in bit nodes:

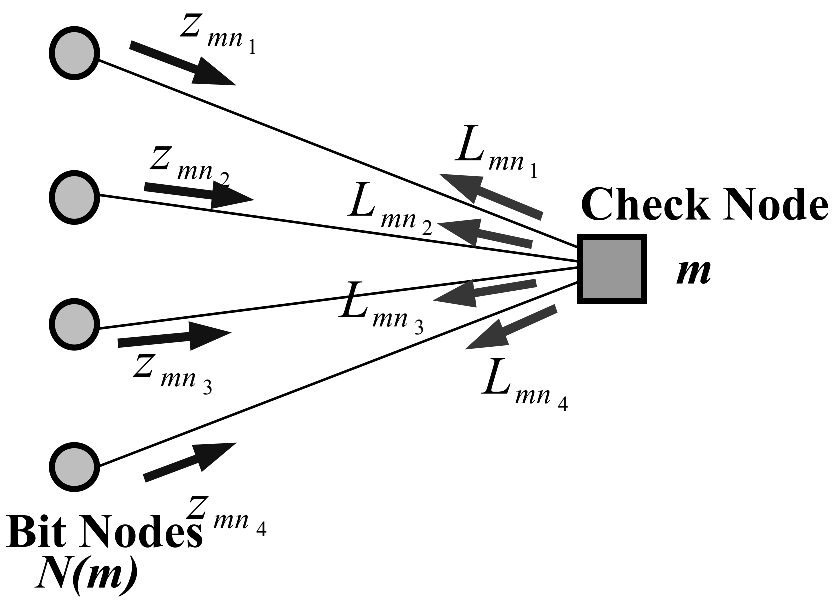


$$z_{mn} = F_n + \sum_{m' \in M(n)\setminus m} L_{m'n}$$

$$z_n = F_n + \sum_{m \in M(n)} L_{mn}\,,$$ for hard decision

BP-Based Algorithm (min-sum)

—— simplification in check node processing

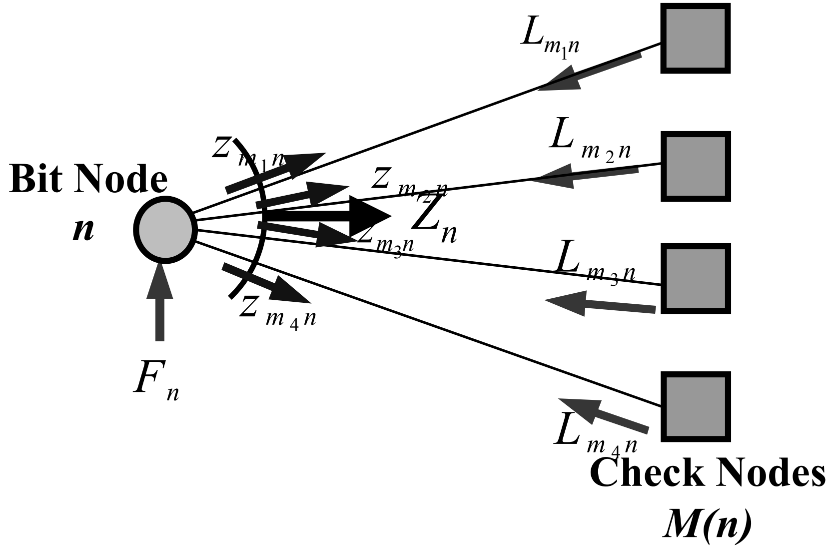


$$L_{mn} = 2\tanh^{-1}\left(\prod_{n' \in N(m)\backslash n} \tanh\left(z_{mn'}/2\right)\right)$$

$$\approx \prod_{n' \in N(m)\backslash n} \operatorname{sgn}(z_{mn'}) \cdot \min_{n' \in N(m)\backslash n} \left|z_{mn'}\right|$$

- Low complexity;
- Independent of channel characteristics for AWGN channels;
- Degradation in performance, especially for geometric LDPC codes.

APP Algorithm
—— simplification in bit node processing

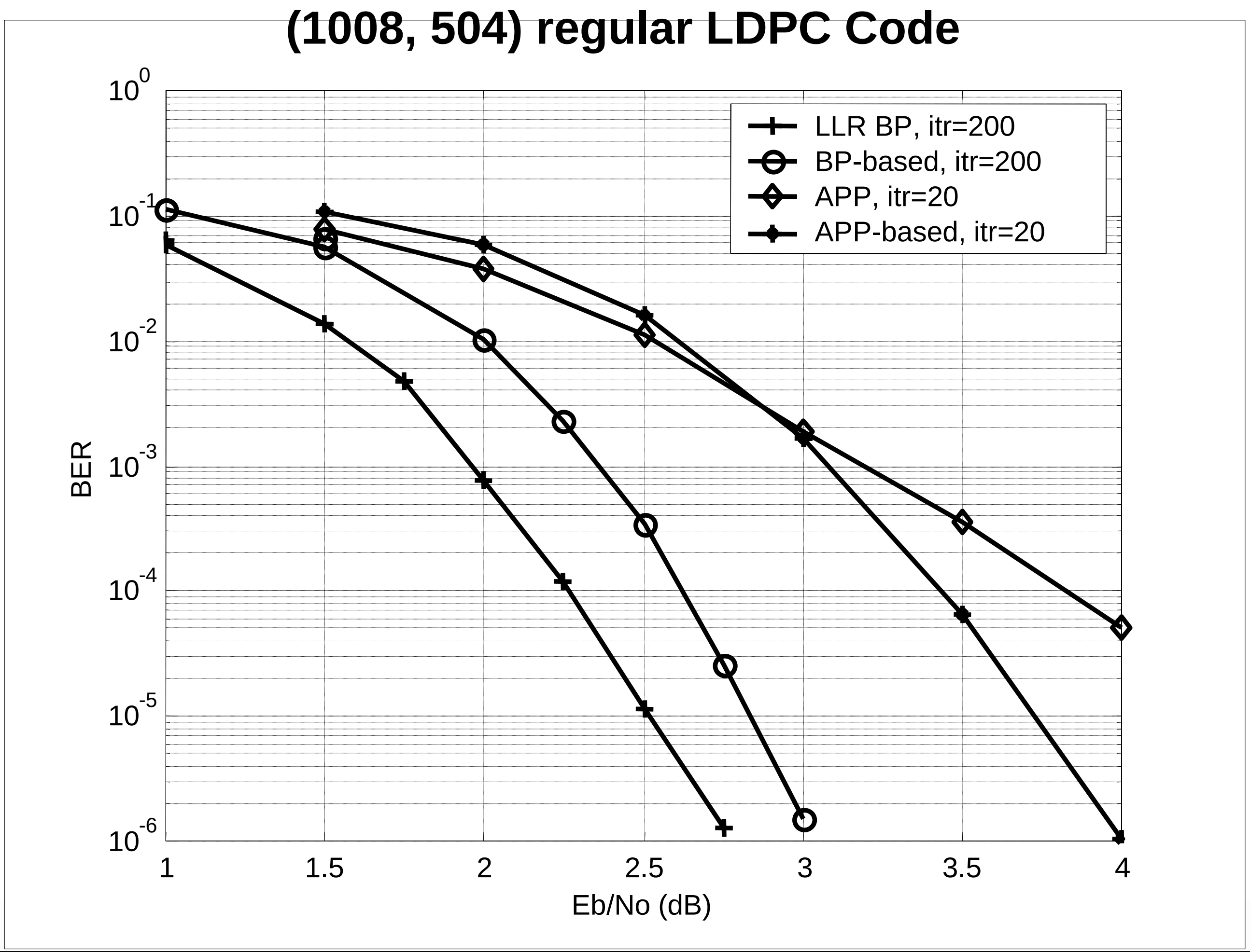


$$Z_n = F_n + \sum_{m \in M(n)} L_{mn}$$

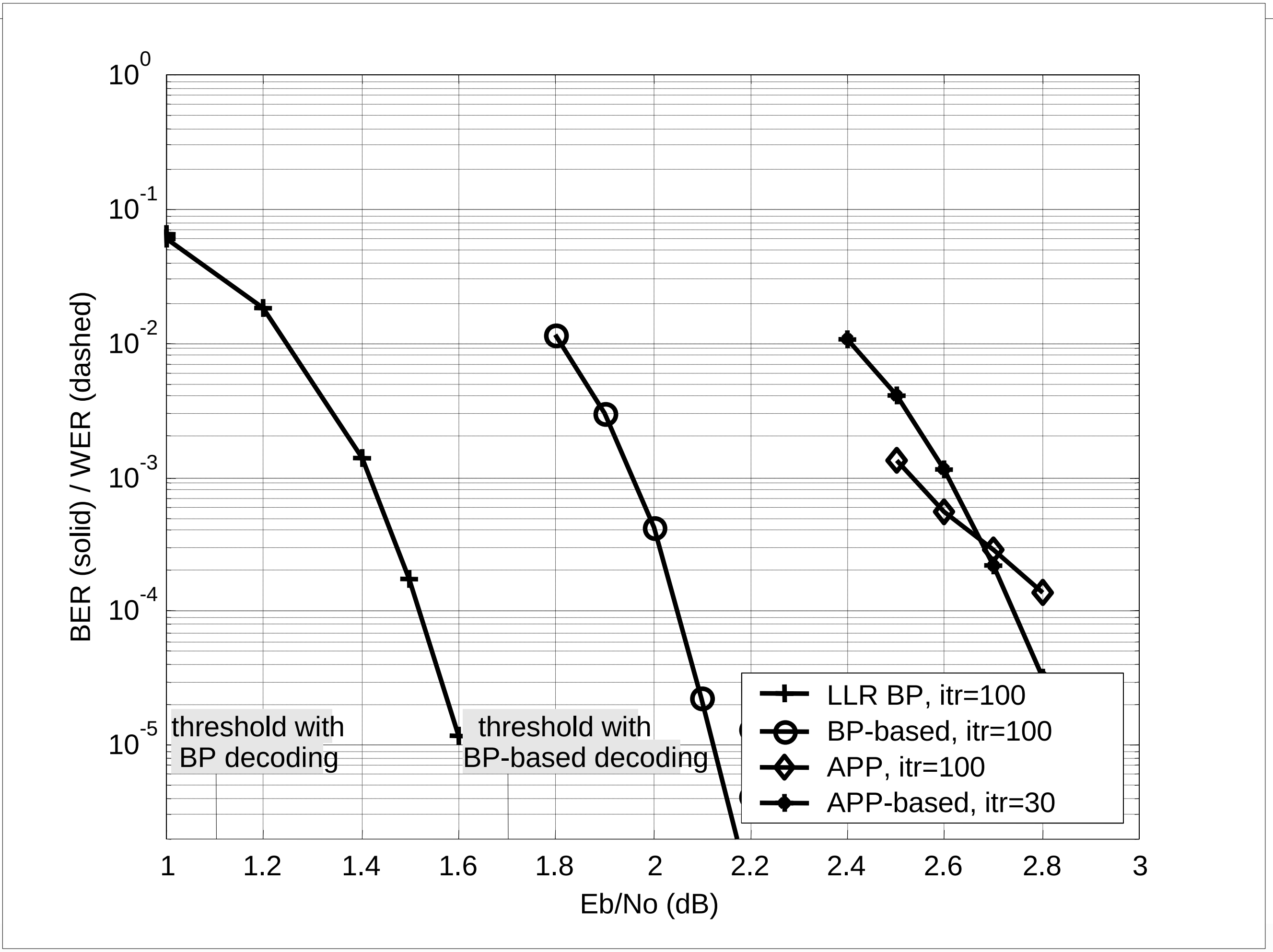
- Z_n is not only for hard decision, but also as a substitution for Z_{mn}.
- Lower computational complexity and storage requirement.
- Introducing correlation in the iterative decoding process.

APP-Based Algorithm —— simplification in both nodes

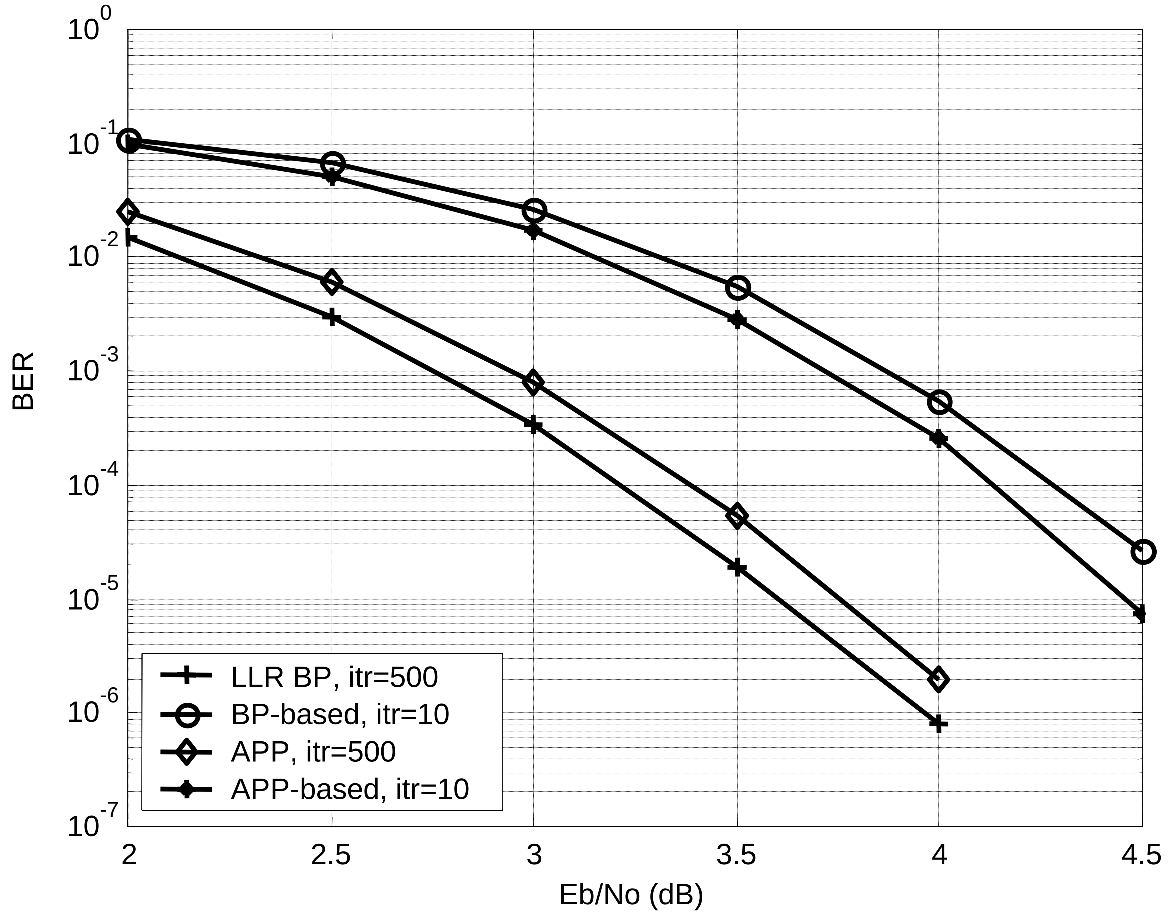
Performance of BP and Its Simplified Versions



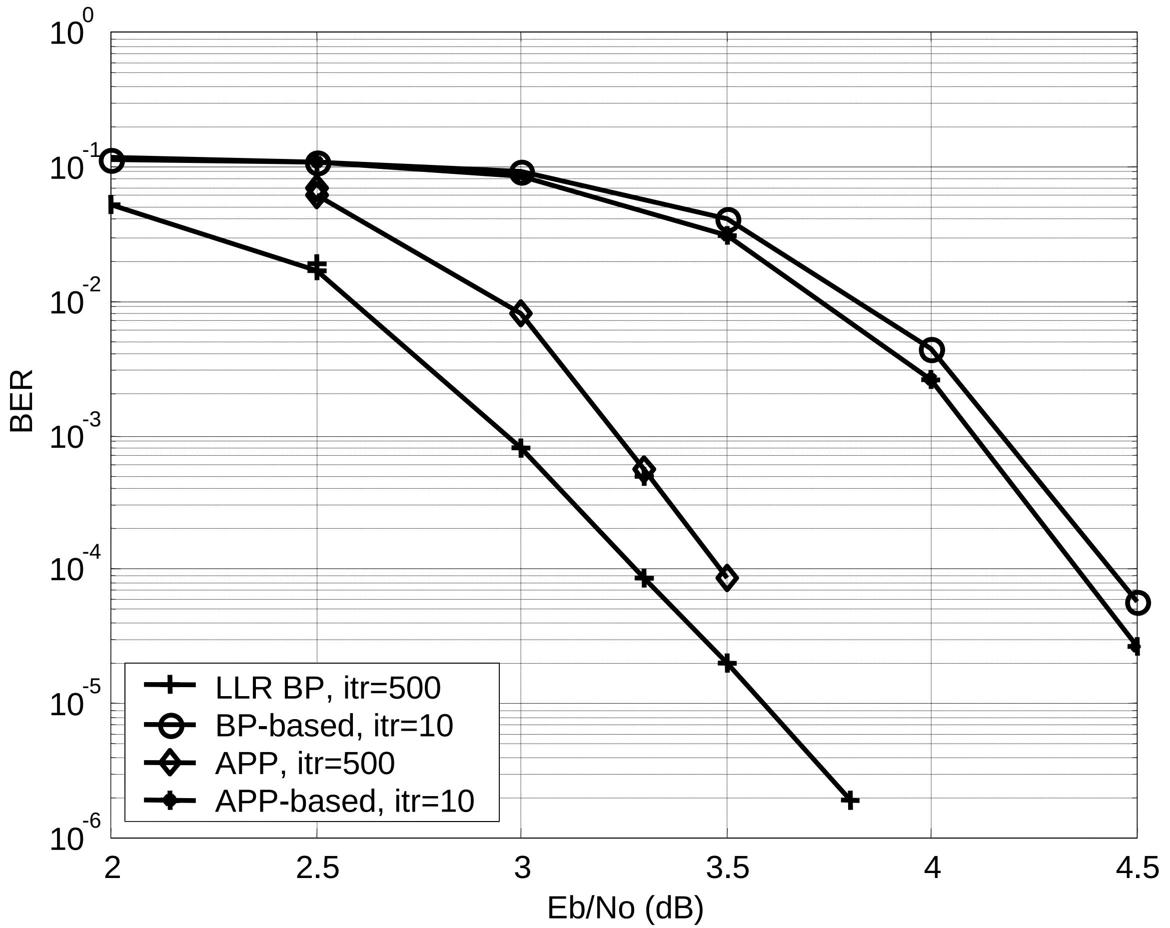
(8000, 4000) Regular LDPC Code



(273, 191) DSC Code

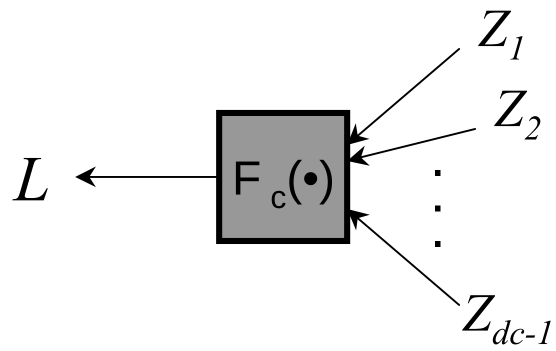


(1057, 813) DSC Code



Improvement of the BP-based algorithm

check node processing in different algorithms



BP:

$$L_1 = 2\tanh^{-1}\left(\prod_i \tanh\left(Z_i/2\right)\right)$$

BP-based:

$$L_2 = \prod_i \operatorname{sgn}(z_i) \cdot \min_i |Z_i|$$

Two statements hold [Chen-Fossorier-COM02]:

1. $\operatorname{sgn}(L_1) = \operatorname{sgn}(L_2)$;
2. $|L_1| < |L_2|$.

Two improvements of the check node processing

Normalized BP-based algorithm:

Divide L_2 by a normalization factor α greater than 1,

$$L_2 \leftarrow L_2 / \alpha \; .$$

Offset BP-based algorithm:

Decreasing $|L_2|$ by a offset value β ,

$$|L_2| \leftarrow \max(|L_2| - \beta, 0) \; .$$

- ❖ Decoder parameters, α's or β's, need to be optimized.

A Simple Way to Determine Normalization Factors [Chen-Fossorier-COM02]

- A good criterion to determine α at a given iteration: Let the mean of $|L_2| / \alpha$ be equal to the mean of $|L_1|$, i.e.,

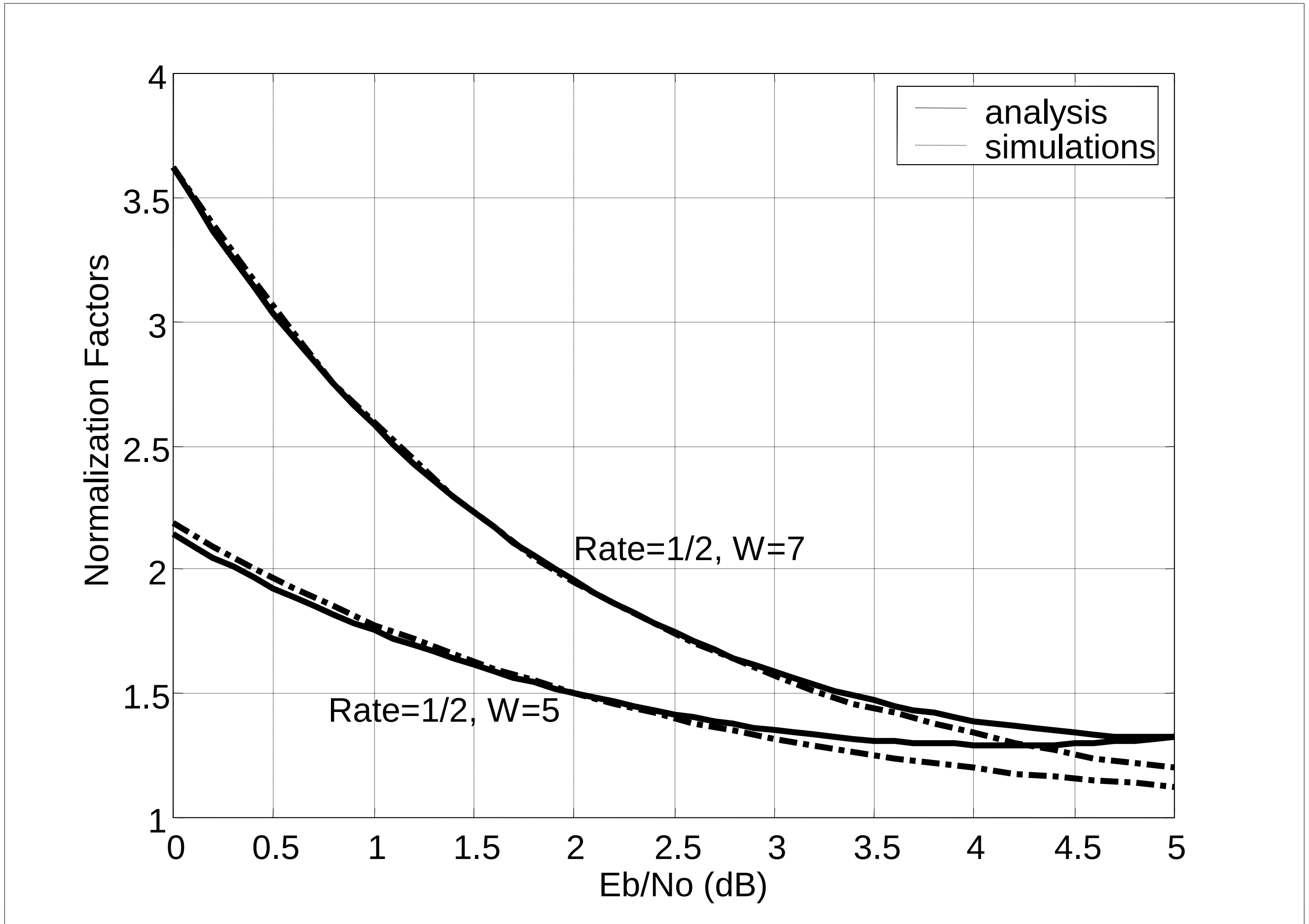
$$\alpha = E(|L_2|) / E(|L_1|)$$

- Normalization factors of the first iteration can be readily determined both by simulation and by the theory of probabilities:

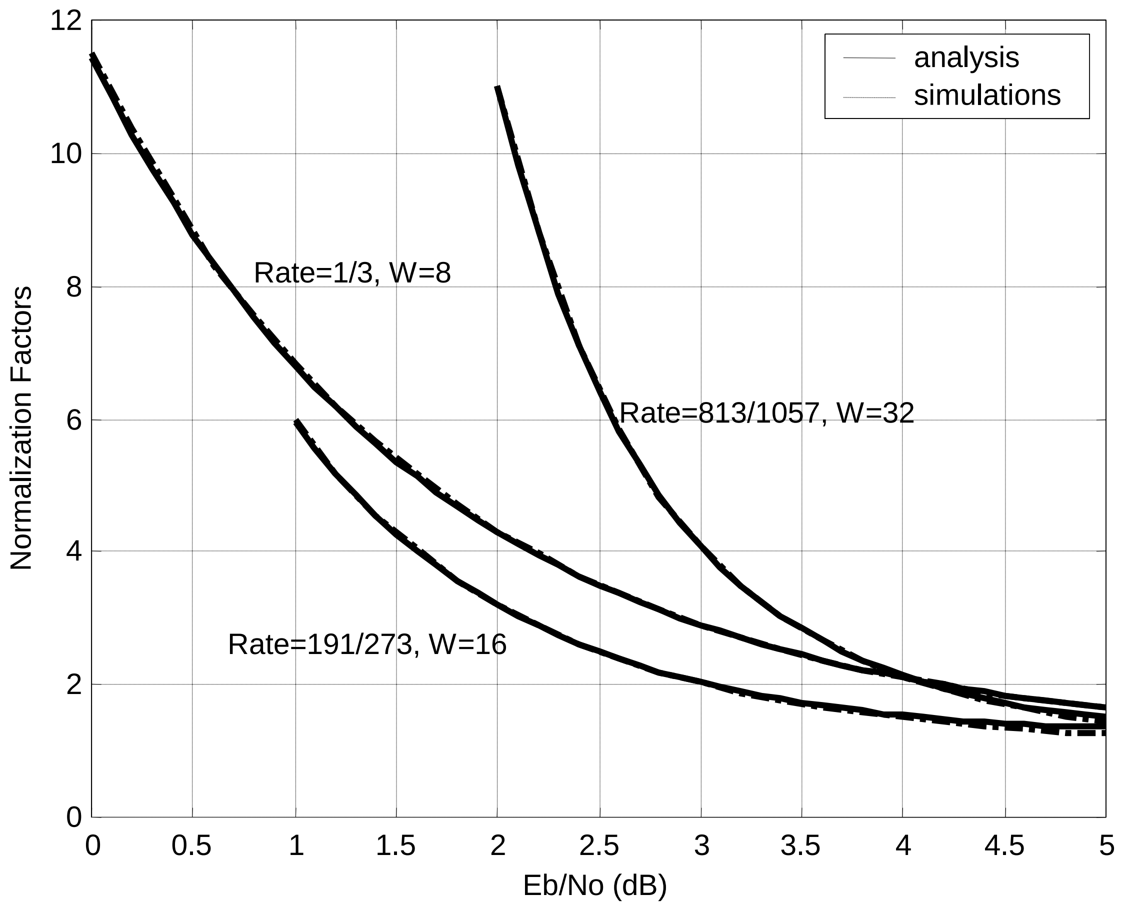
$$E(|L_1|) = E\left(2\tanh^{-1}\left(\prod_i \tanh(|Z_i|/2)\right)\right)$$

$$E(|L_2|) = E\left(\min_i |Z_i|\right)$$

- For a specific code, we can keep the same factor for all the iterations and all SNR values; then the normalized BP-based algorithm becomes independent of the channel characteristics;



analysis
simulations
Rate=1/2, W=7
Rate=1/2, W=5
Normalization Factors
Eb/No (dB)
4
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


analysis
simulations
Rate=1/3, W=8
Rate=813/1057, W=32
Rate=191/273, W=16
Normalization Factors
Eb/No (dB)
0
2
4
6
8
10
12
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

- Normalization factors determined for 3 important DSC codes:

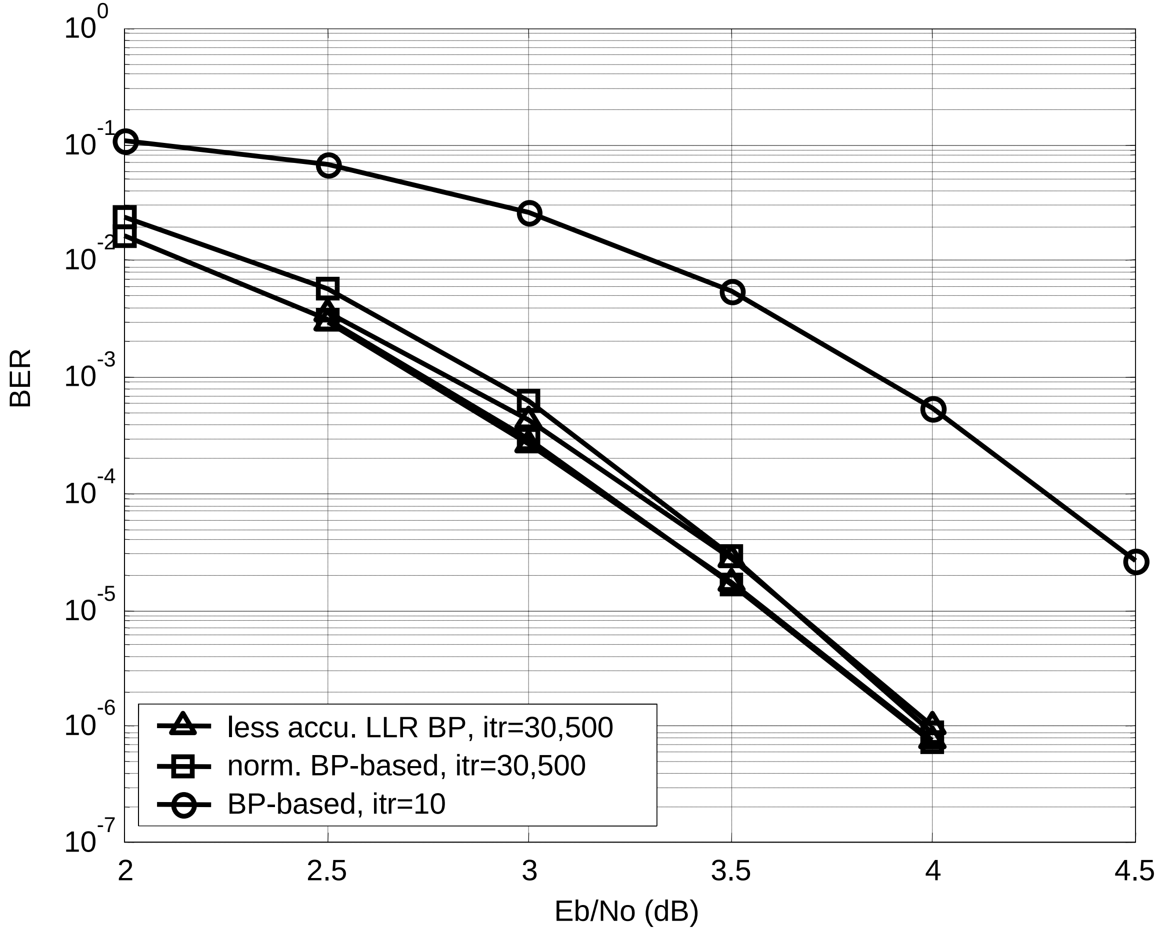
(273, 191) DSC $\rightarrow$ 2.0

(1057, 813) DSC $\rightarrow$ 4.0

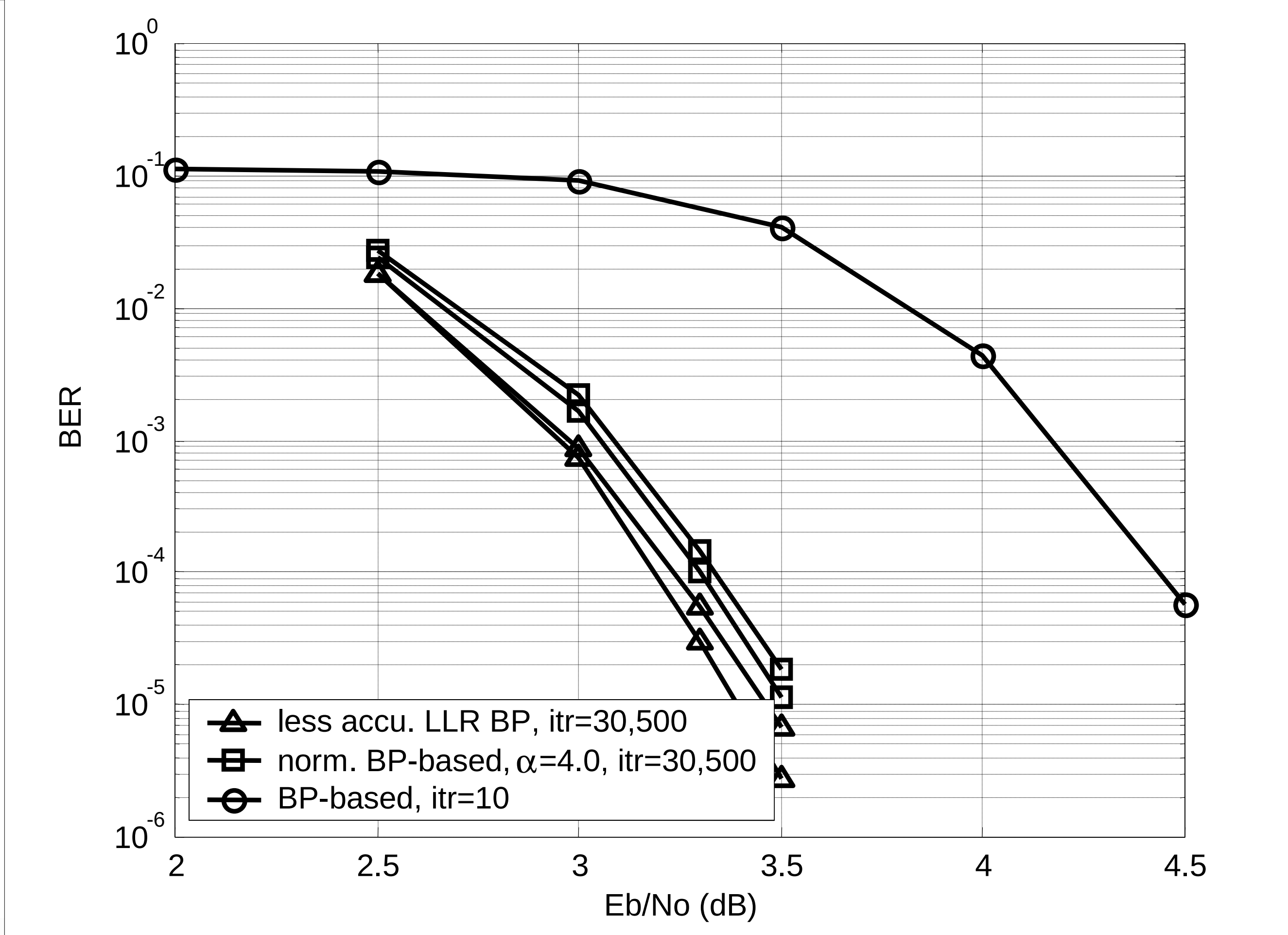
(4161, 3431) DSC $\rightarrow$ 8.0

Normalization can be easily implemented by register shift!

(273, 191) DSC Code, α=2.0



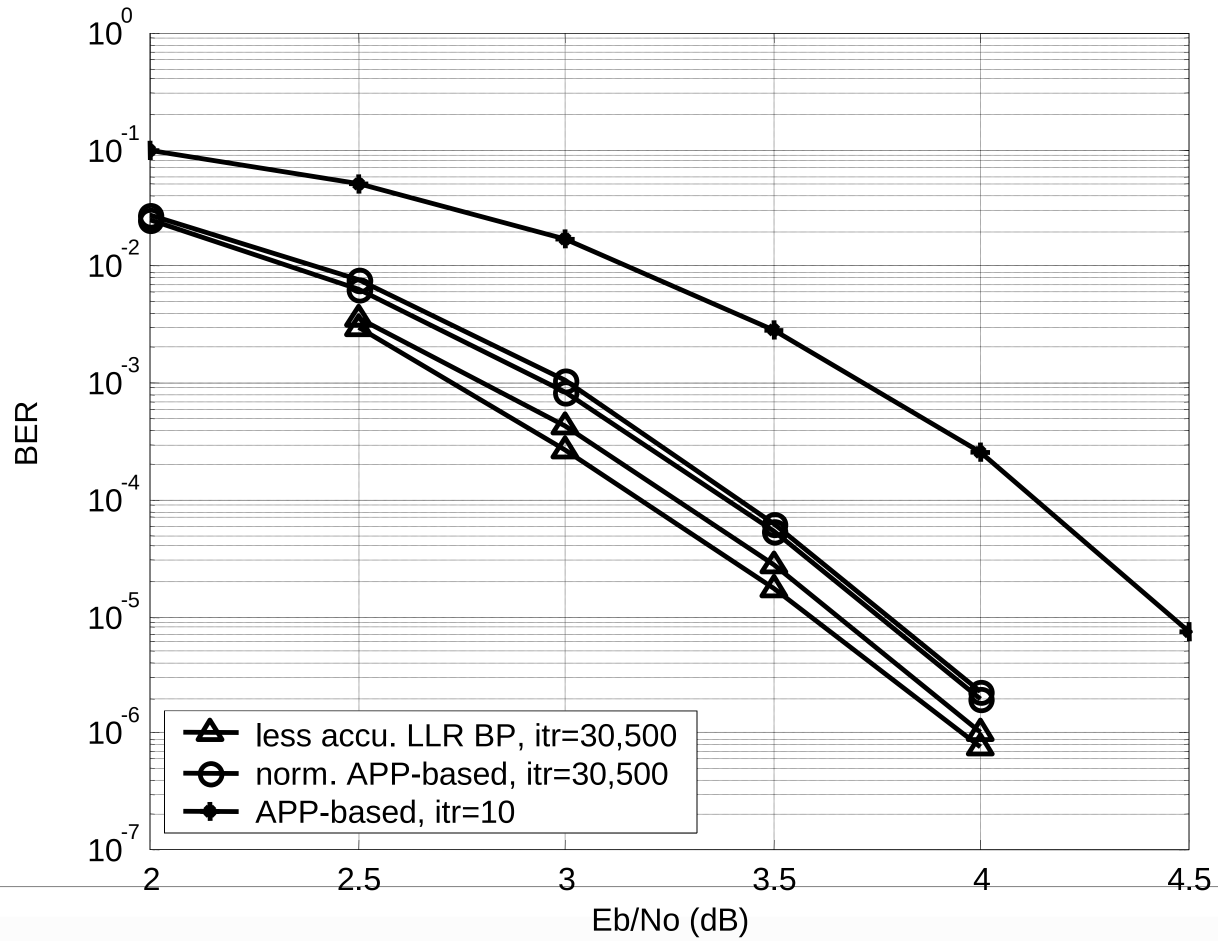
(1057, 813) DSC Code, α=4.0



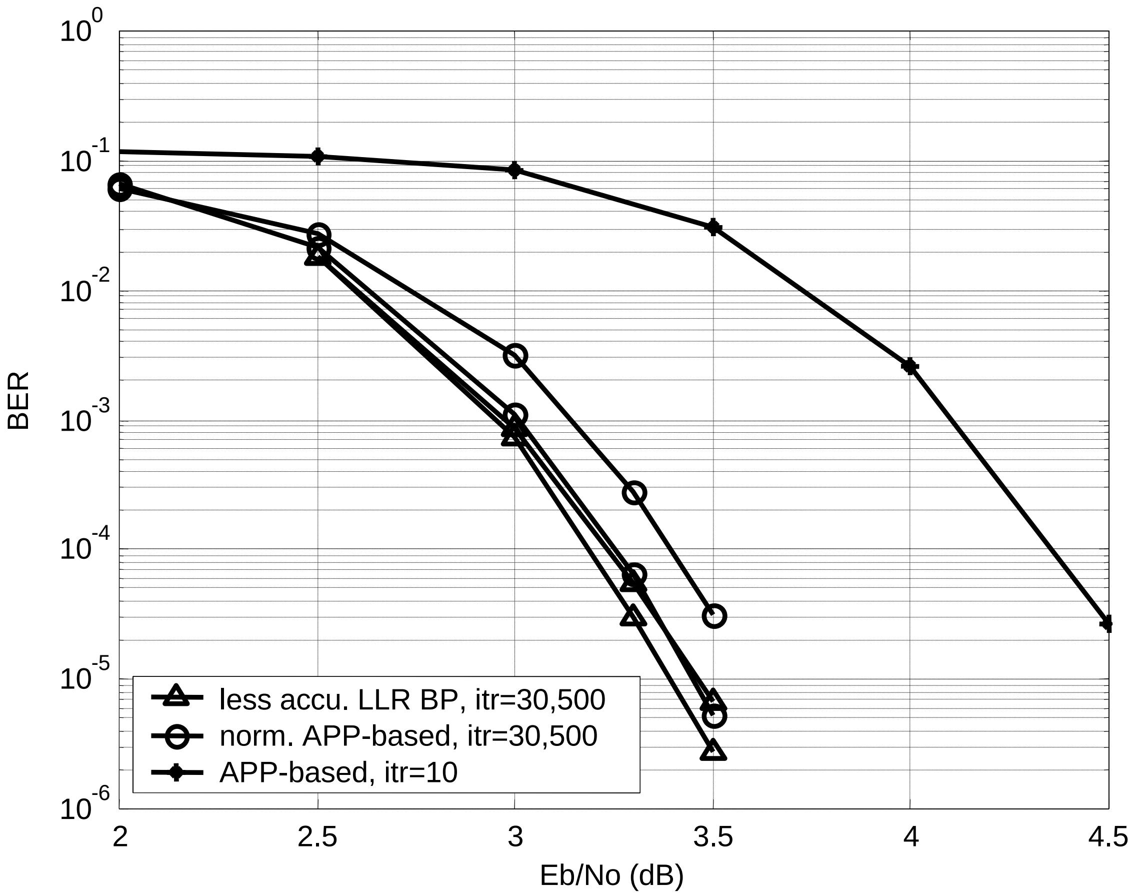
Normalized APP-based algorithm

- APP-based algorithm + normalization in check nodes
  - $\Rightarrow$ normalized APP-based algorithm
- Working well for geometric LDPC codes.

(273, 191) DSC Code



(1057, 813) DSC Code



Optimizing Decoder Parameters by Density Evolution

- **Density evolution** (DE) is a powerful tool to analyze message-passing algorithms of LDPC codes [Richardson-IT01].
- Assumptions:

  (1) symmetric channels (BSC, AWGN, ……);

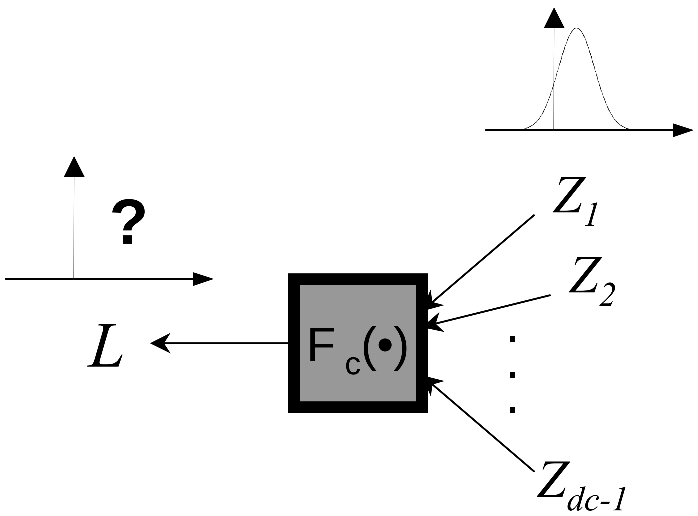
  (2) decoder symmetry;

  (3) all-0 sequences transmitted;

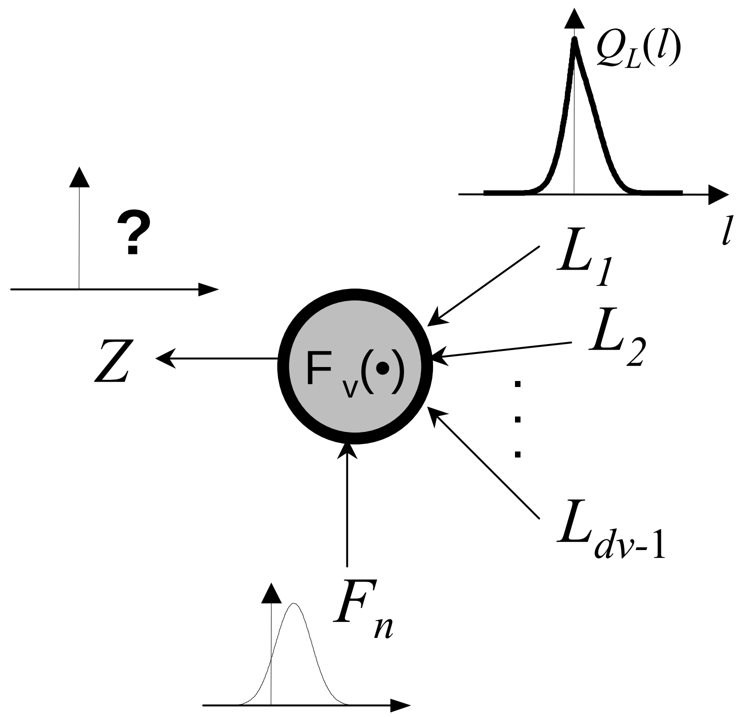
  (4) infinite code length --- loop free.
- Basic idea: numerically derive the probability density functions (pdf) of the messages from one iteration to another, based on decoding algorithms, and then determine the bit error rate.

Density evolution algorithms

Check node processing:



Bit node processing:



Density evolution algorithms for BP and BP-based algorithms

(1) In bit nodes: *SAME*

- Only additions involved in both alogrithms.
- The output pdf is the convolution of the input pdf's.
- Can use FFT to speed up the computation.

(2) In check nodes: *DIFFERENT*

Due to different ways of processing

BP: $$L = 2\tanh^{-1}\left(\prod_i \tanh\left(Z_i/2\right)\right)$$

BP-based: $$L = \prod_i \operatorname{sgn}(Z_i)\cdot \min_i |Z_i|$$

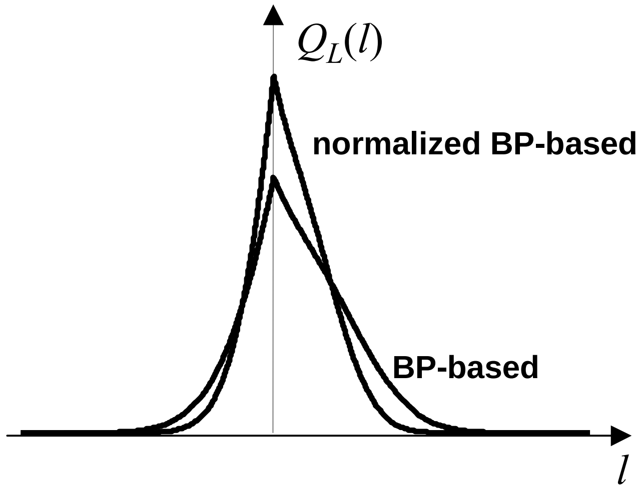
DE for normalized and offset BP-based algorithms

[Chen-Fossorier-CL02]

- Slightly modify the DE algorithm of the BP-based algorithm.

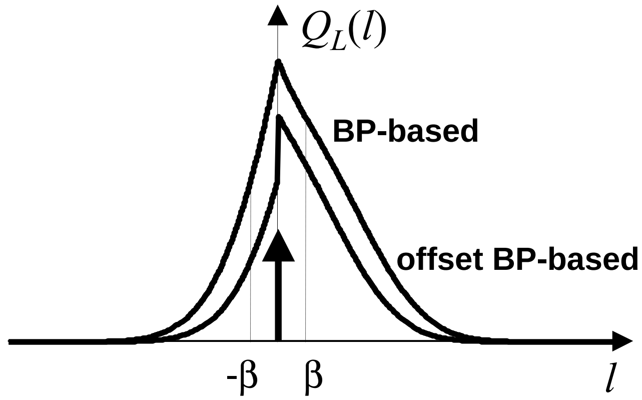
- Normalized BP-based

$$L \leftarrow L/\alpha$$
$$Q_L(l) \leftarrow \alpha\, Q_L(\alpha \cdot l)$$



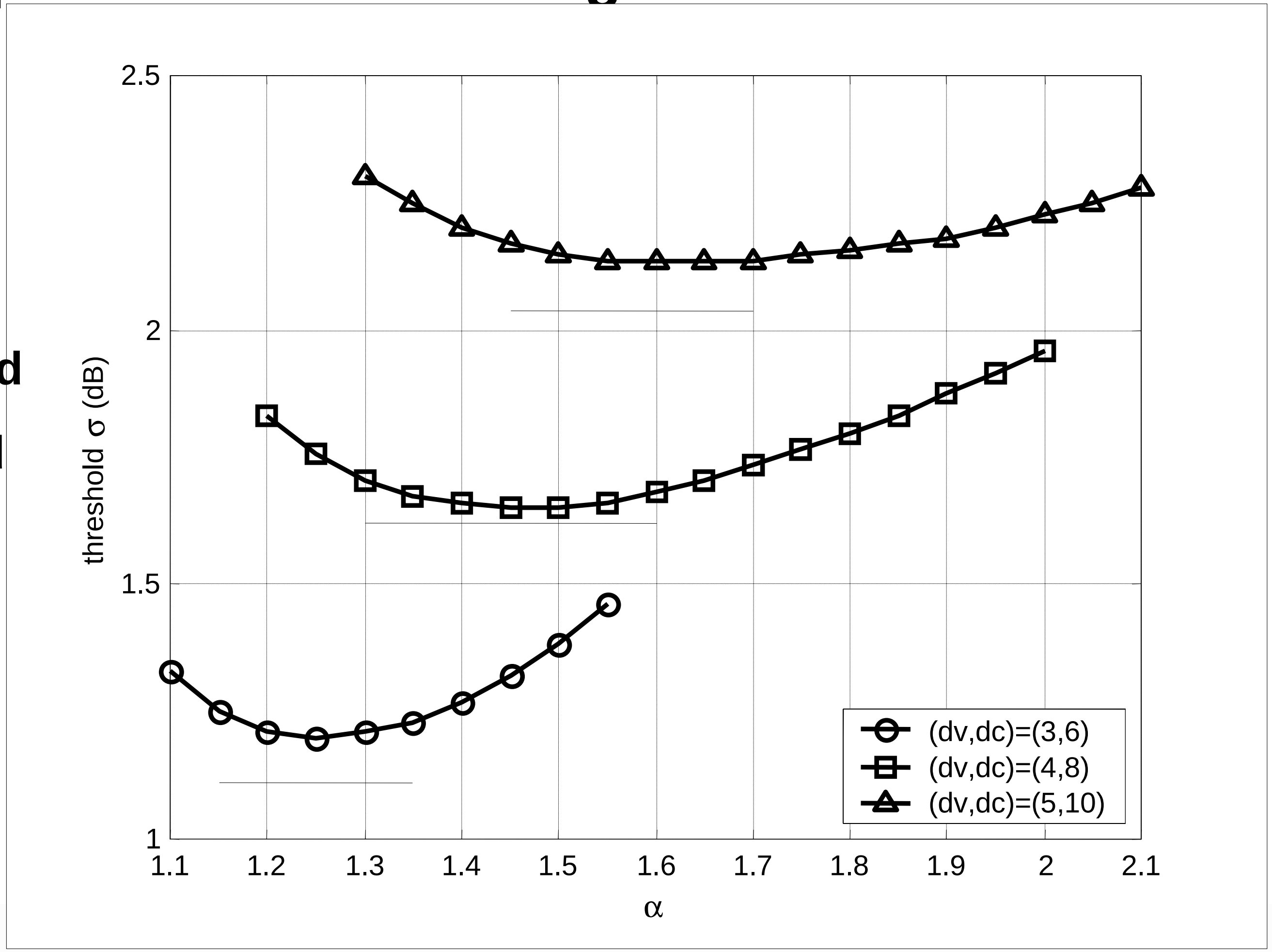
- Offset BP-based

$$|L| \leftarrow \max(|L| - \beta, 0)$$
$$Q_L(l) \leftarrow u(l)\, Q_L(l+\beta) + u(-l)\, Q_L(l-\beta) + \delta(l)\int_{-\beta}^{\beta} Q_L(l)\,dl$$

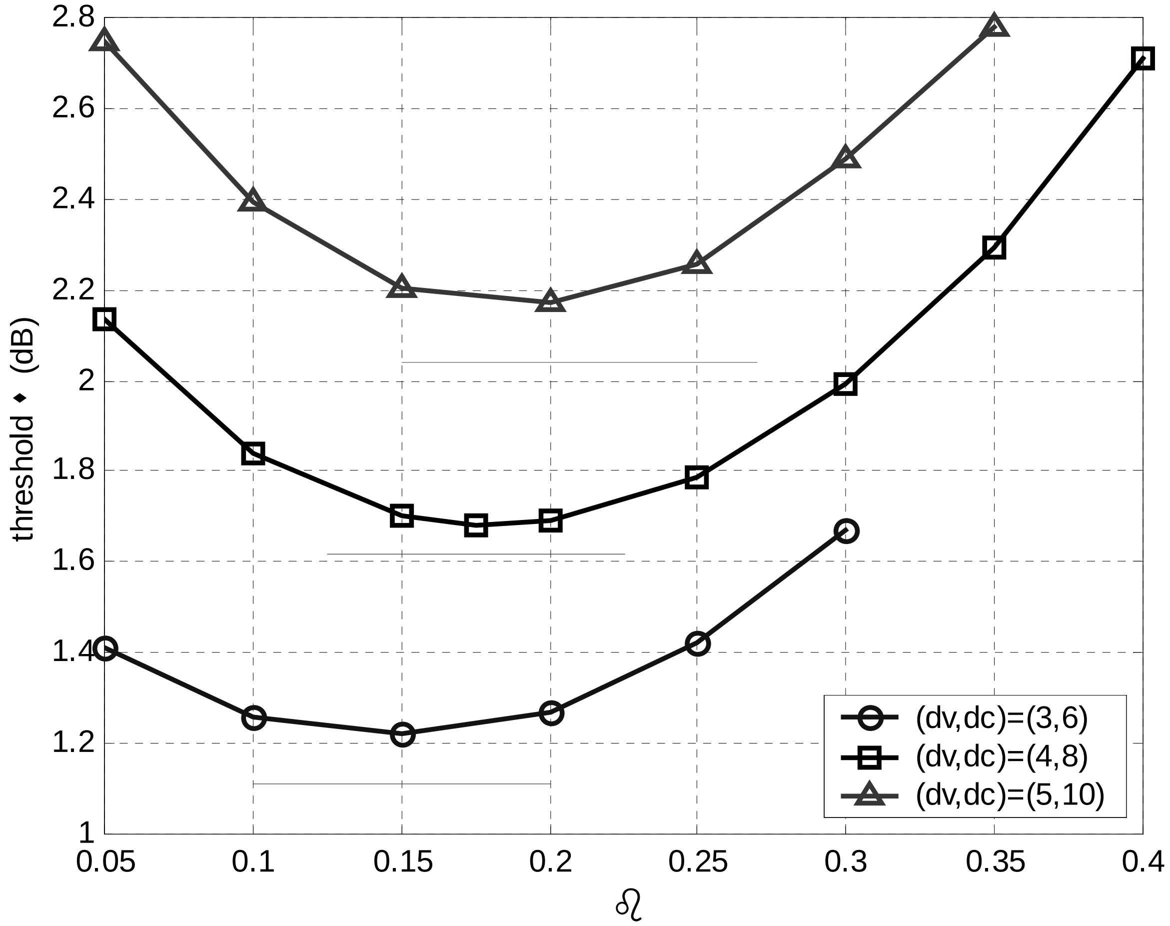


Applying DE to Find Best Decoder Parameters for Improved BP-Based Algorithms

Normalized BP-based



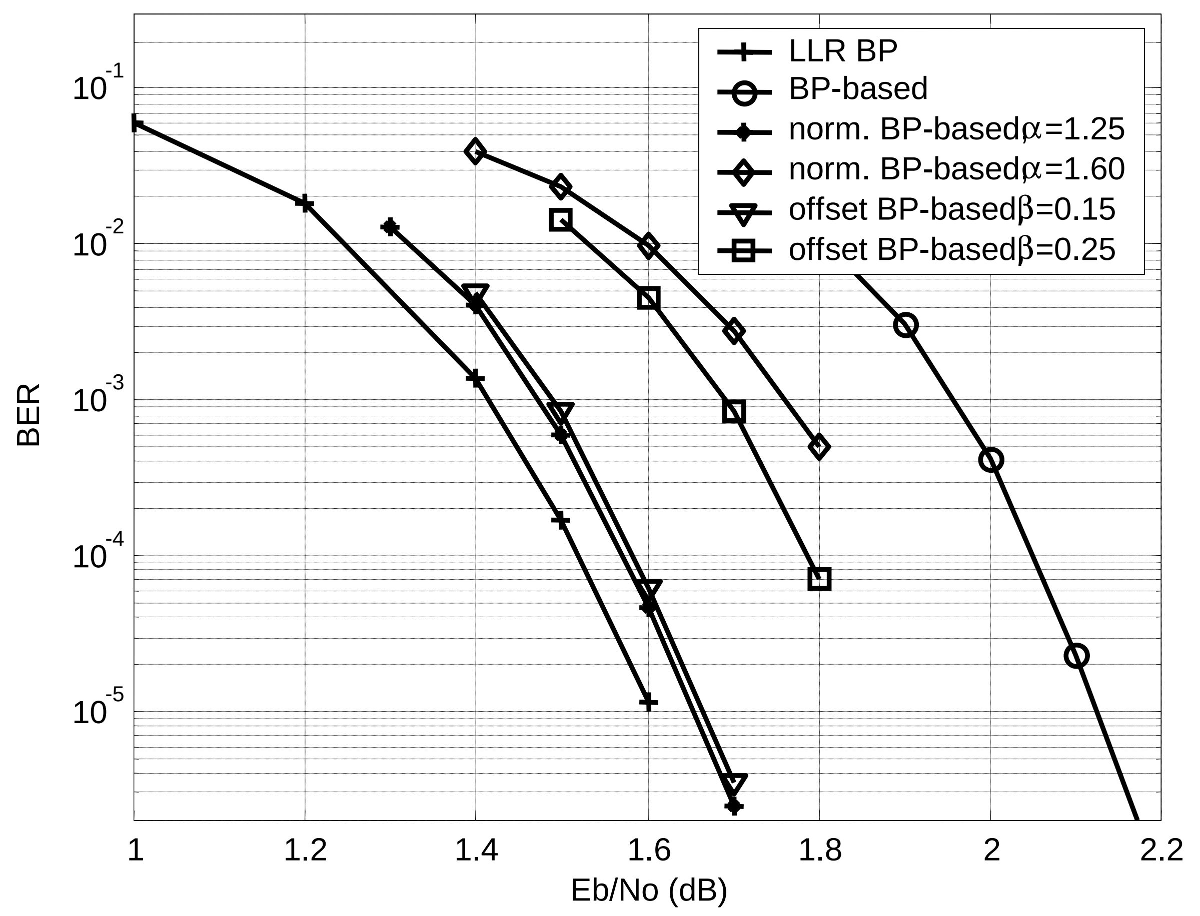
Offset BP-based



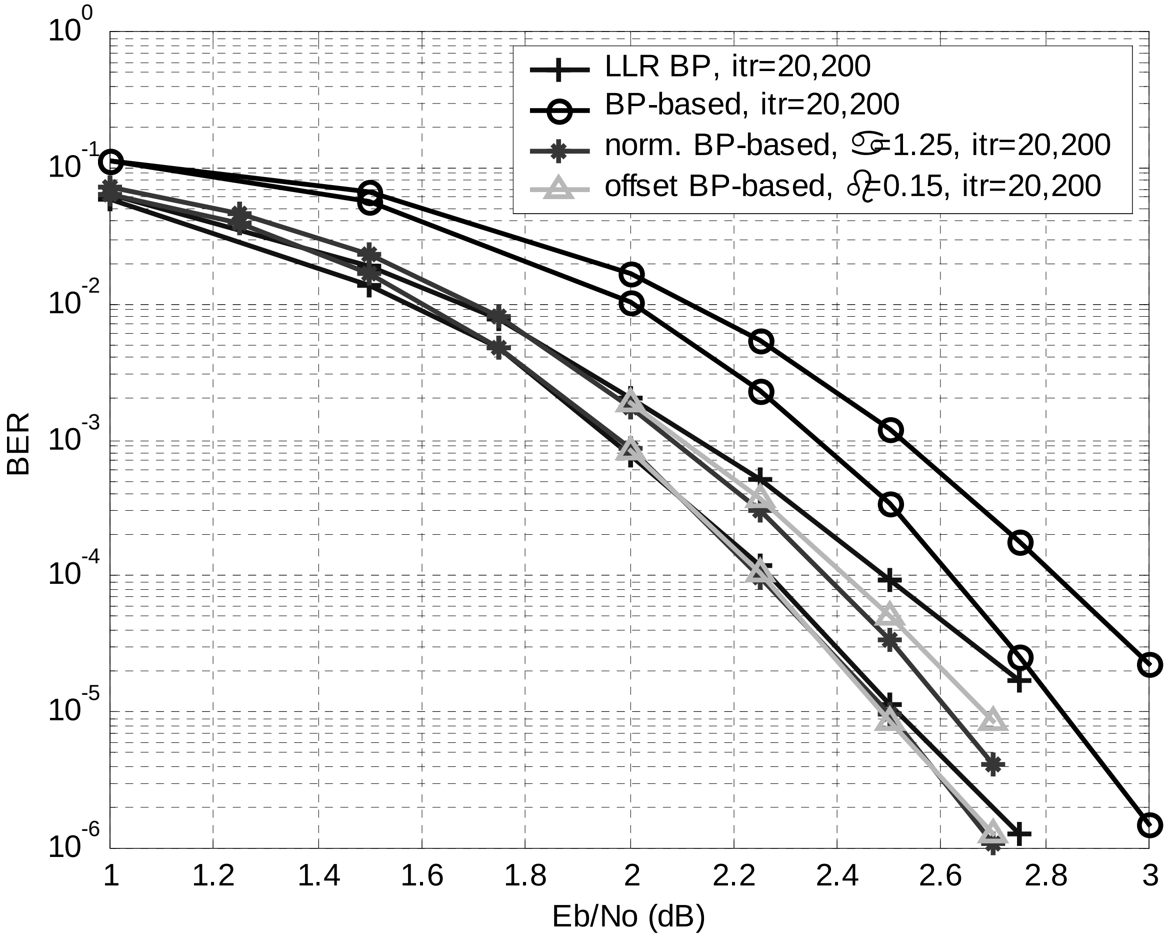
Thresholds (in dB) for various decoding algorithms.

(d_v,d_c)	rate	BP	BP-based	Normalized BP-based		Offset BP-based	
				a	s	ß	s
(3,6)	0.5	**1.11**	**1.71**	1.25	**1.20**	0.15	**1.22**
(4,8)	0.5	**1.62**	**2.50**	1.50	**1.65**	0.175	**1.70**
(5,10)	0.5	**2.04**	**3.10**	1.65	**2.14**	0.2	**2.17**
(3,5)	0.4	**0.97**	**1.68**	1.25	**1.00**	0.2	**1.03**
(4,6)	1/3	**1.67**	**2.89**	1.45	**1.80**	0.25	**1.84**
(3,4)	0.25	**1.00**	**2.08**	1.25	**1.11**	0.25	**1.13**

An (8000, 4000) LDPC code, (dv, dc)=(3,6), 100 iterations.



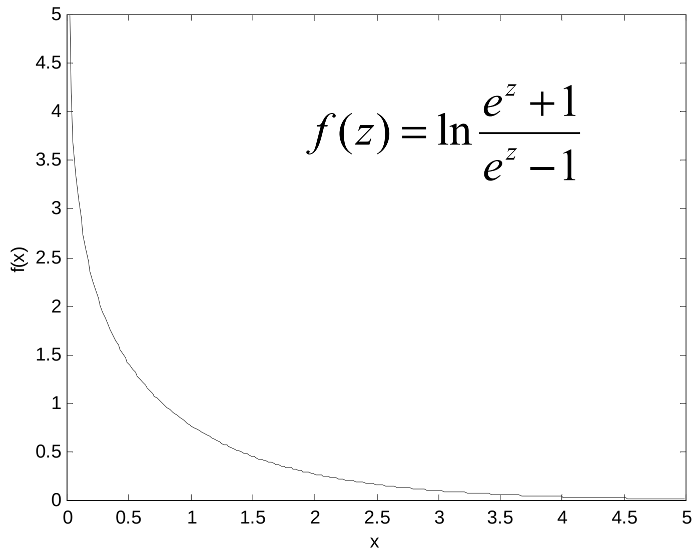
A (1008, 504) , regular LDPC code, (dv, dc)=(3,6)



Hardware Implementation of BP Algorithm

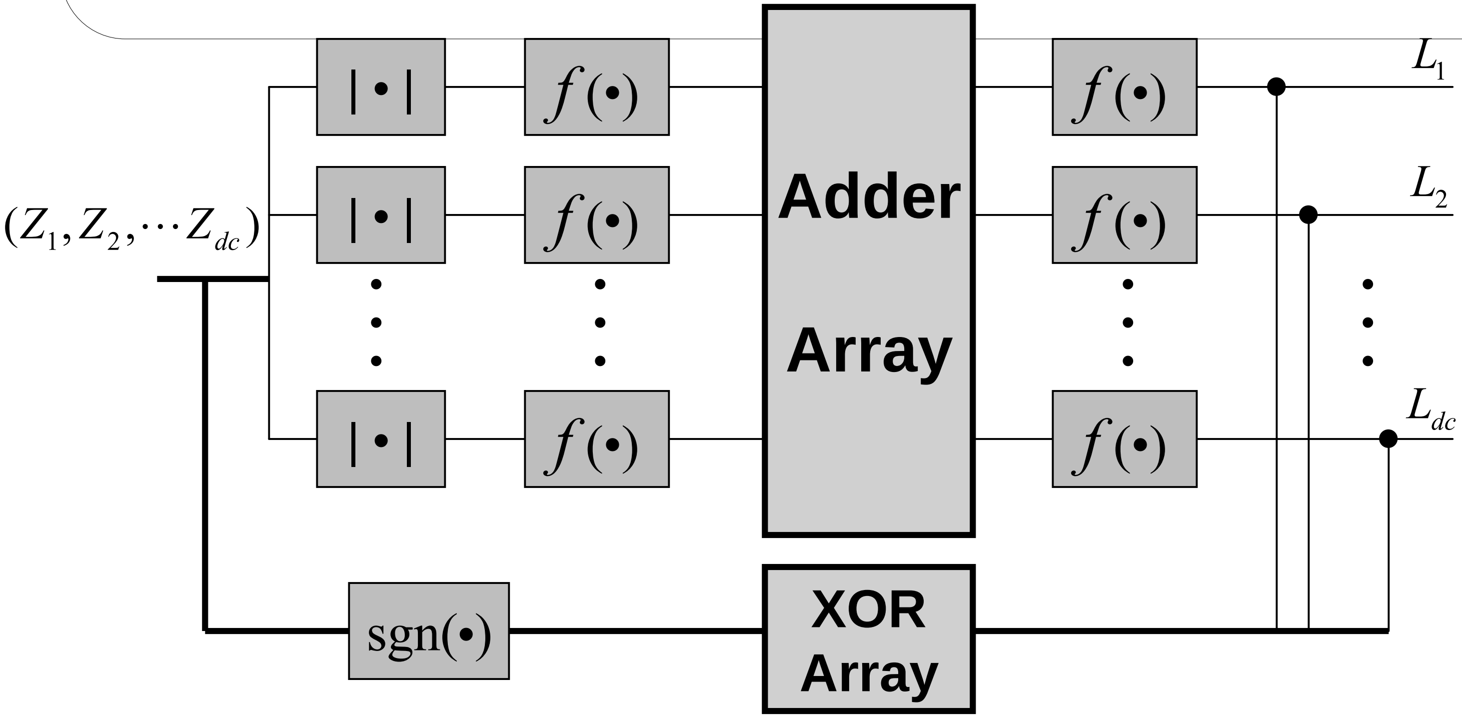
$$L = 2\tanh^{-1}\left(\prod_i \tanh(Z_i/2)\right)$$

$$= \prod_i \operatorname{sgn}(Z_i) \cdot f\left(\sum_i f(|Z_i|)\right)$$



- $f(z)$ can be implemented by look-up table (LUT).
- Only need two kinds of operations: LUT and additions.

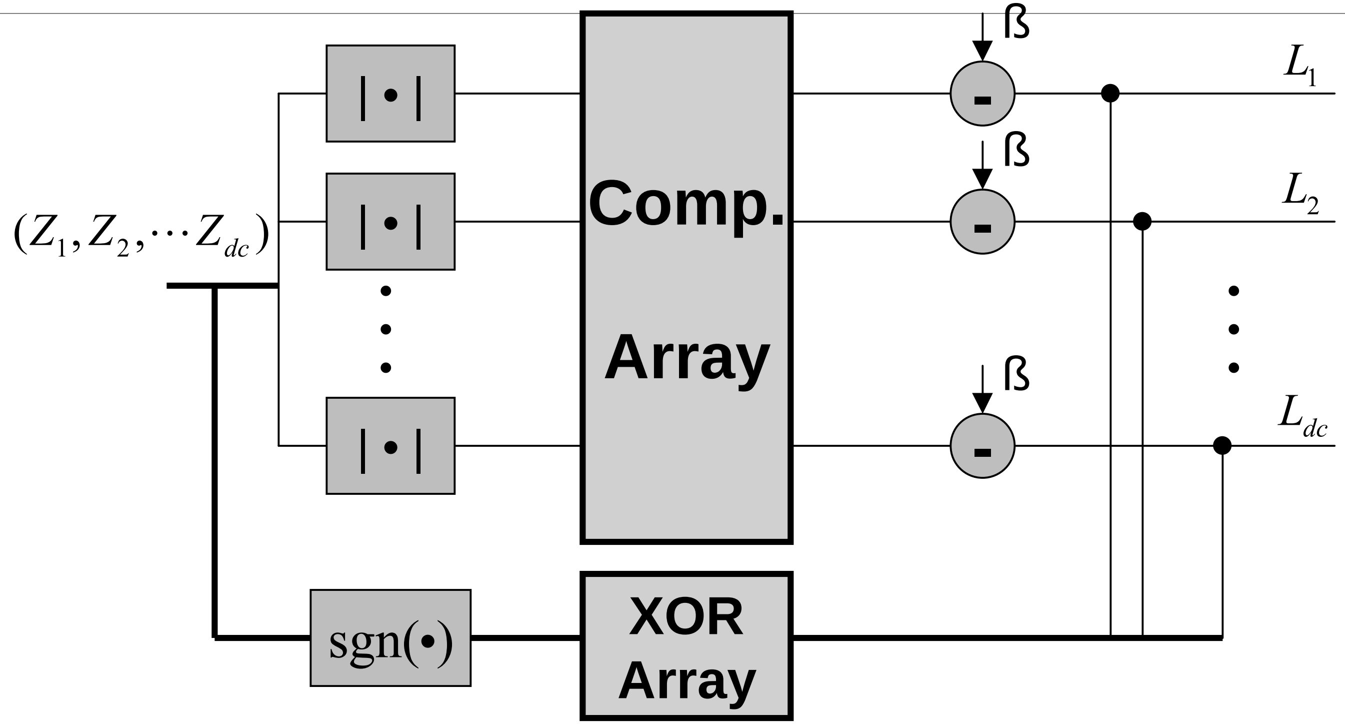
Check node implementation of BP algorithm



Computational complexity in each check node:

- $3(dc\text{-}1)$ additions
- $2dc$ LUT's

Check node implementation of BP-based algorithm and improved versions

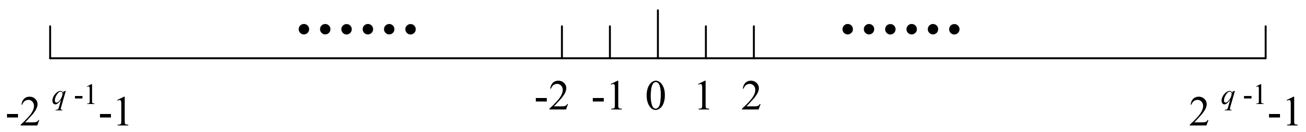


Computational complexity in each check node:

- 3(dc-1) comparisons
- dc offsetting for offset BP-based algorithm

Quantization Effects

***q*-**bit quantization

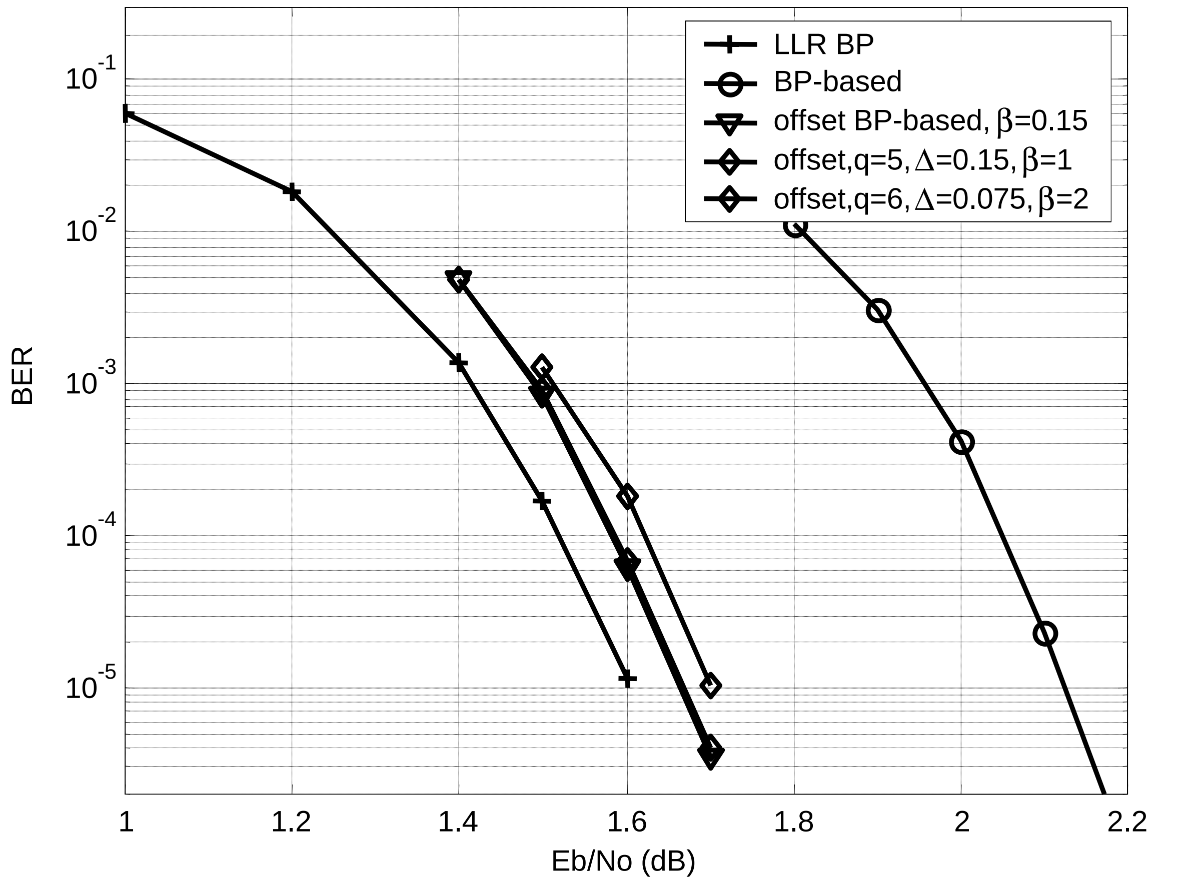


Density evolution algorithms for the BP-based and the normalized BP-based algorithm can be extended to quantized cases.

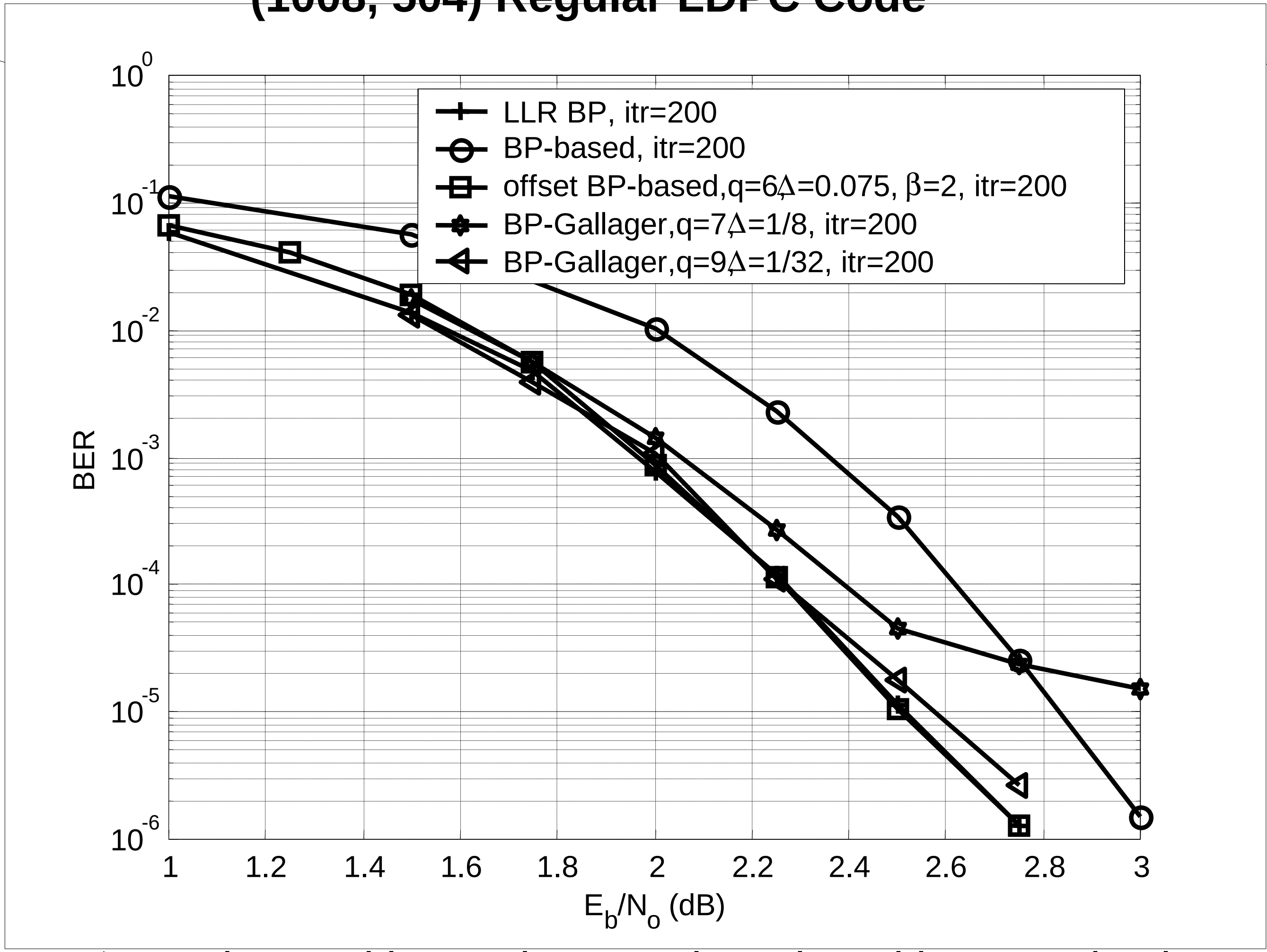
Thresholds for quantized offset BP-based decoding with (dv,dc)=(3,6).

q	?	ß	thresholds(dB)
5	0.15	1	1.24
5	0.075	2	1.60
6	0.15	1	1.24
6	0.075	2	1.22
7	0.15	1	1.24
7	0.075	2	1.22
7	0.05	3	1.22

An (8000, 4000) , regular LDPC code, (dv, dc)=(3,6)

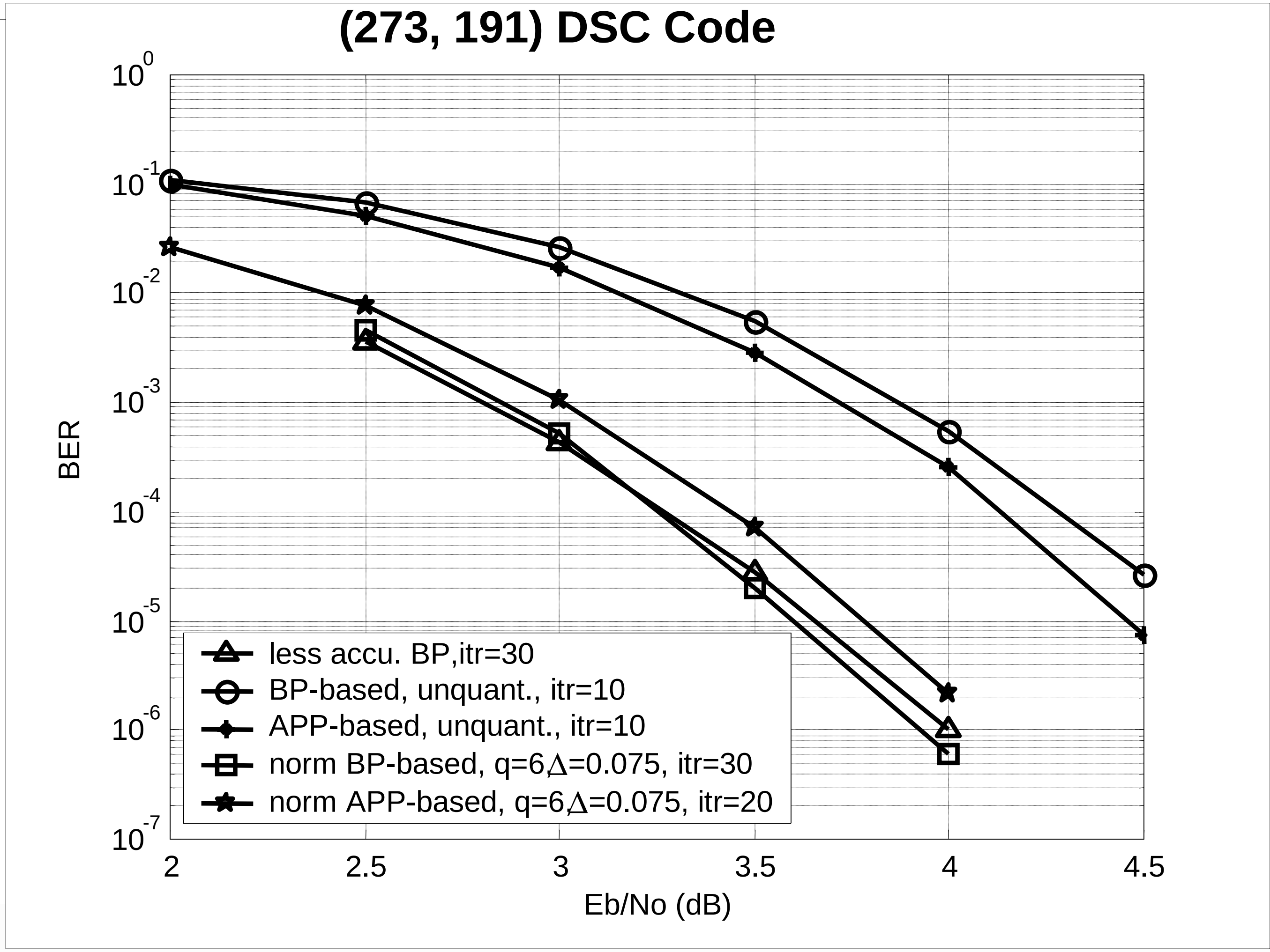


(1008, 504) Regular LDPC Code



❖ BP is sensitive to the error introduced by quantization.

Quantized Decoding Algorithms for DSC Codes



(1057, 813) DSC Code