Performance estimation of 8-PSK turbo-coded modulation over Rayleigh fading channels

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Abstract: *We present new estimation techniques that make it possible to predict the asymptotic performance of turbo-coded modulation over Rayleigh fading channels. These techniques require the knowledge of the minimum Hamming distance of the turbo code. The Error Impulse Method provides this information. The derived expressions present two main advantages over previous work: they are valid for any kind of code interleaver and do not require any information about the component codes.*

Keywords: coded modulation, union bound, fading channel, bit-interleaving, turbo code

1. INTRODUCTION

Transmission in bandwidth-constrained channels can be error-protected if a coded-modulation technique is used. These techniques make channel coding possible without expanding the signal bandwidth. The most popular of these techniques is Trellis-Coded Modulation (TCM), introduced by Ungerboeck [1]. As TCMs attracted the interest of many researchers, they led to considerable research [2]: theory formalization, performance bounds and the search for new TCM schemes.

The evaluation of performance bounds for TCMs in fading channels was considered in [3], where a Chernoff bound was applied to upper-bound the error probability. However, this bound (commonly used because of its simplicity) is too loose for most of the signal-to-noise ratio values, especially for the Rayleigh fading channel. In [4], Slimane *et al* presented new tight bounds for Rayleigh fading channels. Finally, the exact union bound for TCMs over fading channels was evaluated in [5].

Turbo-coded modulation is an alternative to TCM. Different approaches to turbo (trellis)-coded modulation were presented in [6], [7], [8]. Before [9], the only method used to study the performance of turbo-coded modulations over fading channels was computer Monte-Carlo simulations. In [9], the authors presented a methodology to derive bounds for specific turbo-coded modulation schemes. This method requires the knowledge of the weight enumerating function of the component codes. Unfortunately, it is only valid when the interleaver in the turbo code is uniform (i.e. statistically uniform).

An original new method for computing minimum distances of linear codes, in particular turbo codes, was presented in [10]. This method, called the Error Impulse Method (EIM), provides an estimation of the distance spectrum of the turbo code. The prediction of performance at low error rates of turbo code-BPSK/QPSK associations is then straightforward for Gaussian channels. In [11] and [12], the EIM was applied to estimate the performance of bit-interleaved turbo codes associated with high-order modulations over the Gaussian channel.

In this paper, we present the derivation of three expressions to estimate the performance of bit-interleaved turbo-coded modulation at low error rates over the Rayleigh fading channel. Our method is valid for any kind of code interleaver, as it is based on the application of the EIM (in fact, the EIM is a powerful tool when designing good interleavers for turbo codes). Moreover, no information about the component codes is required.

Throughout the paper, we adopt a slow-fading memoryless non-selective channel model. The derived expressions will be particularized for the Rayleigh channel. Coherent detection, maximum-likelihood (ML) decoding and ideal Channel State Information (CSI) are assumed in our study.

The paper is organized as follows. Section 2 presents the principle of the transmission scheme. In Section 3, we present the estimations on the error probability, that are a function of the effective distance of the code and the product distance of the coded modulation. In Section 4, we state the hypotheses that will enable us to apply the EIM to our scheme. The final computable expressions are also given. Finally, Section 5 gives some examples of comparisons between the simulation curves and the estimations.

2. TRANSMISSION SCHEME

We consider the transmission scheme depicted in Fig. 1. This scheme follows the principle of the pragmatic approach [6], together with the principle of Bit-Interleaved Coded Modulation (BICM) [13]. Our scheme contains a turbo code, a random bitwise interleaver and a memoryless 8-PSK modulator. In the emitter, the information sequence is turbo encoded before being bitwise interleaved. The bit-interleaver (π) is assumed to be ideal (i.e. infinite depth and completely random). Its purpose is to break the sequential fading correlations and increase the diversity order to the minimum Hamming distance of the turbo code. We consider an 8-PSK modulation that follows a classical Gray mapping. The constellation and mapping are presented in Fig. 2.

The theoretical study assumes ML decoding. However, in our simulations the decoding performs a symbol-to-bit LLR calculation, followed by bit deinterleaving $(\pi)^{-1}$ and a turbo decoder that uses the Max-Log-MAP algorithm.

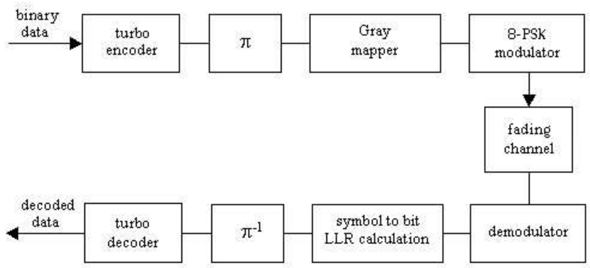

Figure 1: Transmission scheme

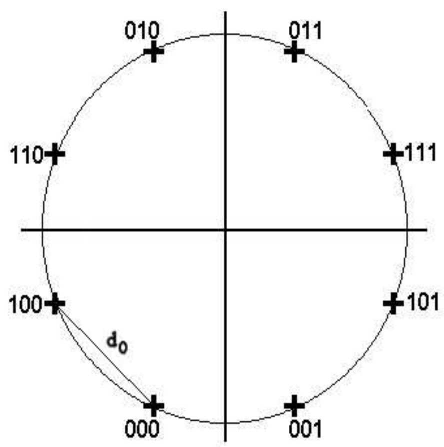

Figure 2: 8-PSK constellation with Gray mapping

3. UNION BOUND FOR TURBO-CODED MODULATIONS

3.1. Notation

In our transmission scheme, the input bits are turbo encoded and then mapped to produce a sequence of signals $\mathbf{s}_l = (s_1, s_2, ..., s_l)$, where l is the number of symbols in the sequence and each signal s_i is a two-dimensional vector chosen from the 8-PSK signal set (see Fig. 2).

Let $\mathbf{s}_l = (s_1, s_2, \cdots, s_l)$ be the transmitted sequence and $\mathbf{r}_l = (r_1, r_2, \cdots, r_l)$ the received sequence. The decoder makes an error if it decodes $\hat{\mathbf{s}}_l = (\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_l)$ instead of $\mathbf{s}_l$ (if $\hat{\mathbf{s}}_l \neq \mathbf{s}_l$). The received signal at time i can be written as:

$$r_i = a_i \cdot s_i + n_i \tag{1}$$

where a_i is the amplitude of the fading process and n_i is a sample of a zero-mean complex Gaussian noise process with variance $N_0/2$. The *pairwise error probability*, denoted by $P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l)$, is the probability that the decoder chooses $\hat{\mathbf{s}}_l$ instead of $\mathbf{s}_l$.

3.2. Bounds on the pairwise error probability

We denote by Q the set of all i for which $s_i \neq \hat{s}_i$ and by l_Q the cardinal of Q. If we use a Chernoff bound, $P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l)$ is upper-bounded as follows [9]

$$P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l) \leq \prod_{i=1}^{l_Q} \frac{1}{1 + \frac{1}{4N_0}|s_i - \hat{s}_i|^2} \tag{2}$$

For high signal-to-noise ratios, the upper-bound can be expressed as

$$P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l) \leq \frac{1}{(\frac{1}{4N_0})^{l_Q} d_p^2(l_Q)} \tag{3}$$

where $d_p^2(l_Q)$ is the squared product distance, given by

$$d_p^2(l_Q) = \prod_{i \in Q} |s_i - \hat{s}_i|^2 \tag{4}$$

In [4], the authors show that, for the case of Rayleigh fading channels with CSI, a tighter bound can be derived. Considering these results, a tight approximation to the *pairwise error probability* for high signal-to-noise ratios is given by

$$P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l) \approx \frac{(2L-1)!!}{2^{L+1}L!} \prod_{i=1}^{L} \frac{1}{1 + \frac{1}{4N_0}|s_i - \hat{s}_i|^2} \tag{5}$$

where $L = min(l_Q)$ is the effective distance of the code and

$$(2L-1)!! = (2L-1) \cdot (2L-3) \cdot ... \cdot 3 \cdot 1 \tag{6}$$

3.3. Estimations on the error event probability

An upper-bound on the error event probability, P_e, is obtained from the union bound [14]. The term with the smallest l_Q and $d_p^2(l_Q)$ dominates P_e

for high signal-to-noise ratios. Let $\gamma(l_Q, d_p^2(l_Q))$ be the average number of sequences having the effective length l_Q and the squared product distance $d_p^2(l_Q)$. By substituting $P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l)$ from equations (2), (3) and (5) in the following expression:

$$P_e \approx \gamma(L, d_p^2(L)) P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l) \quad (7)$$

three approximations to P_e can be obtained. Note that only the case $l_Q = L$ must be considered when using equations (2) and (3).

4. APPLICATION OF THE EIM

4.1. Hypotheses to apply the EIM

The EIM [10] provides an estimation of the Hamming minimum distance, d_{Hmin}, of a turbo code. It also provides an estimation of the average number of coded sequences at distance d_{Hmin}.

Our goal is to compute the approximations in equation (6) by using the information provided by the EIM. Because of bit-interleaving (π in Fig. 1), we can assume the following hypothesis:

Hyp. 1: $\forall i$, if $s_i \neq \hat{s}_i$ there is only one bit that changes between them.

As we consider high signal-to-noise ratios, the following hypothesis can also be assumed:

Hyp. 2: $\forall i$, if $s_i \neq \hat{s}_i$, s_i and $\hat{s}_i$ are adjacent signals in the constellation.

From **Hyp. 1**, it follows that $L = d_{Hmin}$. Denoting by d_0 the minimum Euclidean distance of the constellation, **Hyp. 2** can also be expressed as

$$|s_i - \hat{s}_i| = \begin{cases} 0 & \text{if } s_i = \hat{s}_i \\ d_0 & \text{if } s_i \neq \hat{s}_i \end{cases} \quad (8)$$

Then, the minimum squared product distance of the turbo-coded modulation can be calculated as follows

$$d_p^2(L) = (d_0^2)^{d_{Hmin}} \quad (9)$$

4.2. 8-PSK turbo-coded modulation

For 8-PSK modulation, d_0 and the average signal energy E_s are related by

$$d_0 = 2\sqrt{E_s} \sin \frac{\pi}{8} \quad (10)$$

For simplicity, we denote $\gamma(d_{Hmin}, d_p^2(d_{Hmin}))$ by γ. Taking equations (9) and (10) into account, we can now substitute $P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l)$ from equations (2), (3) and (5) in equation (7) to obtain three final expressions that estimate P_e for 8-PSK turbo-coded modulations. The first computable expression comes from (3):

$$P_e \approx \frac{\gamma}{(\frac{E_s}{N_0} sin^2 \frac{\pi}{8})^{d_{Hmin}}} \quad (11)$$

And by substituting $P_2(\mathbf{s}_l, \hat{\mathbf{s}}_l)$ from (2) and (5), we obtain

$$P_e \approx \frac{\gamma}{(1 + \frac{E_s}{N_0} sin^2 \frac{\pi}{8})^{d_{Hmin}}} \quad (12)$$

$$P_e \approx \frac{(2d_{Hmin} - 1)!!}{2^{d_{Hmin}+1} d_{Hmin}!} \frac{\gamma}{(1 + \frac{E_s}{N_0} sin^2 \frac{\pi}{8})^{d_{Hmin}}} \quad (13)$$

respectively.

5. EXAMPLES

Simulation results and estimations from equations (11)-(13) were obtained for different block sizes. The 8-state duo-binary turbo code adopted for the DVB-RCS/RCT standard [15] [16] was considered. We present in Figures 3 and 4 simulated vs. estimated performance over a Rayleigh slow-fading channel with perfect CSI. Estimations consider a memoryless channel model and assume coherent detection and ML decoding. Simulations use the Max-Log-MAP algorithm (with 8 iterations and 6-bit quantized samples). Figure 3 considers the transmission of 188-byte data blocks, and Figure 4 the transmission of 54-byte data blocks.

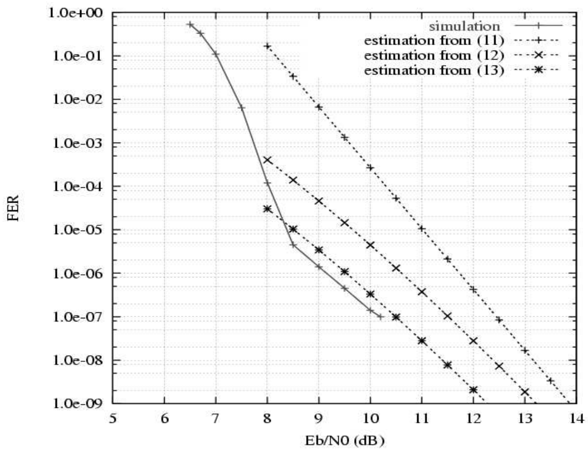

Figure 3: DVB-RCS turbo code. Rate=2/3. 8-PSK turbo-coded modulation. Transmission of 188-byte data blocks. Max-Log-MAP algorithm (with 8 iterations and 6-bit quantized samples).

As expected, the estimation from equation (13) is the closest of the three. For 188-byte data blocks, this estimation is only 0.3 dB from the simulation results, for FER values smaller than 10^{-6}. For 54-byte data blocks, the estimation is about 1 dB from the simulation results, for the same FER values. This

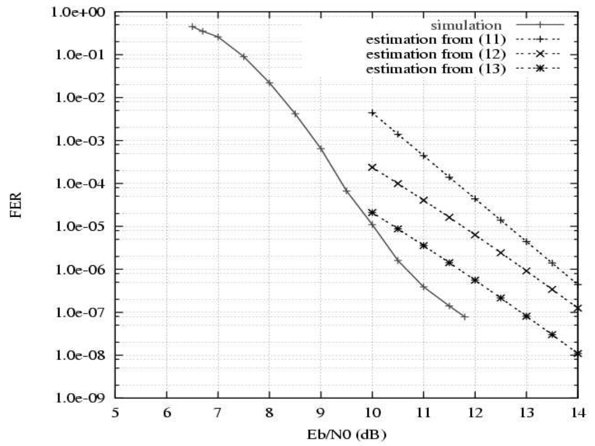

Figure 4: DVB-RCT turbo code. Rate=2/3. 8-PSK turbo-coded modulation. Transmission of 54-byte data blocks. Max-Log-MAP algorithm (with 8 iterations and 6-bit quantized samples).

may be because **Hypothesis 1** no longer holds when considering short data blocks. We also observe that the estimation from equation (11) is too far from the simulation results. This is probably due to the fact that this expression contains two high signal-to-noise ratio approximations.

6. CONCLUSIONS

A new solution to estimate asymptotic performance of turbo-coded modulation has been presented and justified. This solution relies on the application of the Error Impulse Method. The examples show good agreement between estimation and simulation curves. In addition, the estimation expressions enable us to better understand the parameters that dominate turbo-coded modulation performance in fading channels.

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