Application of error impulse method to 16-QAM bit-interleaved turbo coded modulations

L. Conde Canencia, C. Douillard, M. Jézéquel and C. Berrou

The authors propose an estimation technique that makes it possible to predict the performance of 16-QAM bit-interleaved turbo-coded modulations at low error rates over Gaussian channels. This technique relies on knowledge of the impulse spectrum of the turbo code considered, which is provided by the error impulse method.

Introduction: When designing bandwidth efficient coding schemes employing turbo codes, it is difficult to determine their asymptotic behaviour because of the large number of Monte Carlo simulations required to validate performance at low error rates. An original new method for computing minimum distances of linear codes, in particular turbo codes, was recently presented in [1]. This method, called the error impulse method (EIM), is based on the ability of the soft-in decoder to overcome error impulse patterns added to the reference sequence. It thus provides an estimation of the distance spectrum of the code considered without having to enumerate exhaustively all the low-weight codewords. In [2], the EIM was successfully extended to the association of a turbo code and a 8-PSK modulation. In this Letter, we describe the application of the EIM to associations considering 16-QAM modulations.

Transmission scheme: We consider the transmission scheme shown in Fig. 1, which follows the principle of bit-interleaved coded modulation (BICM) [3]. The block designed by π is an ideal bit interleaver (i.e. infinite depth, completely random).

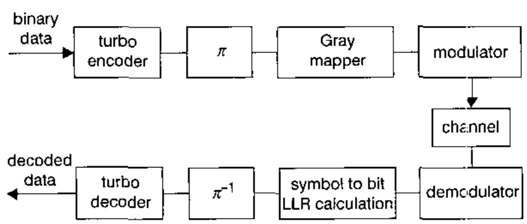

Fig. 1 *Transmission scheme*

Application of EIM to estimate performance: Assuming maximum-likelihood (ML) decoding and transmission over the AWGN channel, an asymptotic estimate of the frame error rate (FER) for high signal-to-noise ratios is:

$$\text{FER} \simeq \frac{1}{2} N(d_{free}) \text{erfc}\left(\frac{d_{free}}{2\sqrt{N_0}}\right) \tag{1}$$

where N_0 is the single-sided noise spectral density, d_{free} is the free Euclidean distance of the coded modulation and $N(d_{free})$ is the average number of nearest neighbours at d_{free} from the transmitted sequence. This expression may not be computable for every coded modulation scheme because d_{free} and/or $N(d_{free})$ may not be known. In our transmission scheme, as the ideal bit interleaver is followed by a Gray labelling, d_{free} can be calculated as:

$$d_{free}^2 = d_{H\min} d_0^2 \tag{2}$$

where d_0 is the minimum Euclidean distance of the constellation (see Fig. 2) and $d_{H\min}$ is the minimum Hamming distance of the code. d_0 is a known parameter that depends only on the constellation. $d_{H\min}$ is obtained when applying the EIM to the considered turbo code. For each position i in the sequence of k information bits ($1 \le i \le k$), the EIM provides a value A_i^*, that is the minimum distance of all codewords with an error at position i. The minimum distance of the code is the smallest in this set of values. An estimation on $N(d_{free})$ can also be obtained with the EIM if the two following hypotheses are considered [1]:

Hypothesis 1: there is only one codeword with distance A_i^* for each position i.

Hypothesis 2: all the distances A_i^* obtained for the whole set of positions i concern distinct codewords.

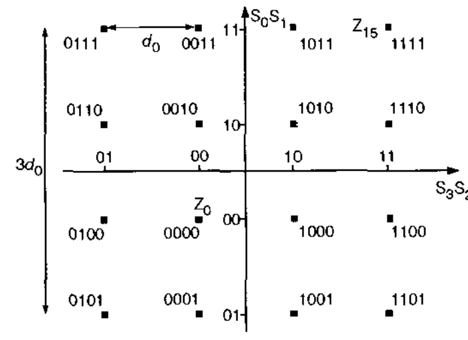

Fig. 2 *16-QAM constellation and mapping*

For 16-QAM modulation, $d_0 = \sqrt{(2/5 \cdot E_s)}$, where E_S is the average signal energy. Then, adopting hypotheses 1 and 2, and once the impulse spectrum (i.e. the whole set of A_i^* values) is obtained for a given code, (1) can be computed as follows:

$$\text{FER} \simeq \frac{1}{2}\sum_{i=1}^{k} \text{erfc}\sqrt{\frac{A_i^* E_S}{10 N_0}} \tag{3}$$

However, this is a loose and pessimistic estimation because it uses d_{free}, i.e. it considers that all the opposed bits in **s** and **s**′ belong to adjacent signal points, where **s** is the transmitted sequence and **s**′ is the wrong sequence chosen by the decoder. Our goal is to find a tighter estimation by taking advantage of the modulation properties.

Constellation and mapping characteristics: The 16-QAM constellation considered follows a classical Gray mapping (Fig. 2). As we assume that the bit-interleaving is ideal, we can add the following hypothesis to hypotheses 1 and 2:

Hypothesis 3: a symbol does not contain more than one opposite bit in **s** and **s**′.

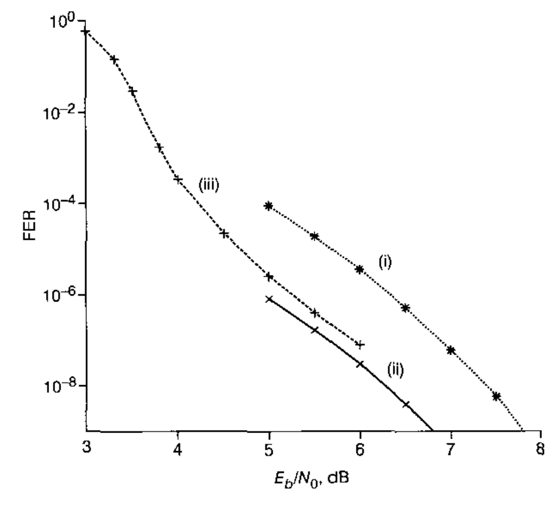

Fig. 3 *Simulation results and analytical estimations, in frame error rate, for 16-QAM modulation associated with 8-state DVB-RCS turbo code*

Rate: 1/2. Spectral efficiency: 2 bit/s/Hz. Simulations use Max-Log-MAP algorithm (with 8 iterations and 6-bit quantised samples) for decoding. Transmission of 188-byte data blocks
(i) estimation from (3)
(ii)estimation from (5)
(iii) BICM simulation

For each bit s_n, $n \in (0,1,2,3)$, and each constellation point Z_m, $m \in (0, \ldots, 15)$, we calculate the Euclidean distance, d, between Z_m and Z'_m, where Z'_m is the signal point such that s_n has an opposite value in Z_m and in Z'_m. Note that, thanks to hypothesis 3, s_n is the only bit that changes from Z_m to Z'_m. It follows that, statistically, $d = 3d_0$ for 1/4 of the bits in a sequence and $d = d_0$ for the others, where d_0 is the minimum Euclidean distance of the constellation. The Euclidean distance between $\mathbf{s}$ and $\mathbf{s}'$, denoted $d_E(\mathbf{s}, \mathbf{s}')$, can be expressed as a function of $d_{H\min}$ and j, where j denotes the number of opposite bits belonging to constellation symbols at distance $3d_0$ in $\mathbf{s}$ and $\mathbf{s}'$:

$$d_E(\mathbf{s}, \mathbf{s}') = j \cdot (3d_0)^2 + (d_{H\min} - j) \cdot d_0^2 = d_0^2 \cdot (8j + d_{H\min}) \quad (4)$$

Note that if $j = 0$, $d_E(\mathbf{s},\mathbf{s}') = d_{free}$ (which agrees with the definition of d_{free}).

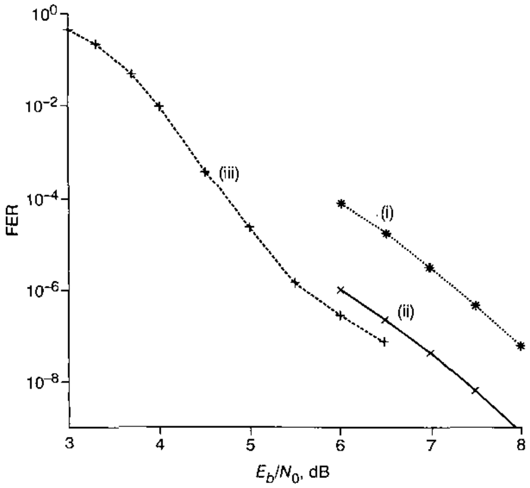

Fig. 4 *Simulation results and analytical estimations, in frame error rate, for 16-QAM modulation associated with 8-state DVB-RCT turbo code*

Rate: 1/2. Spectral efficiency: 2 bit/s/Hz. Simulations use Max-Log-MAP algorithm (with 8 iterations and 6-bit quantised samples) for decoding. Transmission of 54-byte data blocks
(i) estimation from (3)
(ii)estimation from (5)
(iii) BICM simulation

Considering the statistical properties described above and the impulse spectrum obtained from applying the EIM, a tighter estimation of the asymptotic FER performance is then expressed as:

$$\text{FER} \simeq \sum_{i=1}^{k} \sum_{j=0}^{A_i^*} \binom{A_i^*}{j} \left(\frac{1}{4}\right)^j \left(\frac{3}{4}\right)^{A_i^*-j} \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_S}{10N_0}(8j + A_i^*)} \quad (5)$$

Practical issues: Hypothesis 3 no longer holds when considering too short blocks: it is more likely to find a constellation symbol that differs by two (or more) bits in $\mathbf{s}$ and $\mathbf{s}'$. It follows that d_{free} is increased and performance is better than estimated.

Examples: To illustrate this heuristic, simulation results and estimations (3) and (5) were obtained for different block sizes. Simulations considered the 8-state duo-binary turbo code adopted for the DVB-RCS/RCT standards [4, 5]. Fig. 3 considers the transmission of 188-byte data blocks. The simulation curve is 0.3 dB from (5), for FER values lower than 10^{-6}. Fig. 4 considers the transmission of 54-byte data blocks. In this case, estimation (5) upper-bounds the simulation curve.

Acknowledgments: The authors would like to thank France Telecom R&D's DMR/DDH laboratory for financial support.

29 January 2003
Electronics Letters Online No: 20030386
DOI: 10.1049/el:20030386

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Iterative decoding for reducing cyclic prefix requirement in OFDM modulation

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Orthogonal frequency division multiplexing (OFDM) requires a cyclic prefix extension to combat the detrimental effects of excess delay in the channel. A cyclic prefix length of 25% of total OFDM symbol length is specified for next generation wireless local area network systems (IEEE standard 802.11a). Whilst combating multipath and enabling single tap equalisation in the frequency domain, the additional overhead has a detrimental effect on data throughput. Presented is the performance of low density parity check encoding at the transmitter and iterative ('turbo') decoding at the receiver to remove the cyclic prefix. It is shown that with an initial log likelihood operation at the receiver, iterative decoding is able to overcome the intersymbol interference introduced by the channel.

Introduction: In addition to bandwidth efficiency, orthogonal frequency division multiplying (OFDM) is an attractive modulation format since it utilises the computationally efficient inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) algorithm for modulation and demodulation. Furthermore, by employing a cyclic prefix with each OFDM symbol that is longer than the excess delay in the channel, the receiver is effectively able to remove the detrimental effects of multipath propagation in the wireless channel by removing the cyclic prefix before demodulation. This technique renders a frequency selective channel into a number of parallel frequency non-selective channels. Thus, assuming estimation of channel state information (CSI), equalisation becomes a low complexity process performed in the frequency domain [1]. Recent advances in coding theory have lead to development of so-called 'turbo' codes [2]. Moreover, the recent rediscovery of low density parity check (LDPC) codes [3] and the applicability of the 'turbo' principle to their decoding have resulted in a low complexity iterative decoding framework. The performance of LDPC codes with OFDM have been examined [3, 4] in the context of frequency and spatial diversity, in both cases with cyclic prefixes long enough to remove intersymbol effects. In this Letter, it is shown that by application of powerful codes and iterative decoding that the need for a cyclic prefix can effectively be removed, leading to increased data throughput at the expense of computational complexity.

System model: Consider an OFDM symbol consisting of N subcarriers: the ith transmitted data block is sent by modulating the subcarriers of the ith OFDM symbol. Conventionally, to mitigate the effects of delay spread within the channel, a cyclic prefix is placed at the beginning of each OFDM symbol. The cyclic prefix is a repetition of the last time domain samples of the OFDM waveform. OFDM symbol modulation, cyclic prefix insertion, channel convolution, cyclic prefix removal and OFDM symbol demodulation can be represented compactly in matrix mathematics as [5]:

$$\mathbf{y}_i = \mathbf{F}\mathbf{T}_{CPR}\mathbf{H}_0\mathbf{T}_{CP}\mathbf{F}^{-1}\mathbf{x}_i + \mathbf{F}\mathbf{T}_{CPR}\mathbf{H}_1\mathbf{T}_{CP}\mathbf{F}^{-1}\mathbf{x}_{i-1} + \mathbf{F}\mathbf{T}_{CPR}\boldsymbol{\eta}_i \quad (1)$$