

Computation on Unreliable Architecture

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Pr. Michel Jézéquel Télécom Bretagne.



Université
de Bretagne-Sud



Electronic Devices

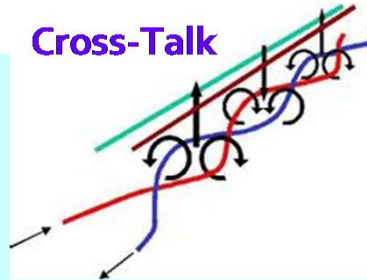


Major Concerns: Permanent Faults and Transient Faults

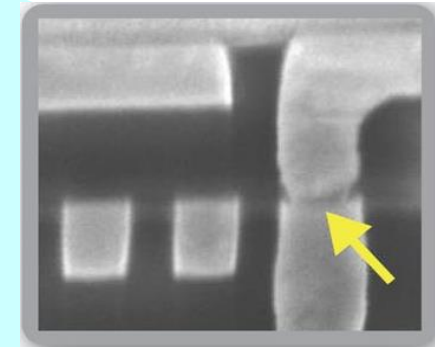
Electromagnetic Interference



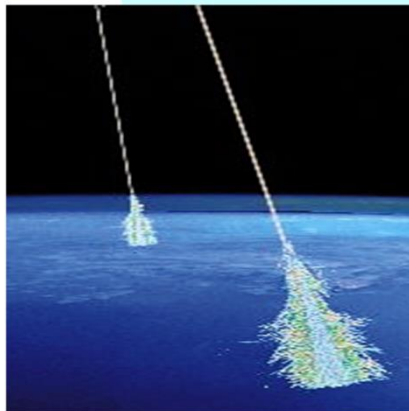
Cross-Talk



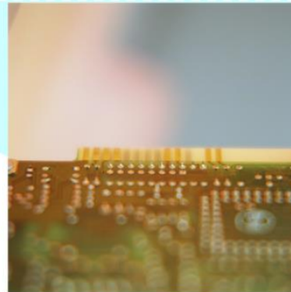
Manufacturing variability



High-Energy Neutrons Earth's atmosphere



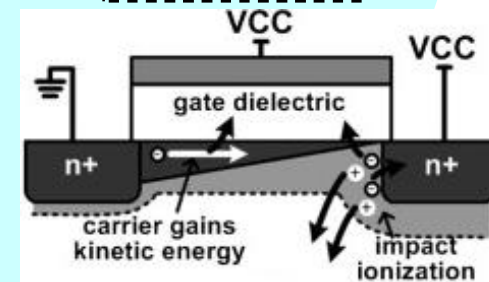
Alpha Particles Metal in device



Temperature Sensitivity



CMOS Aging

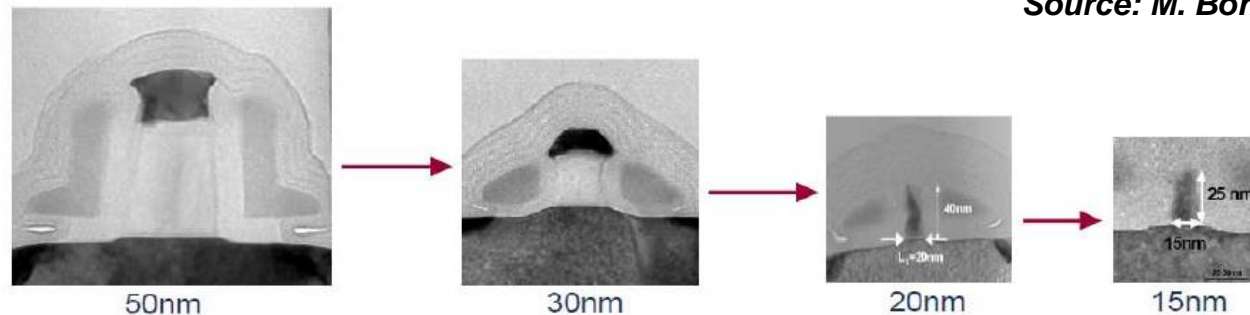
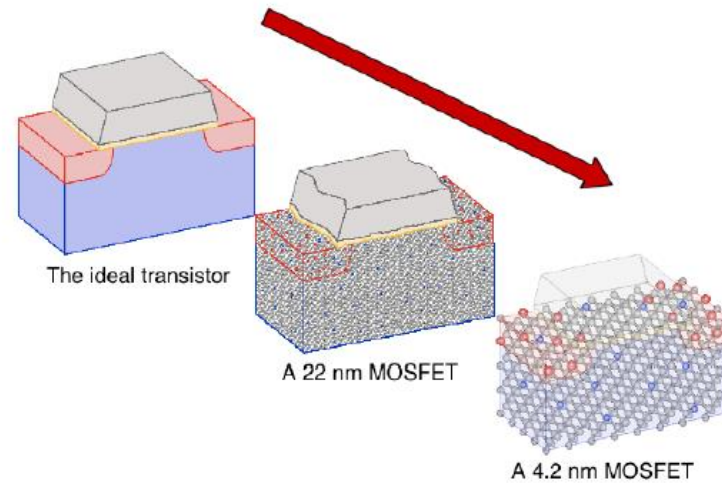


Size Shrinking in CMOS

Scaling to atomistic scale devices...

Nanometer CMOS scaling problems:

- Noise problems (signal integrity)
- Leakage (digital)
- Channel length modulation
- Velocity saturation
- Mobility degradation
- Drain induced barrier lowering (DIBL)
- Parasitic effects
- IC reliability**
- ...



Source: M. Bohr, Intel, IRPS 2003

Roadmap 2011 Edition

International Technology Roadmap for Semiconductors 2011 Edition



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- **The Reliability Issue becomes major concern to semiconductors.**
- **In fact, with deep-micro or nanoscale:**
 - **Error free component becomes more and more expensive**
 - **More and more Transient Faults**

Outline

- Introduction
- Formal Models of Embedded ECC
- Architectural Approaches
- Experimental Results
- Conclusion & Future Perspectives

Question on Reliable Computation...



◆ Can we make reliable communication?



◆ YES! According to Shannon's theory (with a rate limit)

Ref: Mathematical theory of communication, C. E. Shannon, 1948.



Permanent/Transient Faults

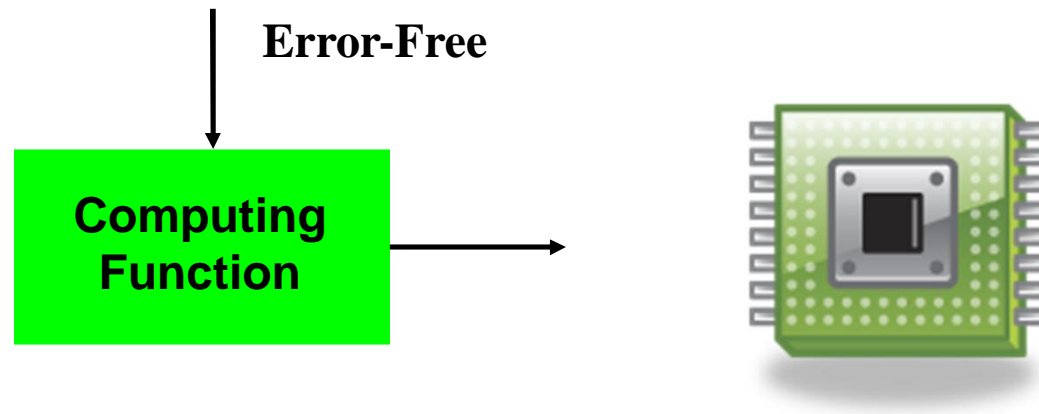


◆ Can we make reliable computation?

◆ Uh, still an open question

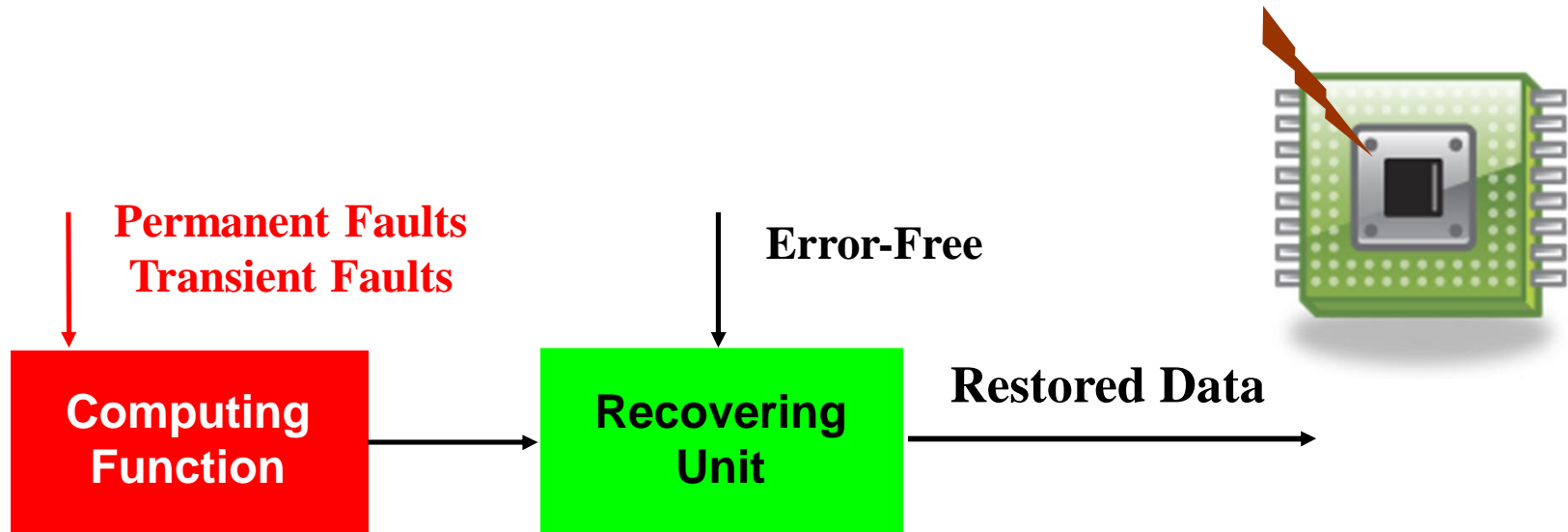
Phases of Computation on Circuit

- **Phase 1: Reliable, Computation is Error-Free.**



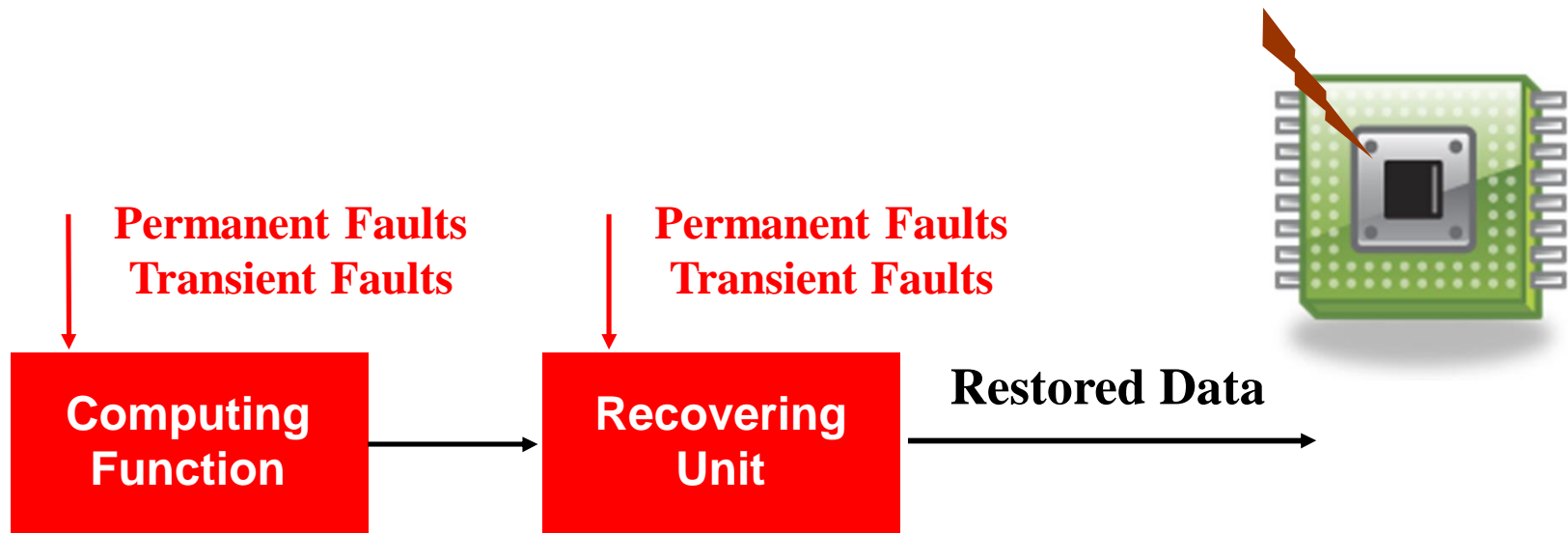
Phases of Computation on Circuit

- **Phase 2: Computation is Erroneous, recovering is Error-Free.**



Phases of Computation on Circuit

- **Phase 3: Computation is Erroneous, recovering is Erroneous as well.**



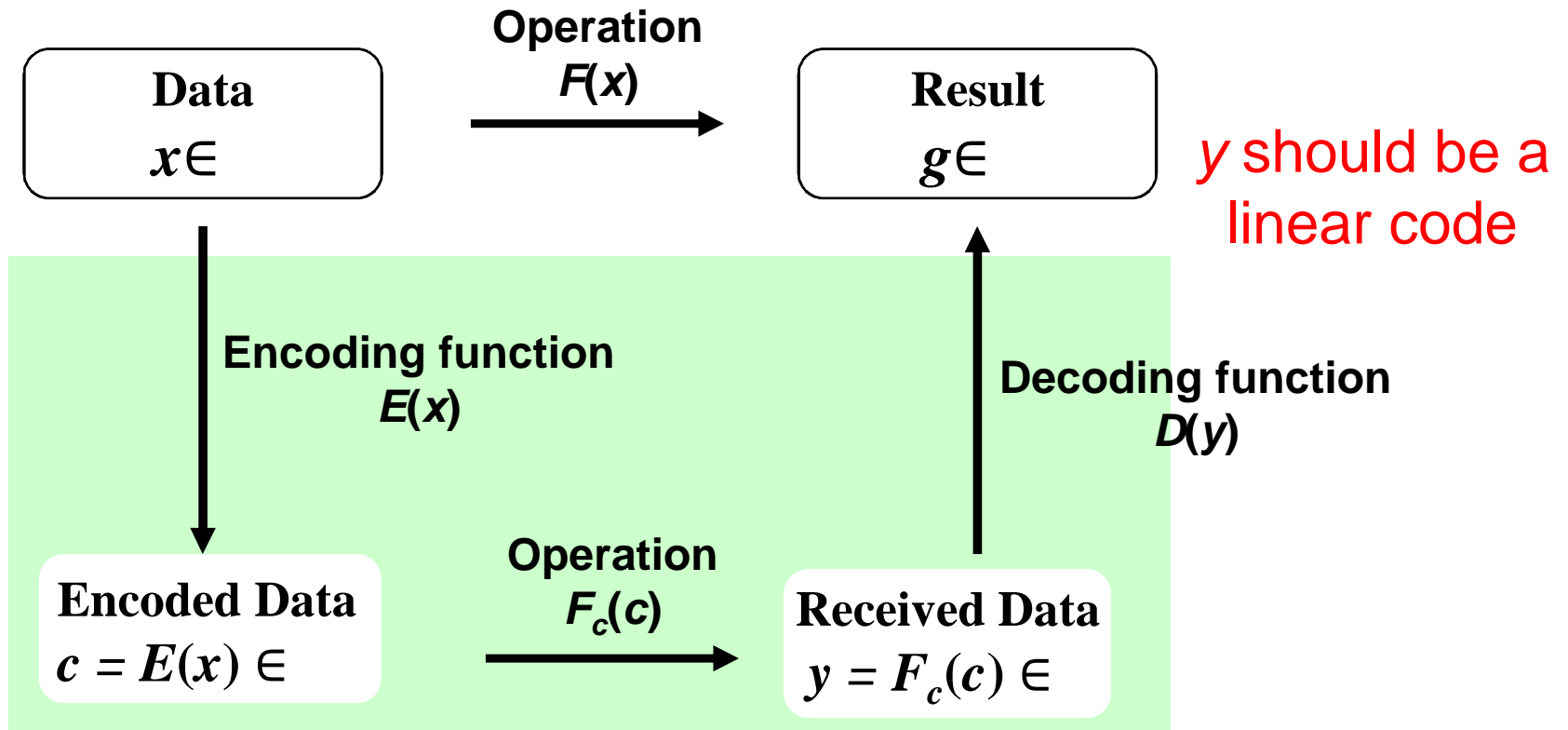
Objective

- **Approaches for reliable computation on unreliable circuit**
- **To design an efficient error-resilient architecture by ECC**
- **For contemporary logics and future nanoelectronic systems**

Outline

- Introduction
- **Formal Models of Embedded ECC**
- Architectural Approaches
- Experimental Results
- Conclusion & Future Perspectives

Formal Model for the application of ECC



Solution 1: Group Homomorphism From $E(x)$ and $F_c(c)$

➤ What is a group Homomorphism?

Let (G, \star) and $(H, *)$ be two groups with an internal composition law.

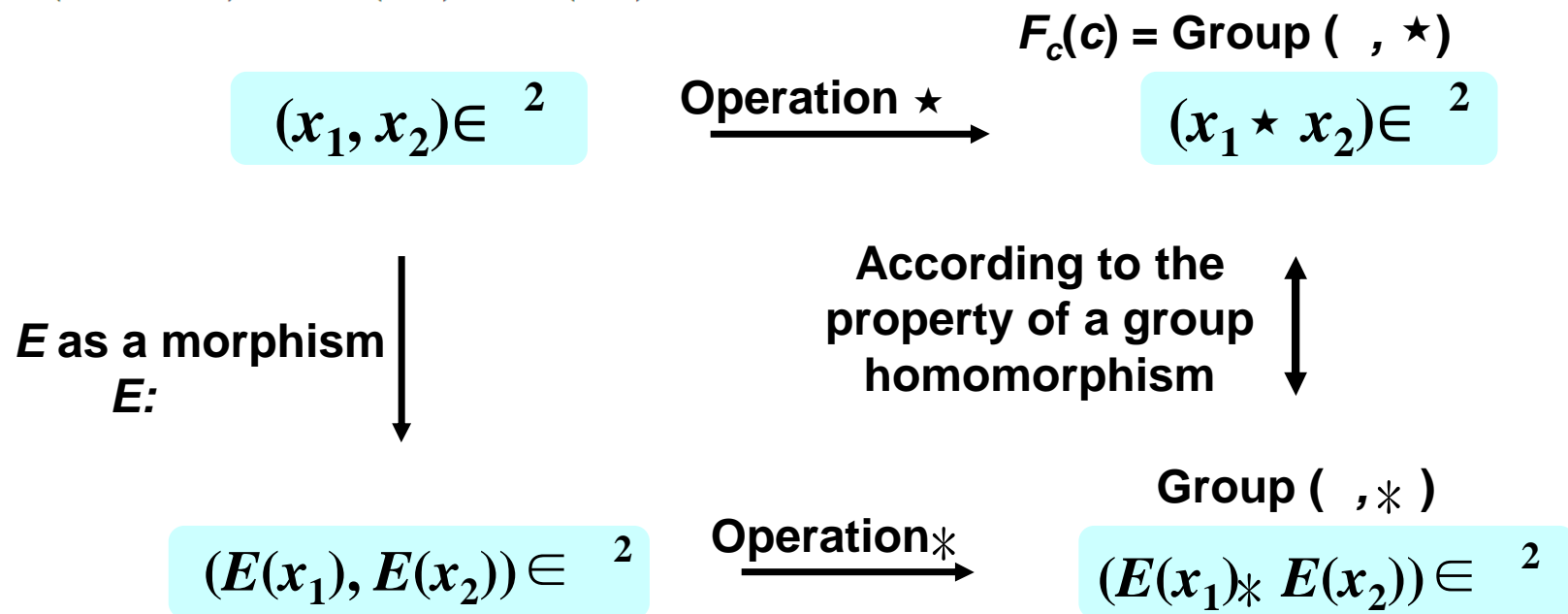
A **group homomorphism** from (G, \star) and $(H, *)$ is a morphism $E: G \rightarrow H$ such that
 $\forall (x_1, x_2) \in G^2,$

$$E(x_1 \star x_2) = E(x_1) * E(x_2)$$

Solution 1: Group Homomorphism From $E(x)$ and $F_c(c)$

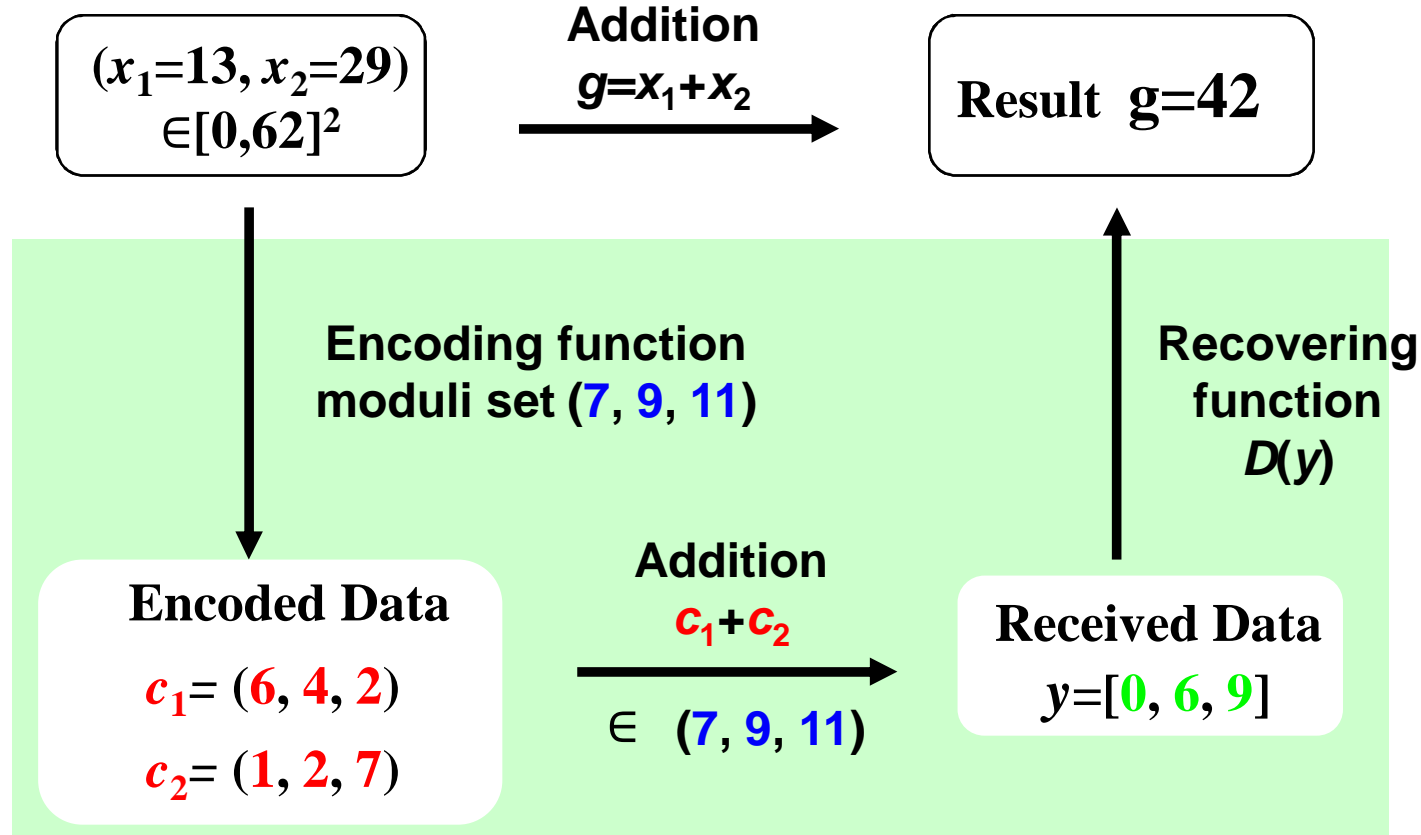
➤ **A group Homomorphism**

$$E(x_1 \star x_2) = E(x_1) * E(x_2)$$



Solution 1: Example – Residue Number System (RNS)

$$42 = (42 \bmod 7 = 0, 42 \bmod 9 = 6, 42 \bmod 11 = 9)$$



$$c_1 = (13 \bmod 7 = 6, 13 \bmod 9 = 4, 13 \bmod 11 = 2)$$

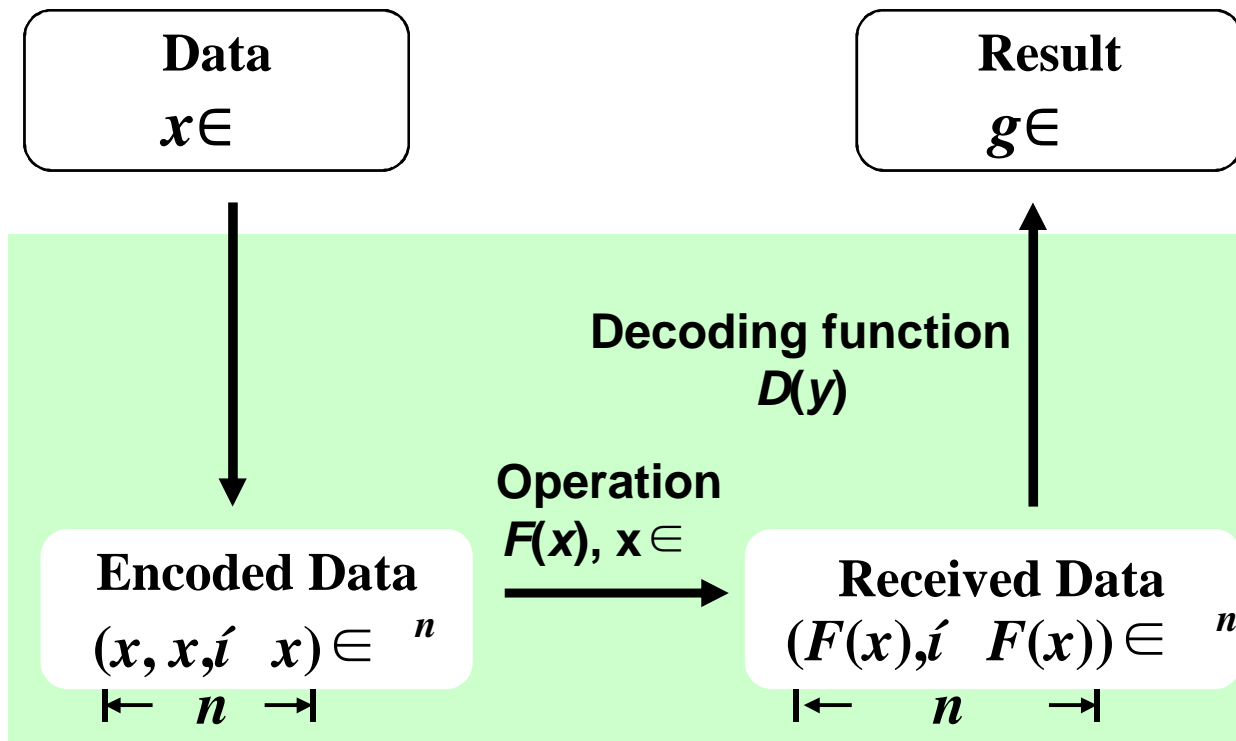
$$c_1 + c_2 = (6 + 1 \bmod 7 = 0, 4 + 2 \bmod 9 = 6, 2 + 7 \bmod 11 = 9)$$

Solution 1: Contribution-RNS based Fault-Tolerant method

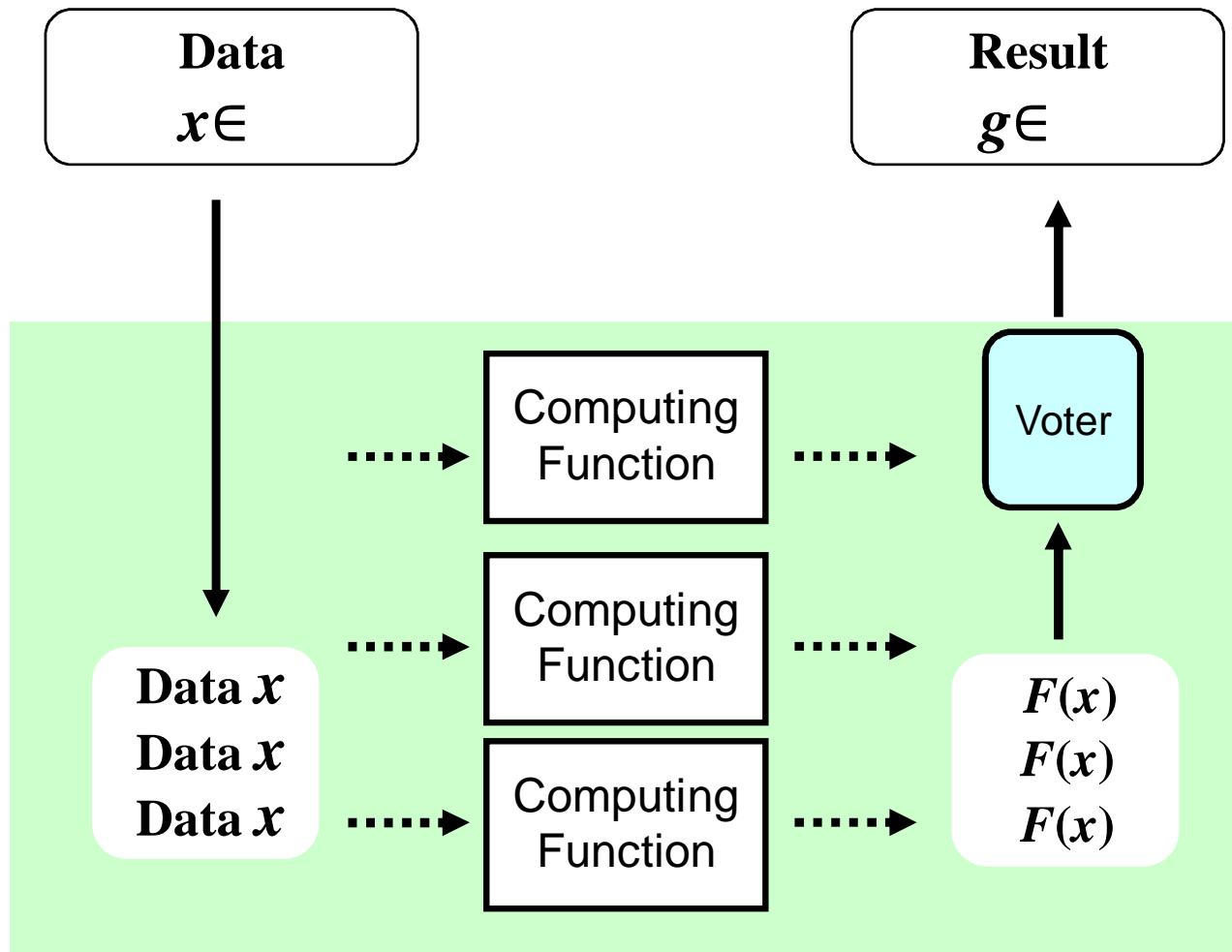
- A contribution of this thesis work:
 - An arithmetic fault-tolerant method
 - Redundant Residue Number System, called BRRNS
 - Fast computation & Error-resilience

- *Yangyang Tang, Emmanuel Boutillon, Christophe Jégo, and Michel Jézéquel*
 öA new single-error correction scheme based on self-diagnosis residue number arithmetic,ö Design and Architectures for Signal and Image Processing (DASIP), 2010.

Solution 2: Repetitions of Computation

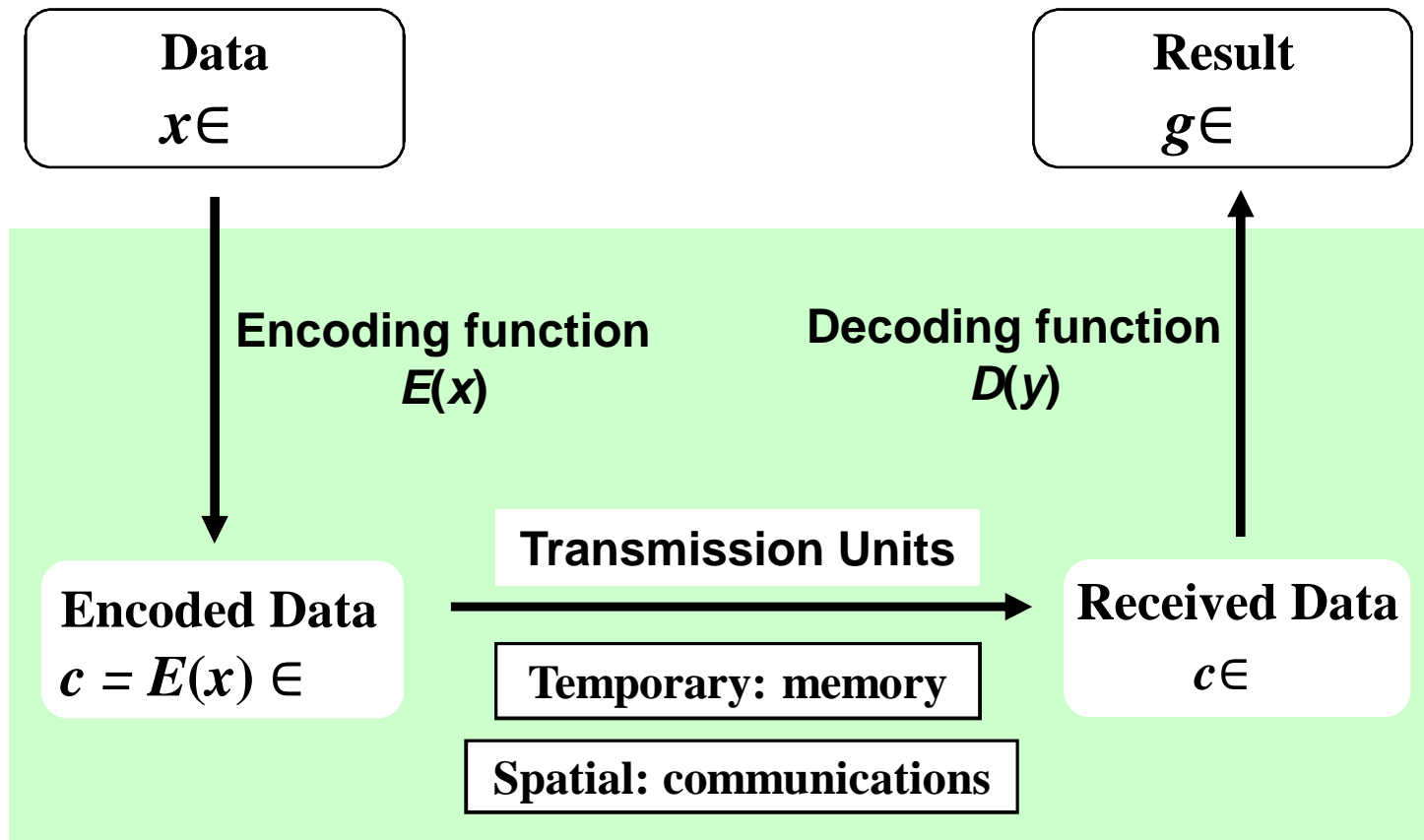


Solution 2: Example – Triple Modular Redundancy (TMR)

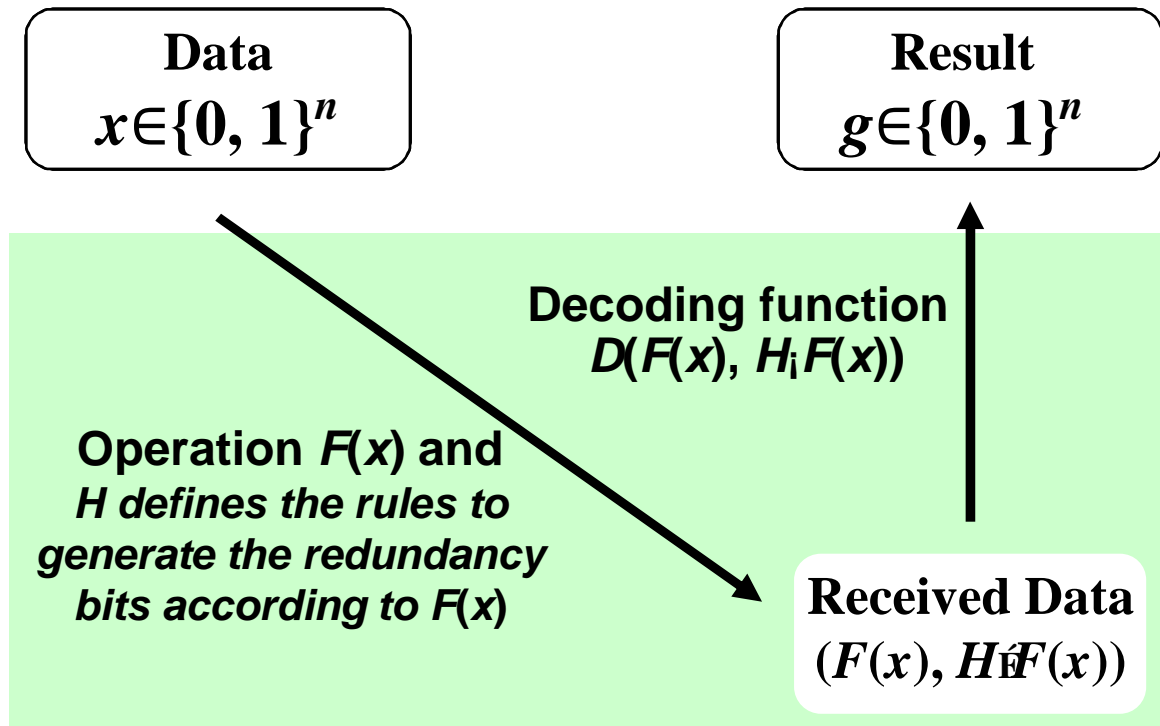


Ref: R. Lyons et al., IBM Journal, 1962

Solution 3: Applying ECC Directly



Solution 4: Function & Parity-Mapped Function & ECC



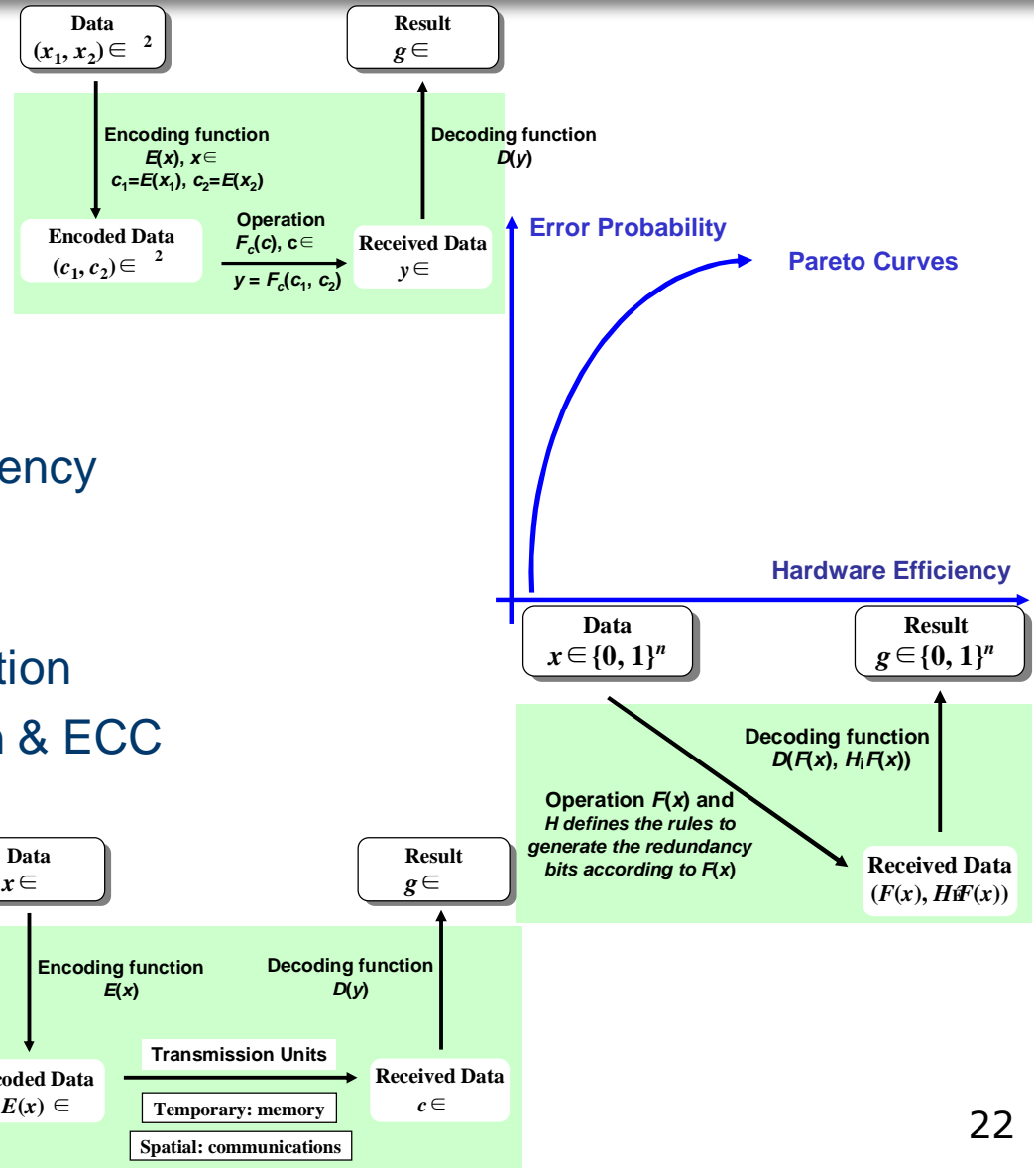
Contributions

- Contribution 1:
 - A RNS based fault-tolerant method
 - Fast computation & Error-resilience

- Contribution 2:
 - Reliability-Efficiency Criteria
 - Estimations on Reliability and Efficiency

- Contribution 3:
 - A fault-tolerant system for computation
 - Function & Parity-Mapped Function & ECC

- Contribution 4:
 - A decoder against internal faults
 - Efficient Error-correction in the presence of high error rate noise



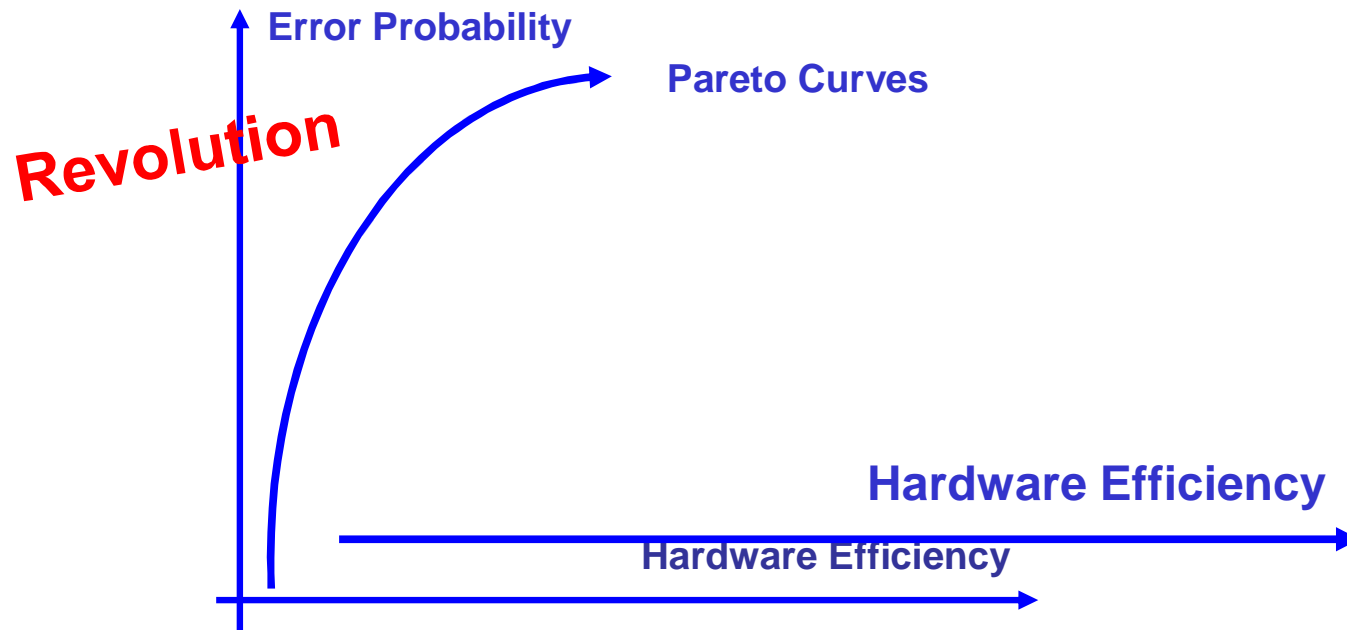
Outline

- Introduction
- Formal Models of Embedded ECC
- Architectural Approaches
 - Reliability-Efficiency Criteria (RE-Criteria)
 - A Fault-Tolerant System for Computation (cDMR)
 - A Decoder Against Internal Faults (MCD)
- Experimental Results
- Conclusion & Future Perspectives

Contribution 2: Reliability-Efficiency Criteria (RE-Criteria)

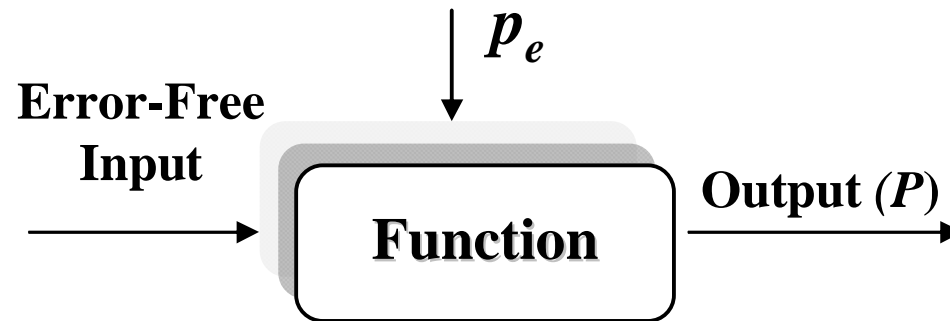
Ref: Y. Tang et al., SiPS, 2011

- **Reliable Electronics:**
 - ✓ Designer objective in signal processing applications
Efficiency = Number of operation / (Area unit x Time unit)
- **Unreliable Electronics: (Occurrence of transient errors)**
 - ✓ New designer objective
Compromise of Efficiency to Reliability (Error Probability)



Contribution 2: Reliability Criterion

➤ Model of Error (Worst-Case)



Hypothesis 1 – Isotropy: an overall $\hat{1}$ unitary error probability p_e as constant (in an unity area during one clock cycle)

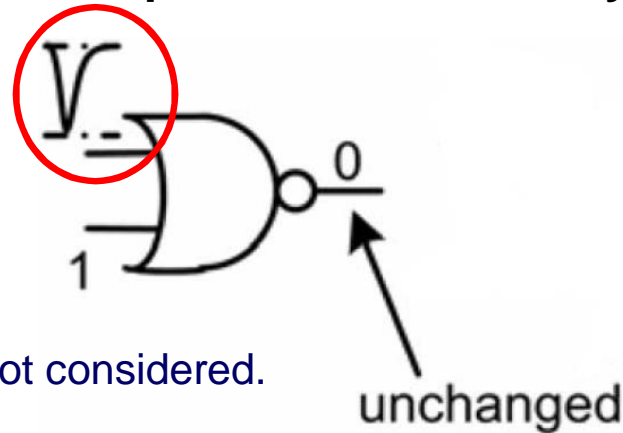
Hypothesis 2 – Contamination: All errors lead to an output error.

Hypothesis 3 – Irreversibility: Two successive errors cannot lead to a correct result.

Contribution 2: Worst-Case hypothesis

Hypothesis 2 $\dot{\text{E}}$ Contamination:

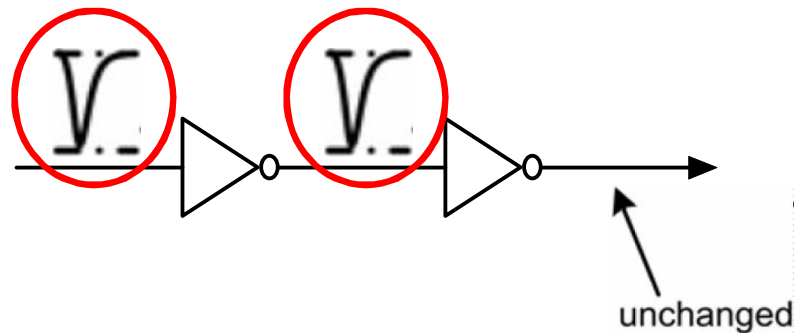
All errors lead to an output error is not always true



e.g., logical mask is not considered.

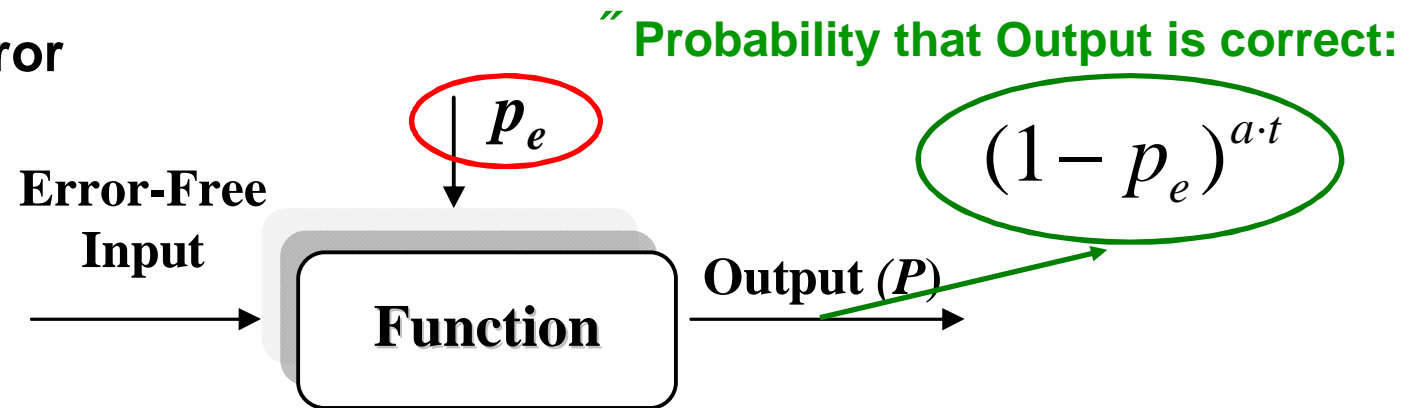
Hypothesis 2 $\dot{\text{E}}$ irreversibility:

Two successive errors cannot lead to a correct result.



Contribution 2: Reliability Criterion

➤ Model of Error

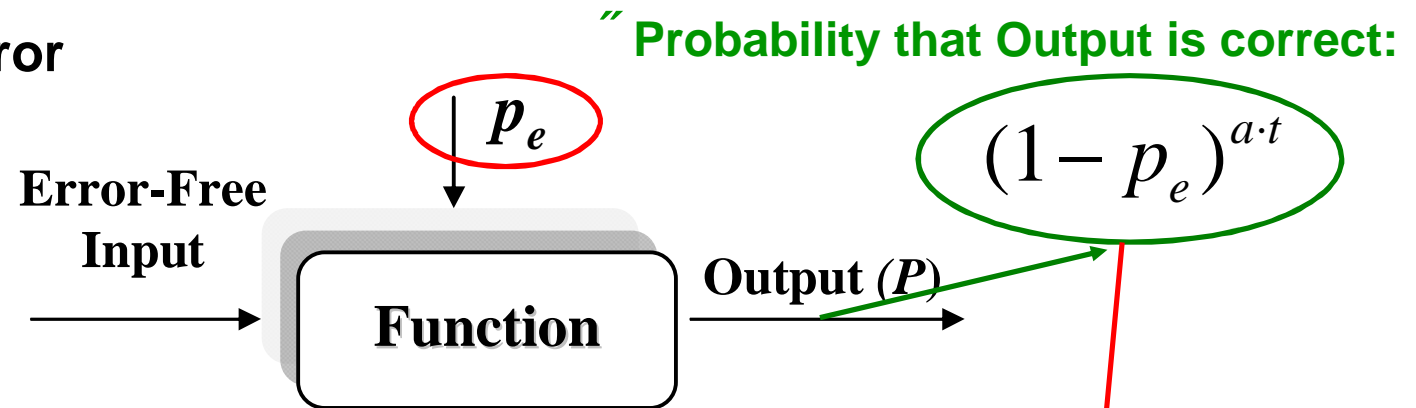


“ Í unitaryĤ error probability p_e (in an unity area during one clock cycle)

“ A function of a unity area requiring t clock cycles to generate an output

Contribution 2: Reliability Criterion

➤ Model of Error



“ Í unitaryĤ error probability p_e (in an unity area during one clock cycle)

“ A function of a unity area requiring t clock cycles to generate an output

“ Resulting error probability

$$P_{(a, t)} = 1 - (1 - p_e)^{a \cdot t}$$

Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

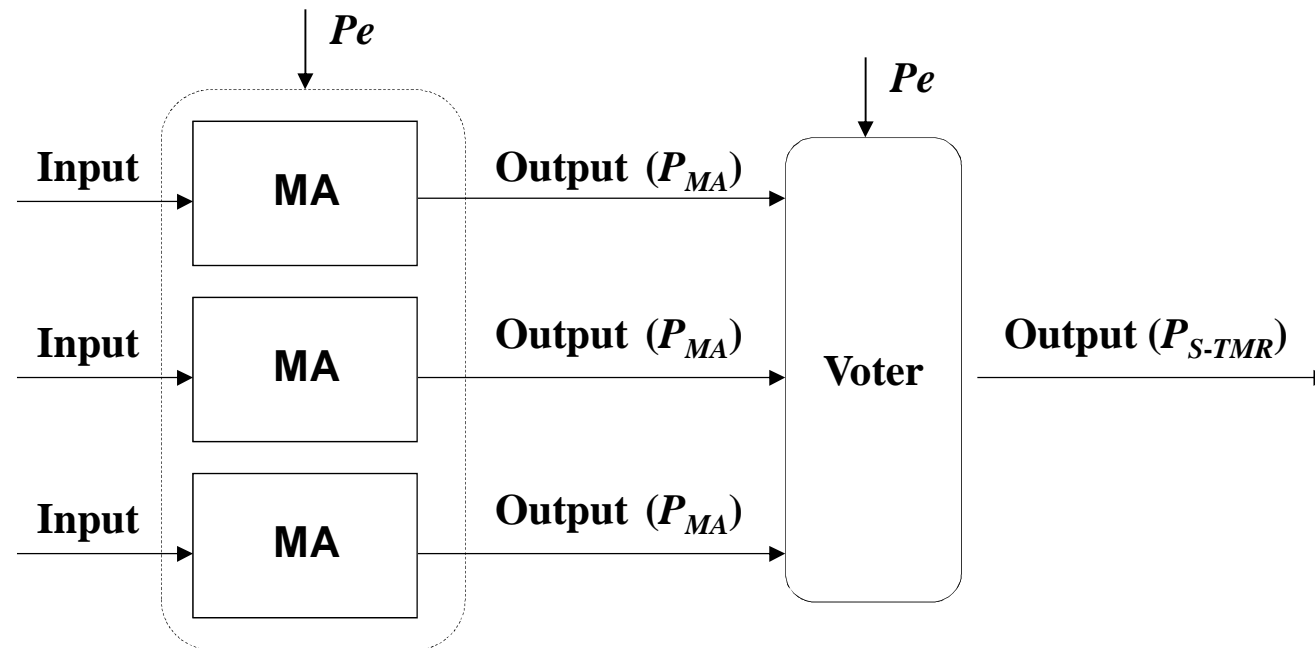
- A pipeline Moving Average (MA) filter is considered as a case study.



$$y(n) = \sum_{i=0}^N x(n-i)$$

Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

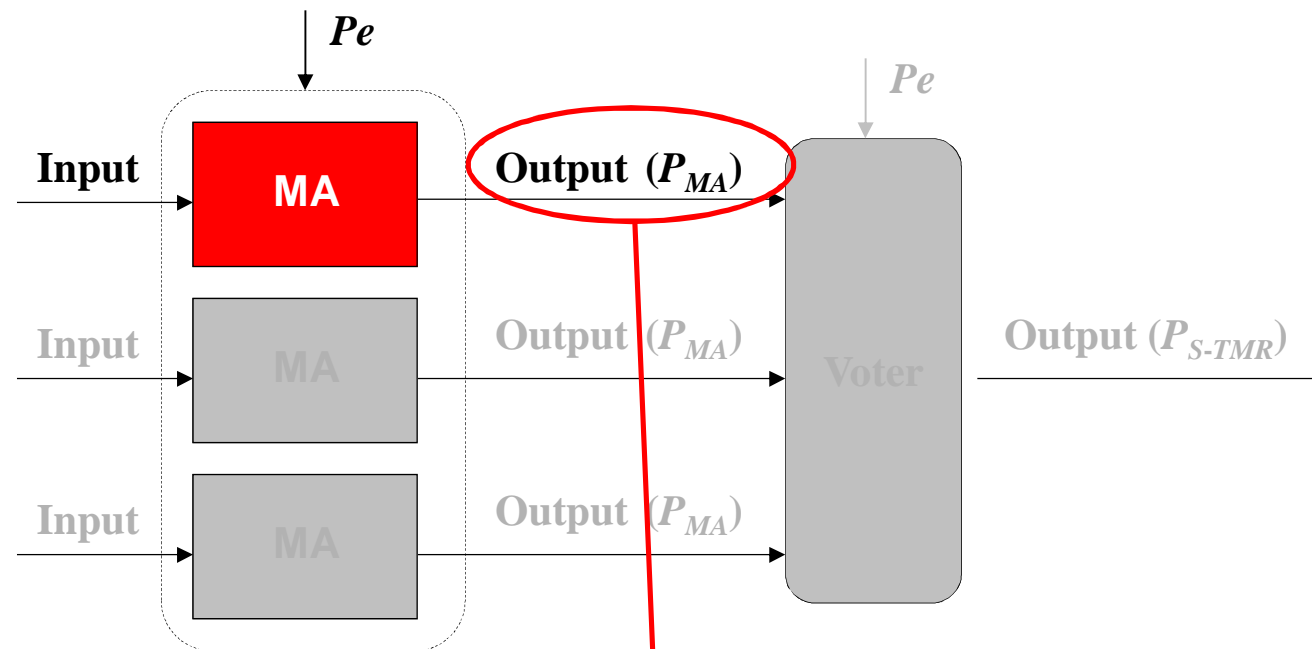
- A pipeline Moving Average (MA) filter is considered as a case study.



Spatial-TMR based MA filter structure

Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

“ Let P_{MA} be the error probability in a single module of size a that performs a computation in t cycles.

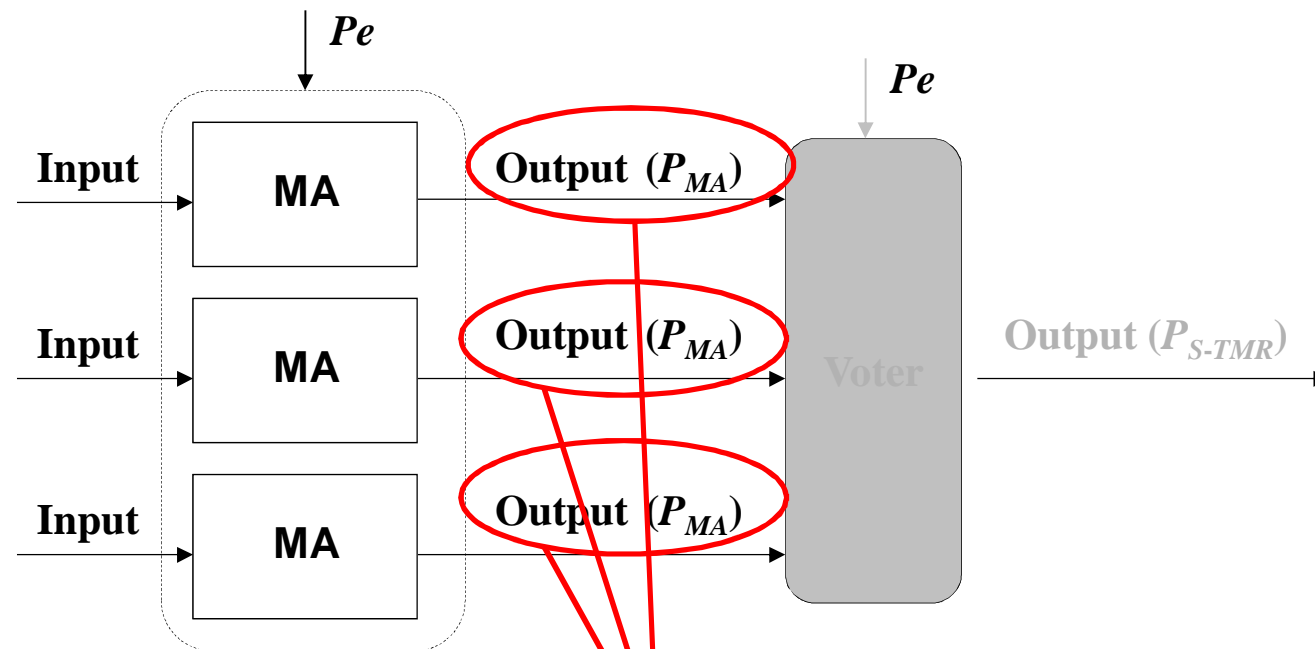


“ By Hypothesis:

$$P_{MA} = 1 - (1 - p_e)^{a \cdot t}$$

Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

“ Let P_{MA} be the error probability in a single module of size a that performs a computation in t cycles.

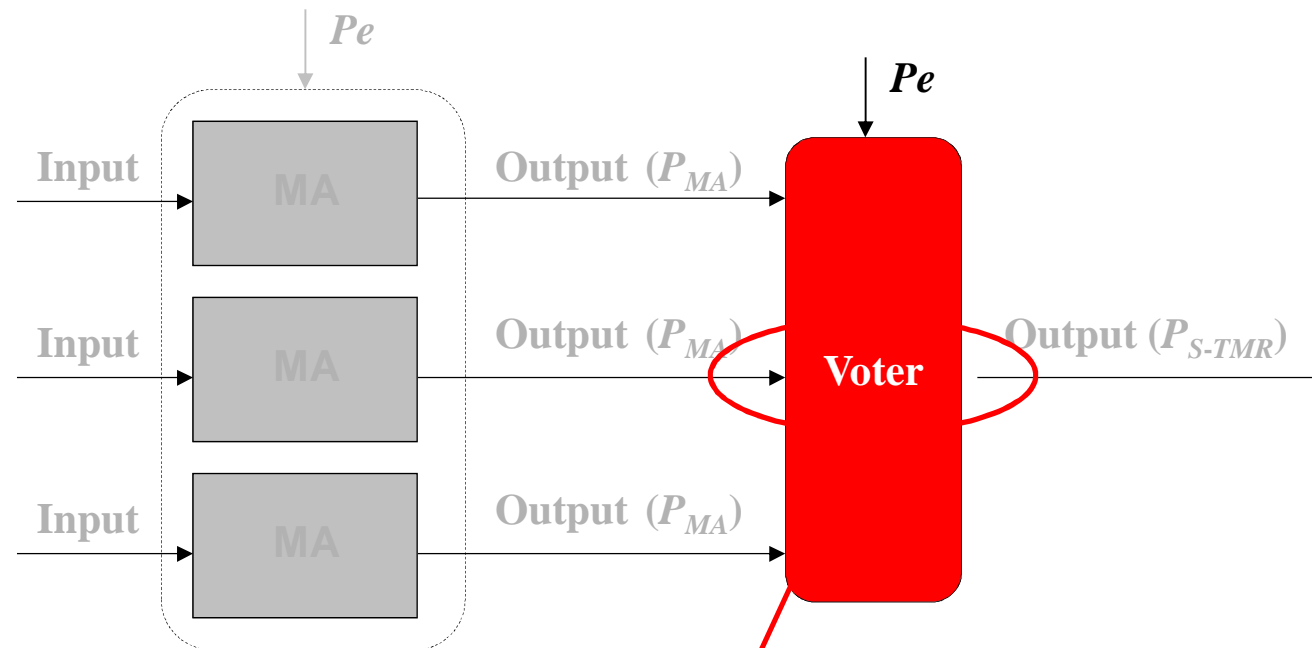


“ By Hypothesis:

$$P_{MA} = 1 - (1 - p_e)^{a \cdot t}$$

Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

“ Event of probability V : voter is faulty and a_v as the area unit of voter.



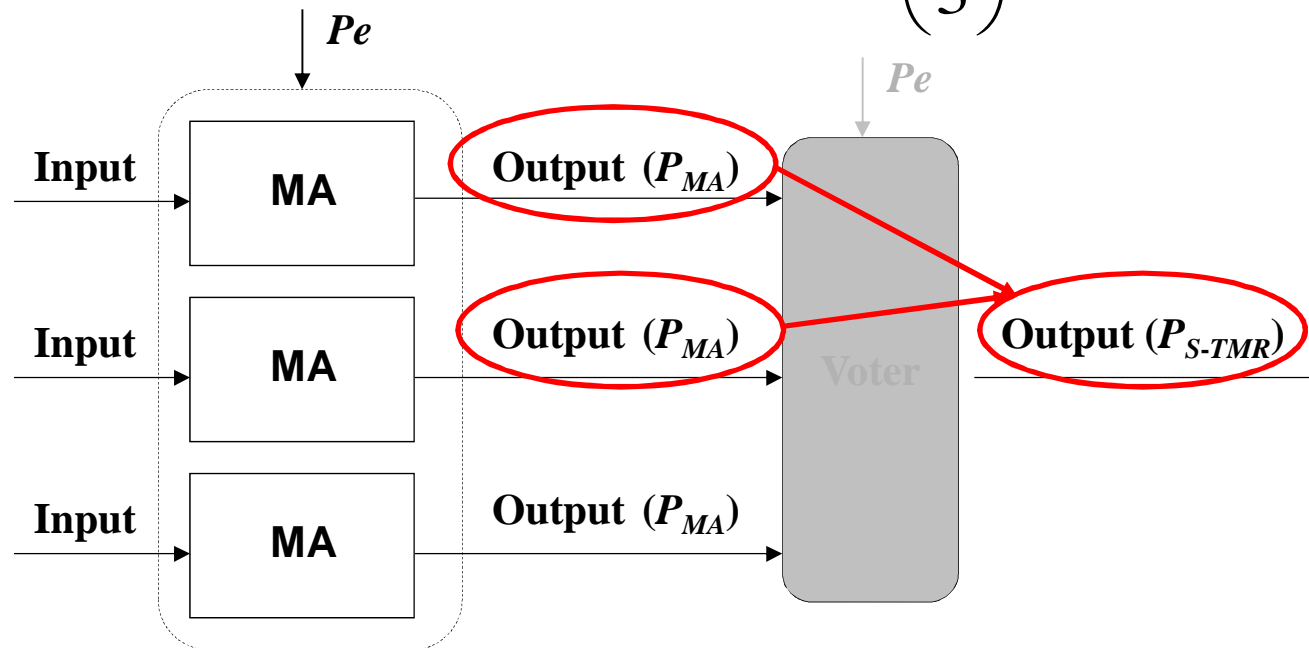
“ By Hypothesis:

$$V = 1 - (1 - p_e)^{a_v} .$$

Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

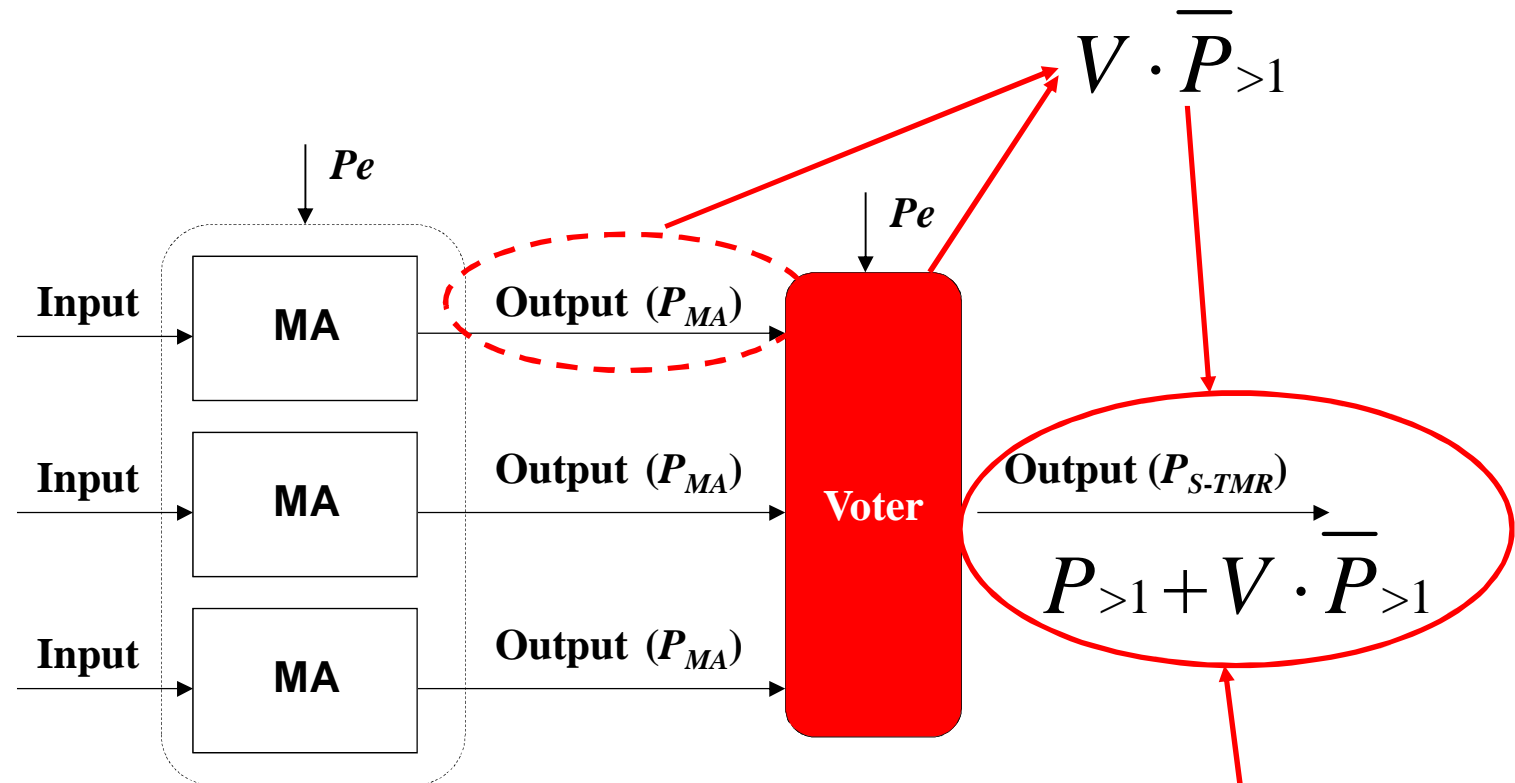
“ Event of probability $P_{>1}$: at least two faulty modules

$$P_{>1} = \binom{2}{3} \cdot P_{MA}^2 \cdot \bar{P}_{MA} + P_{MA}^3$$



Contribution 2: Example of Formal Derivation of $P_{(a,t)}$

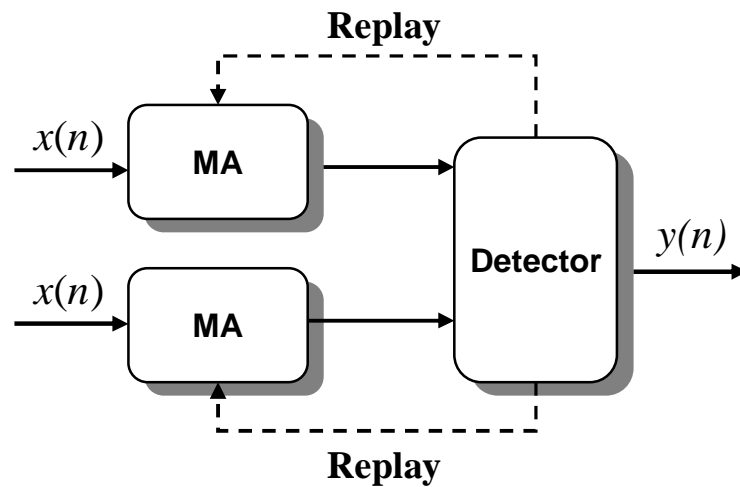
“ Event of probability: only one faulty module and voter is faulty



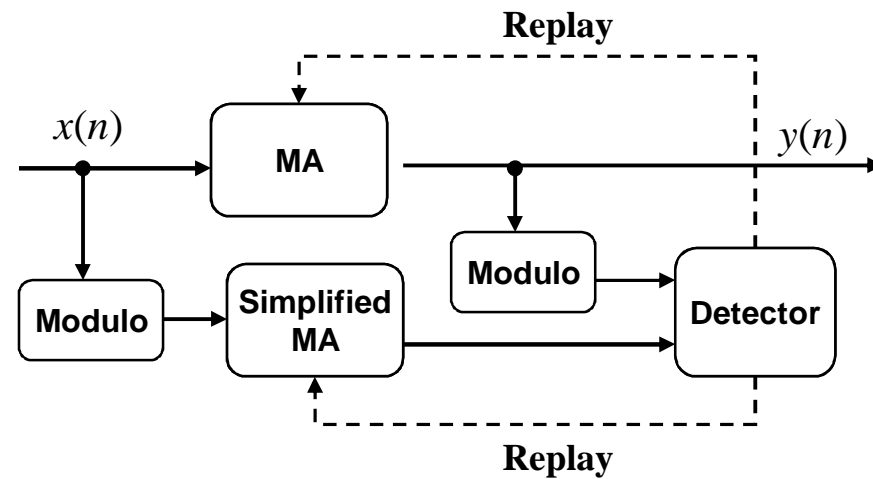
“ Event of probability $P_{>1}$: at least two faulty modules

$$P_{>1} = \binom{2}{3} \cdot P_{MA}^2 \cdot \bar{P}_{MA} + P_{MA}^3$$

Contribution 2: Strategy of Error-Correction for MA Filter



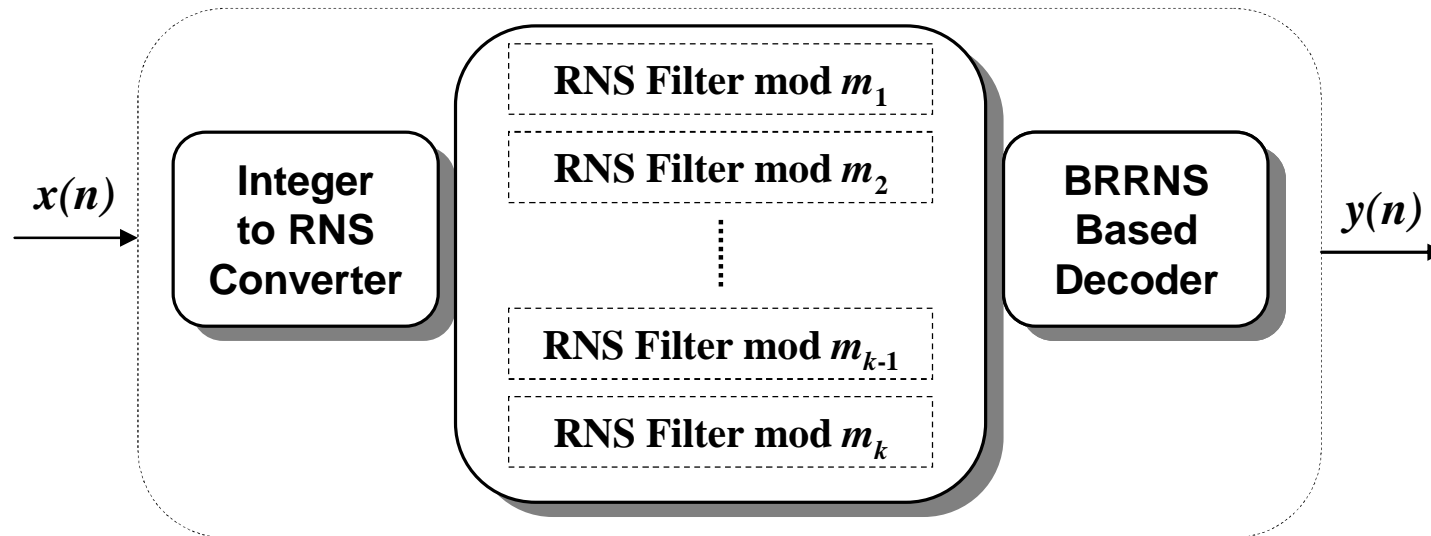
Dual Modular Redundancy (DMR) based ARQ



Modulo operation detecting based ARQ

Ref: S. Lin et al., IEEE Mag., 1984

Contribution 2: Strategy of Error-Correction for MA Filter

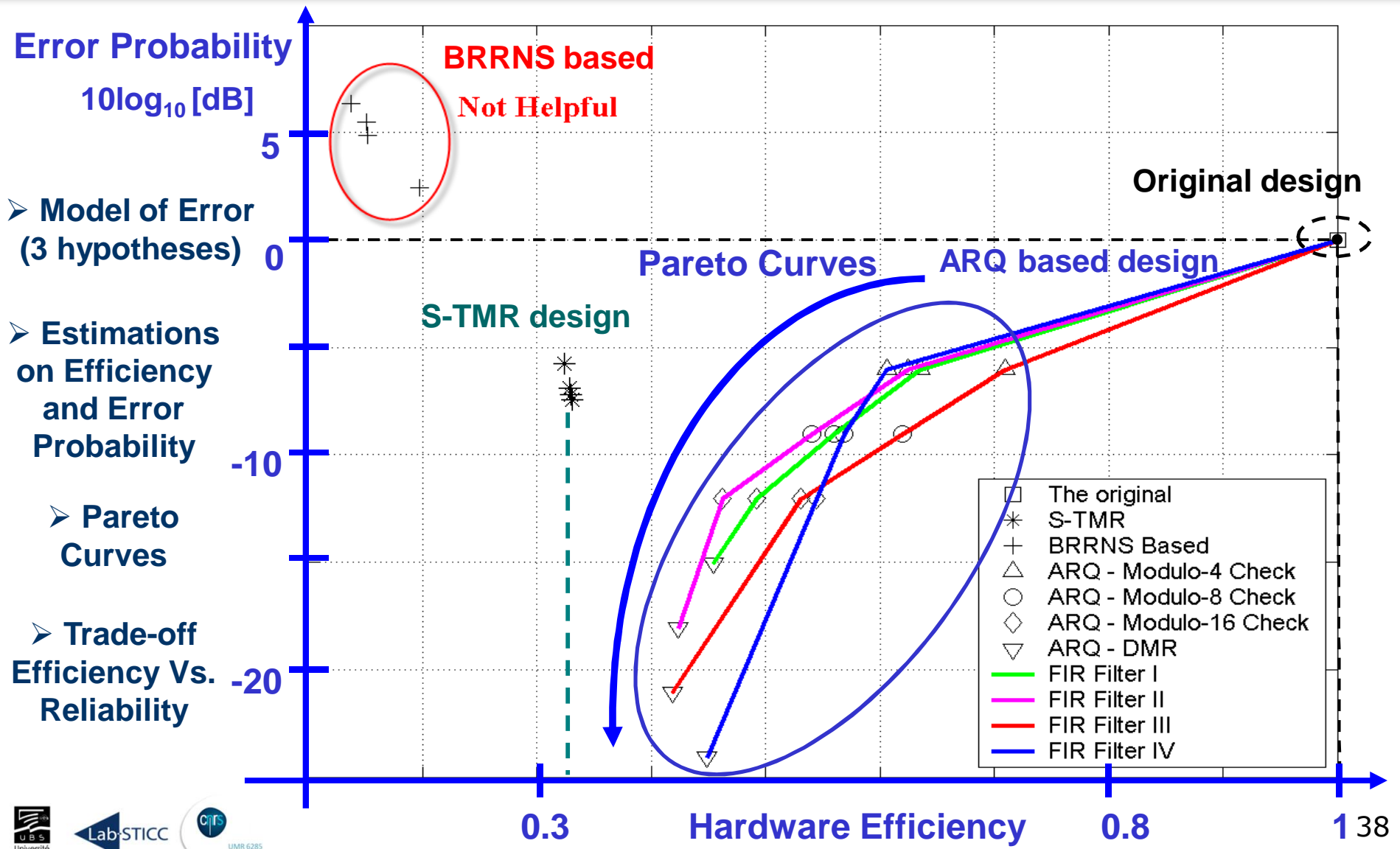


Filter implemented in RNS arithmetic with a BRRNS based decoder

Ref: Y. Tang et al., DASIP, 2010

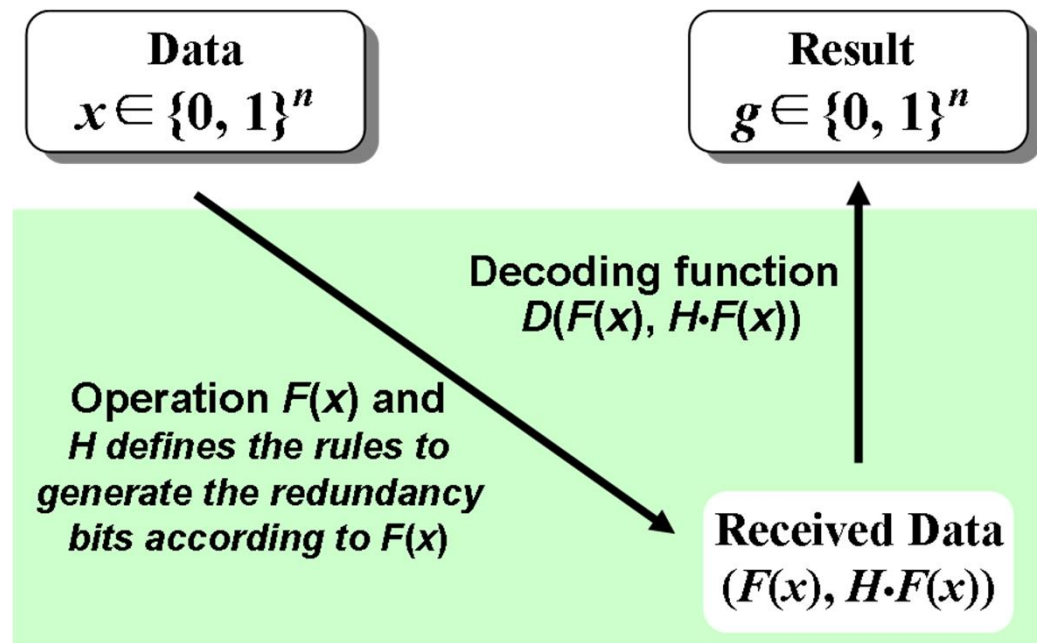
Ref: Y. Tang et al., SiPS, 2011

Contribution 2: Experimental Results – Pareto Curves

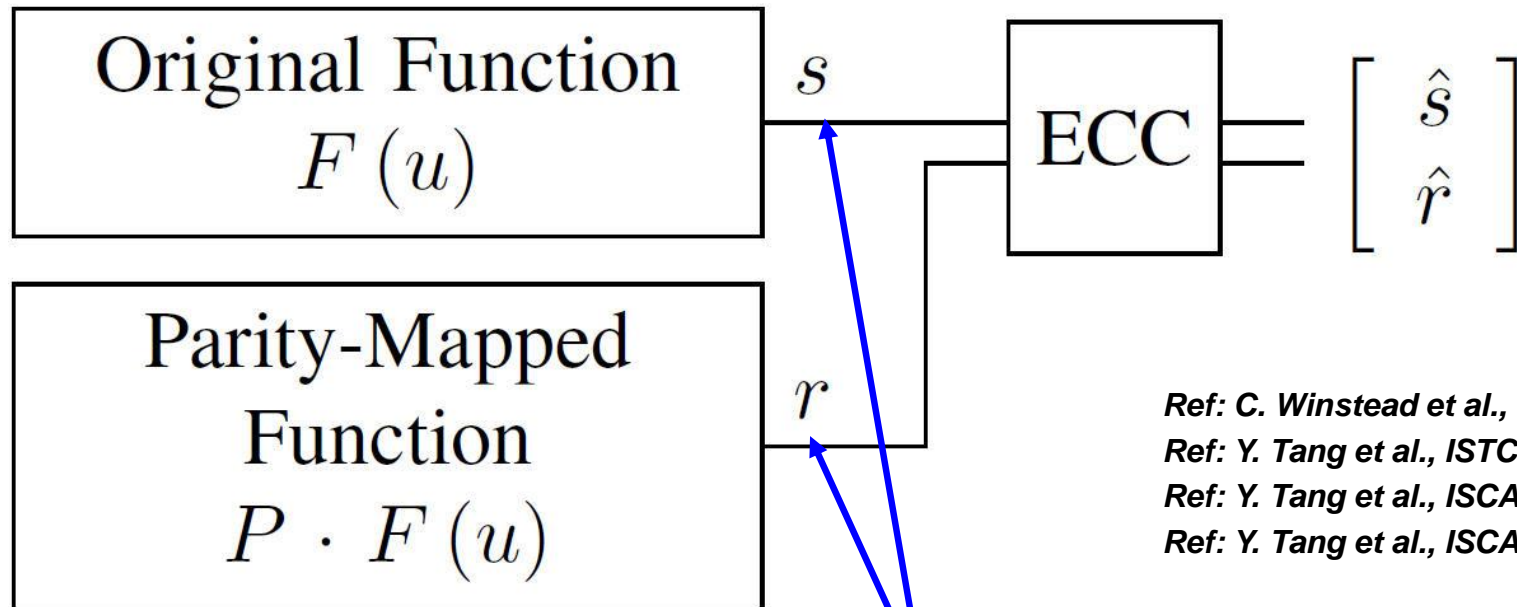


Proposed Work

- Contribution 3:
- A fault-tolerant system for computation
- Function & Parity-Mapped Function & ECC



Contribution 3: Coded Dual Modular Redundancy (cDMR)



Ref: C. Winstead et al., TCAS-II, 2009

Ref: Y. Tang et al., ISTC, 2012

Ref: Y. Tang et al., ISCAS, 2012

Ref: Y. Tang et al., ISCAS, 2013

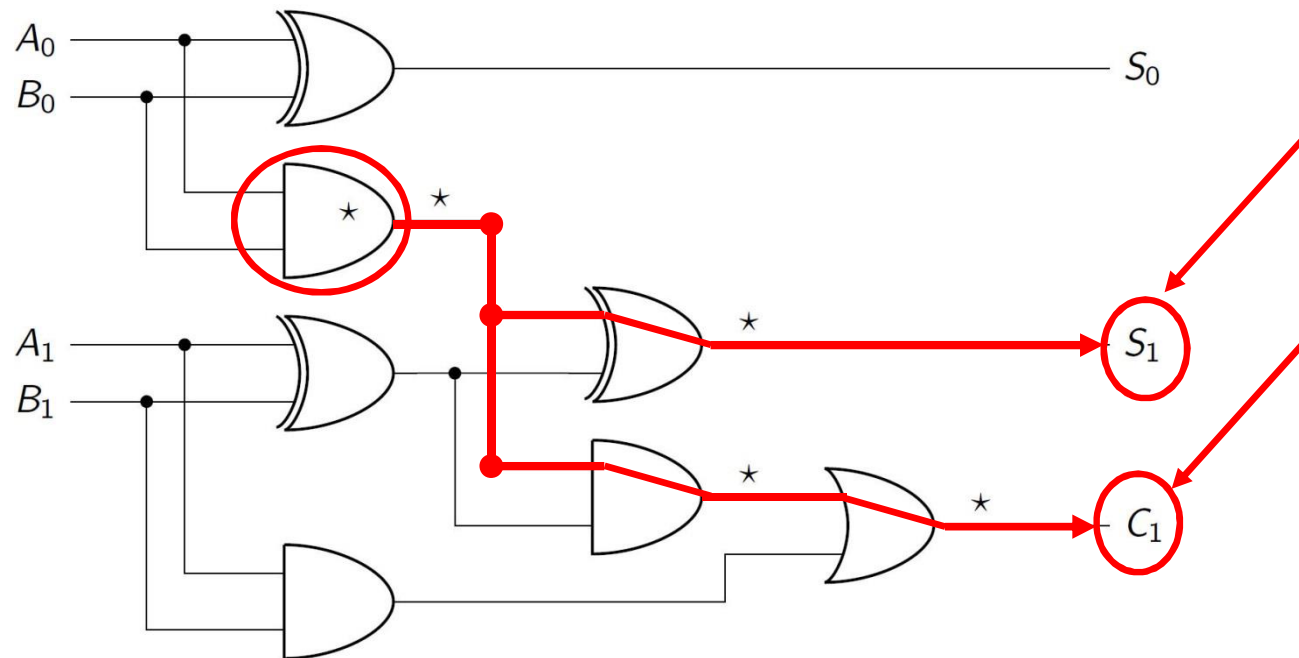
Need to avoid
correlated errors

“The original logic function $F(u)$ is composed with a block ECC code to create the parity-mapped function $P \cdot F(u)$.”

Contribution 3: Traditional Design

Original Function
 $F(u)$

= Two-bit Adder



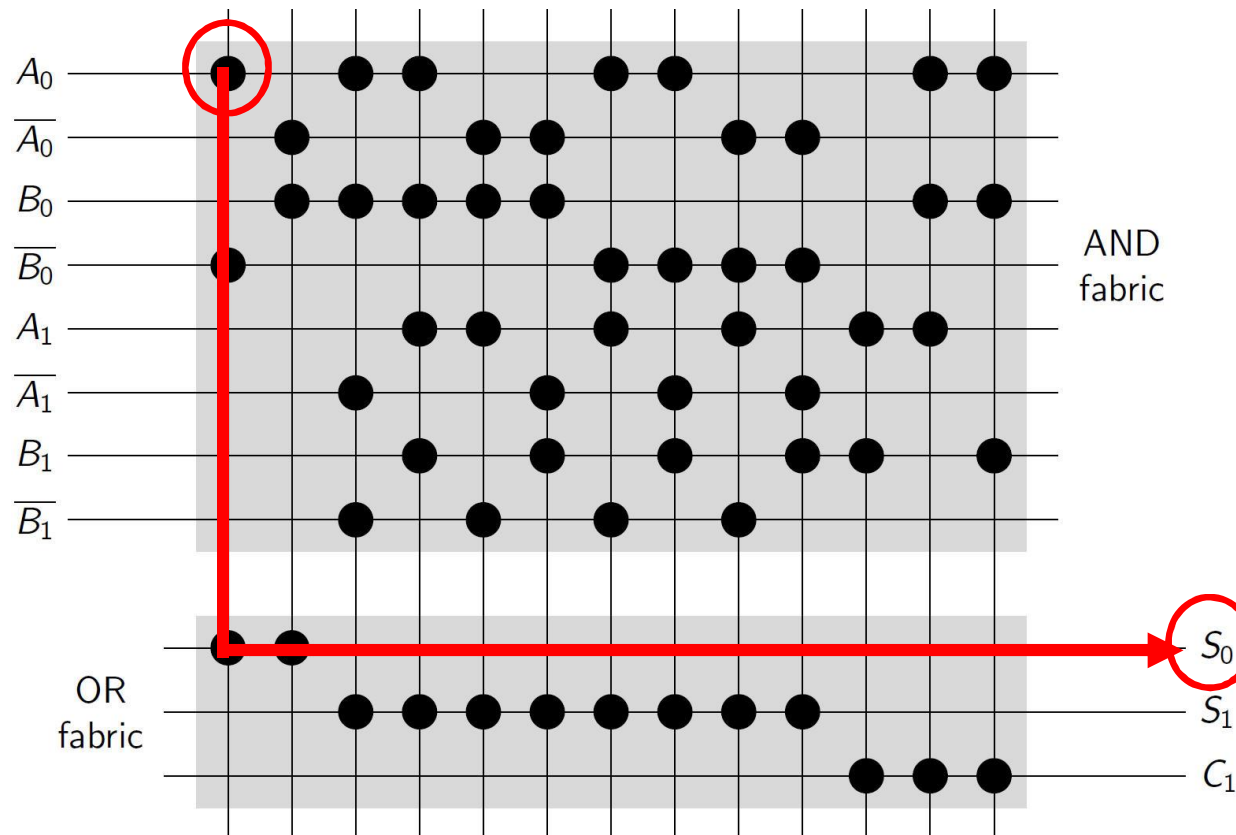
Traditional design, * symbol indicates the occurrence of an error.

Contribution 3: Cross-Bar Technique

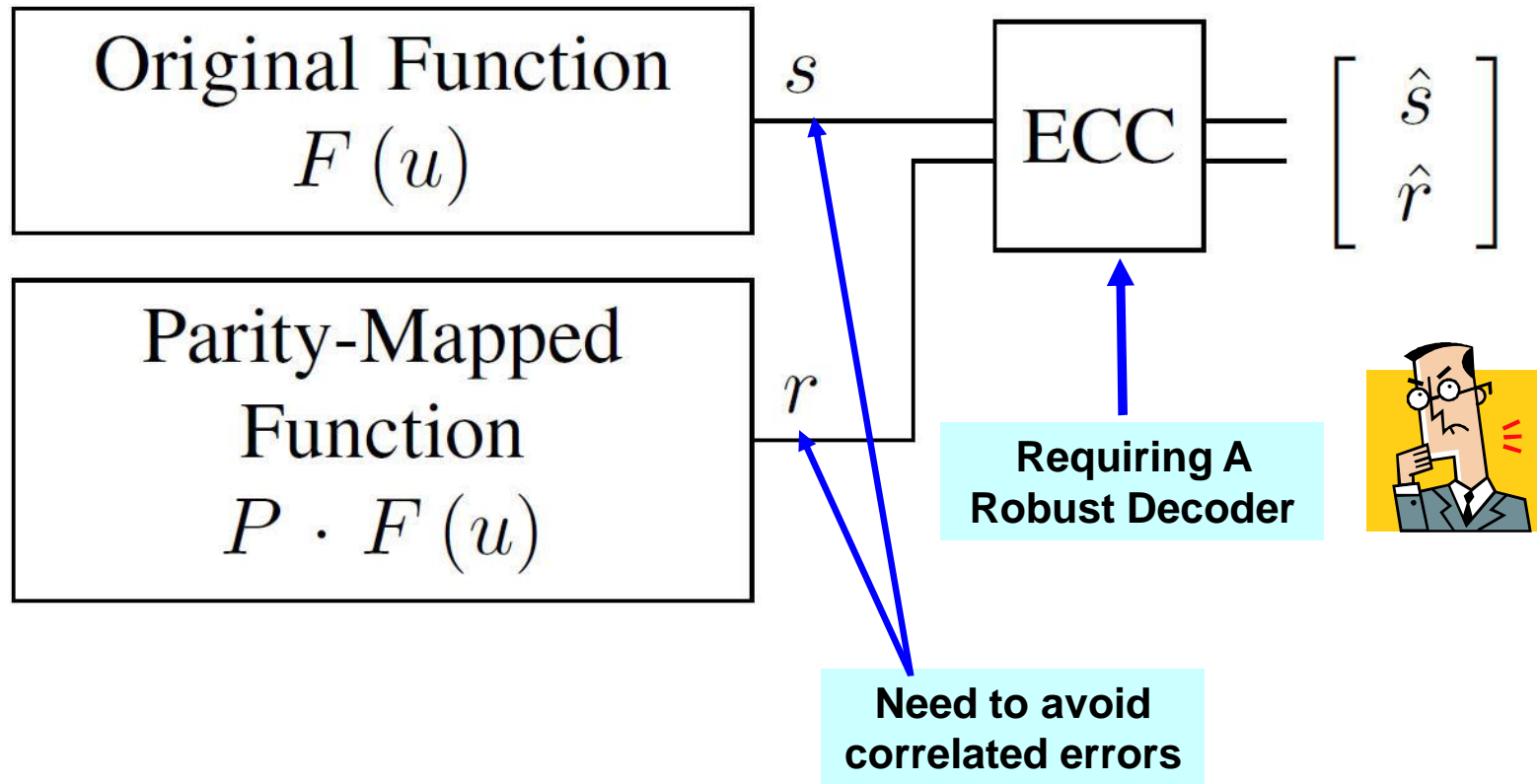
Original Function
 $F(u)$

= Two-bit Adder

Crossbar design suitable for some
nanoelectronic device families.

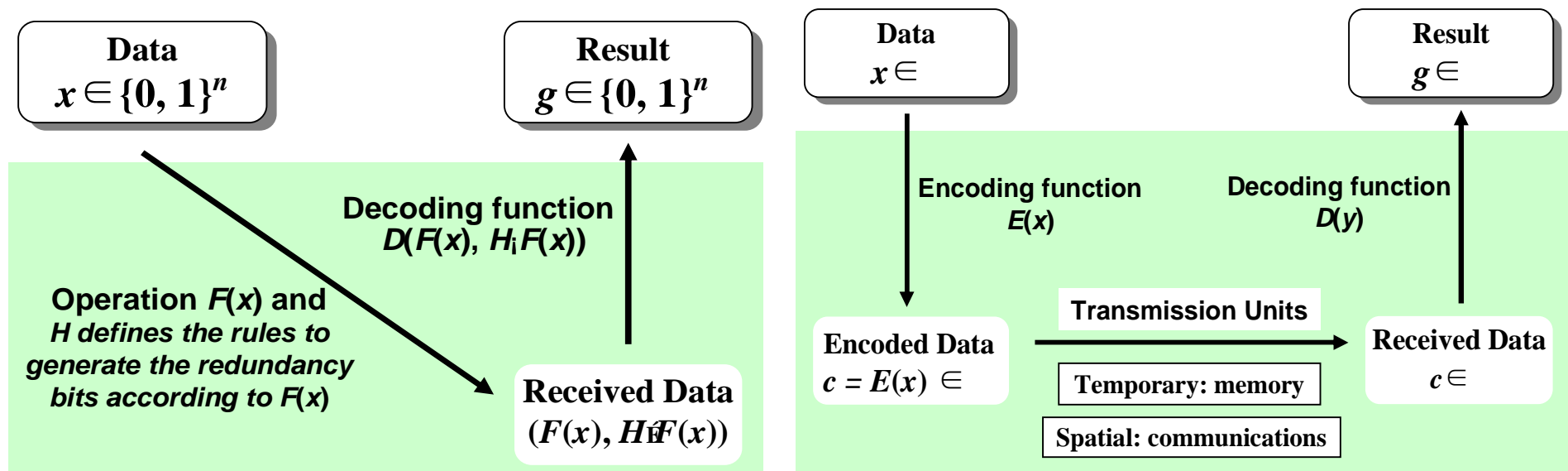


Contribution 3: Coded Dual Modular Redundancy (cDMR)

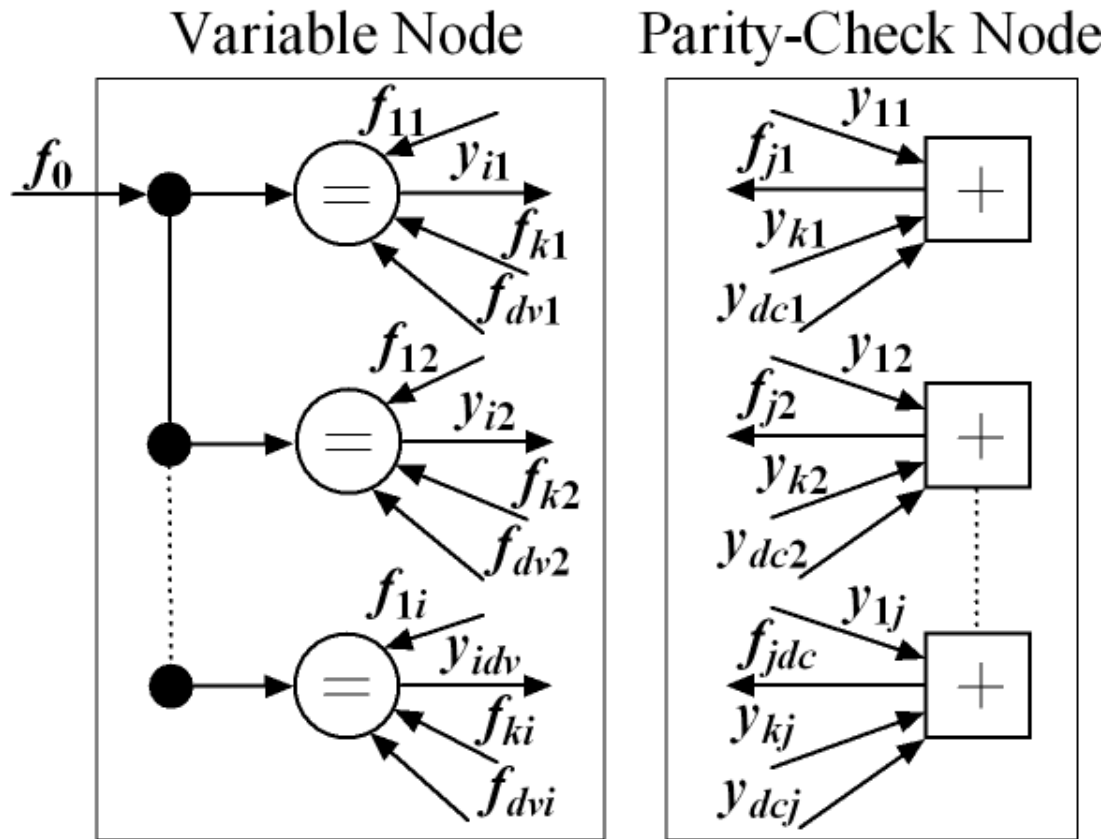


Contribution 4: Muller C-element based Decoder (MCD)

- Contribution 4:
- A decoder against internal faults
- Efficient Error-correction in the presence of high error rate noise



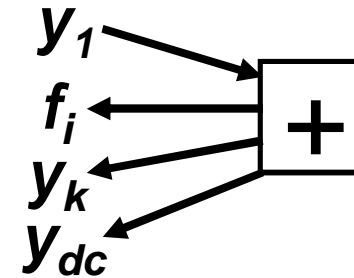
Related Work - Message Passing Decoding Method



Gallager-A Decoder

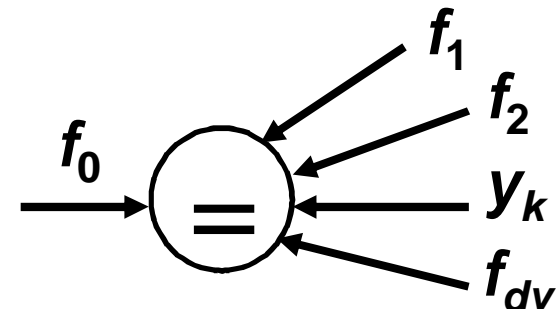
Check Node Processing

$$f_i = \oplus y_{k'} \quad k' \in \mathcal{C}_i$$



Variable Node Processing

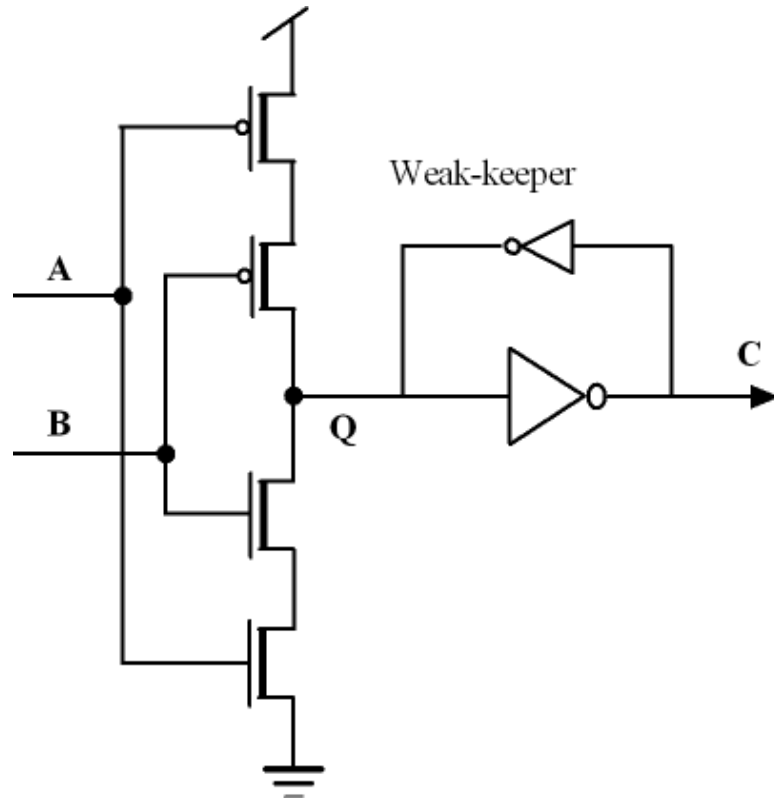
$y_k = f_0$ if $(f_i, i \in \mathcal{C}_k)$ disagree,
 f_1 otherwise



Ref: message-passing decoding, T. Richardson and R. Urbanke, 2001.

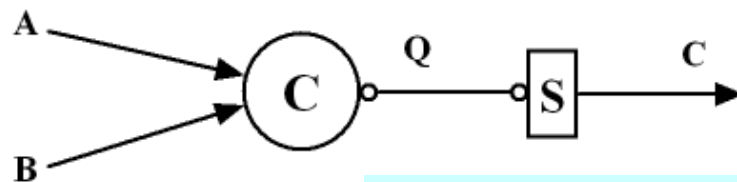
Contribution 4: Muller C-element based Decoder (MCD)

Ref : D. E. Muller and W. S. Bertky, 1959



Truth-Table of Binary C-element

A	B	C
0	0	0
0	1	C_{n-1}
1	0	C_{n-1}
1	1	1

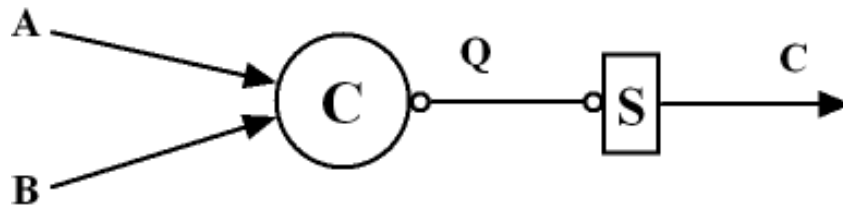


CMOS Implementations of binary C-element

" C_{n-1} denotes the state maintained via weak feedback.

Contribution 4: C-element – Error-Resilience in Nature

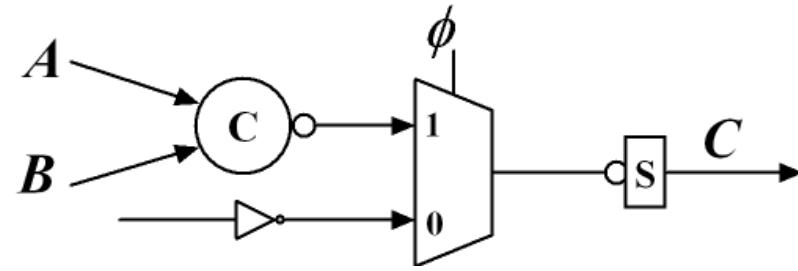
C-element



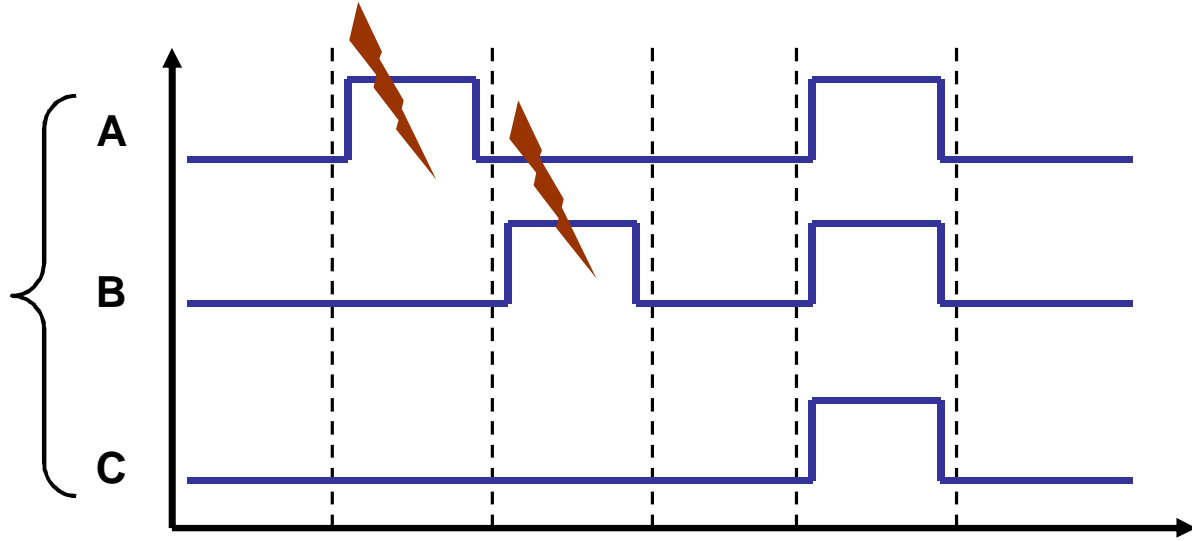
Truth-Table of Binary C-element

A	B	C
0	0	0
0	1	C_{n-1}
1	0	C_{n-1}
1	1	1

“Initializing the state memory.”



Error-Resilience in Nature

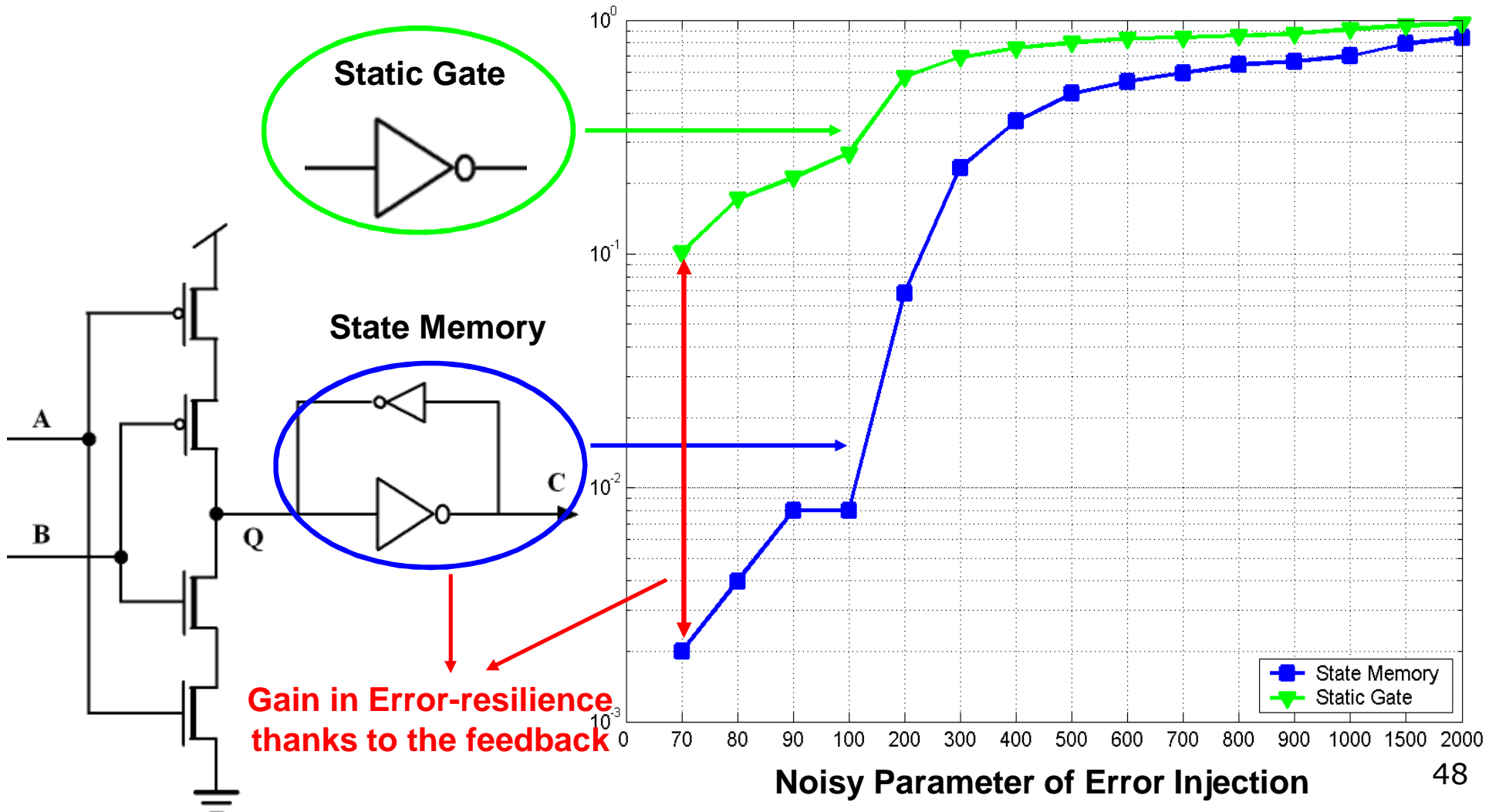


Contribution 4: C-element – Error-Resilience in Nature

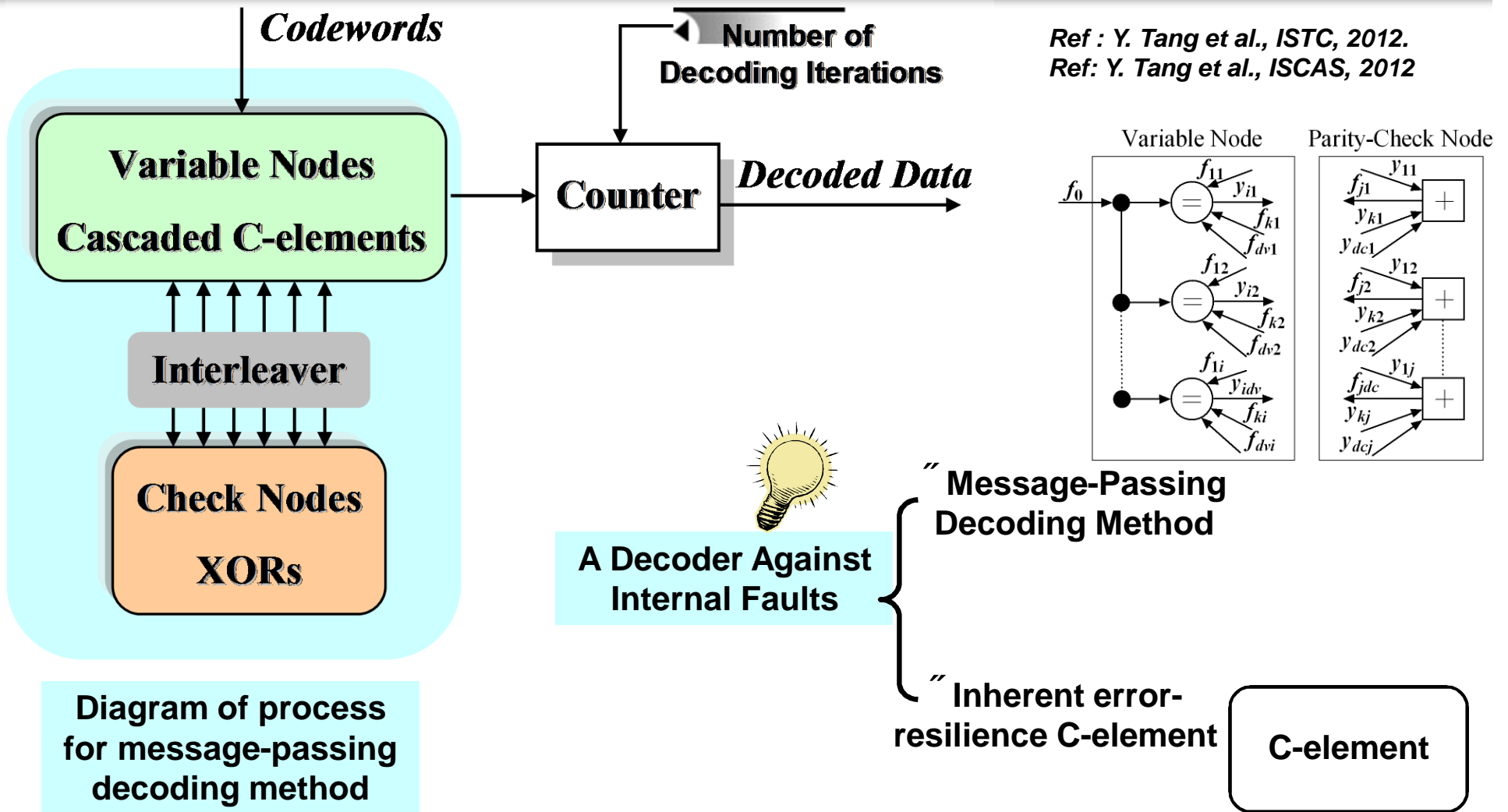
➤ Monte Carlo Simulation in Cadence Virtuoso Spectre

Ref : Y. Tang et al., ISCAS, 2013.

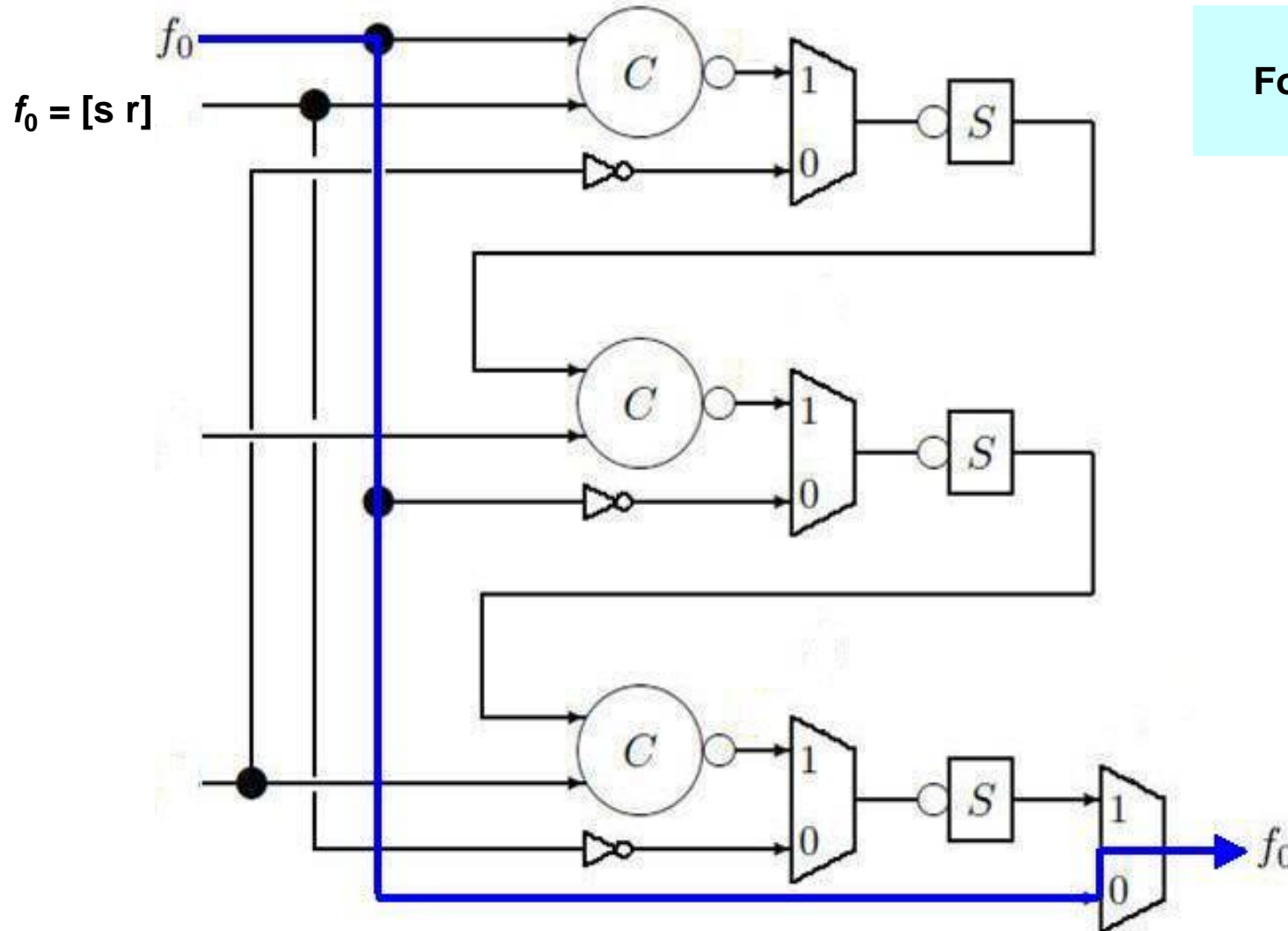
Error Probability



Contribution 4: Muller C-element based Decoder (MCD)



Contribution 4: Variable Node in MCD Phase 0



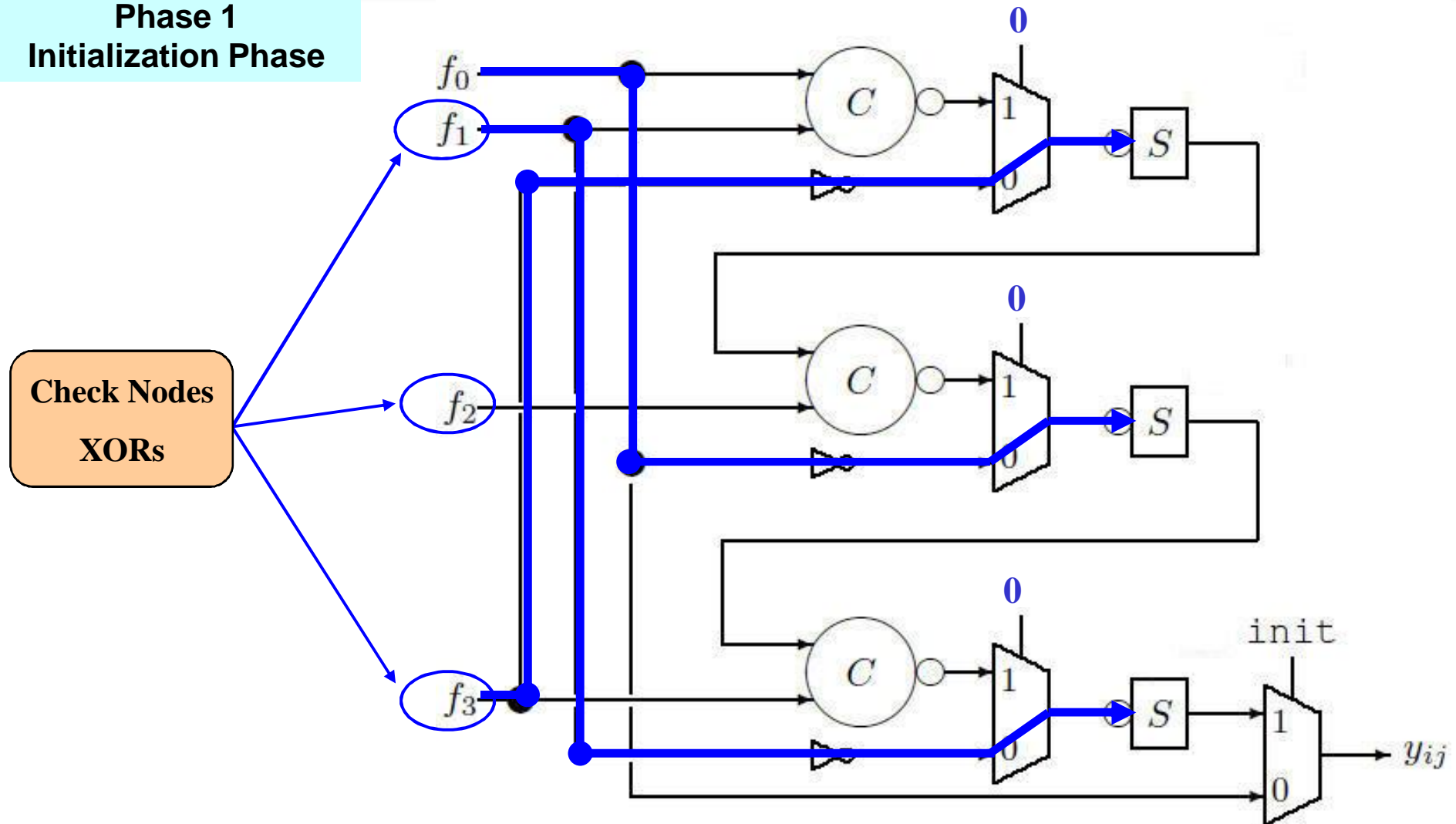
Phase 0
 Forward Channel
 Message

Check Nodes
 XORs

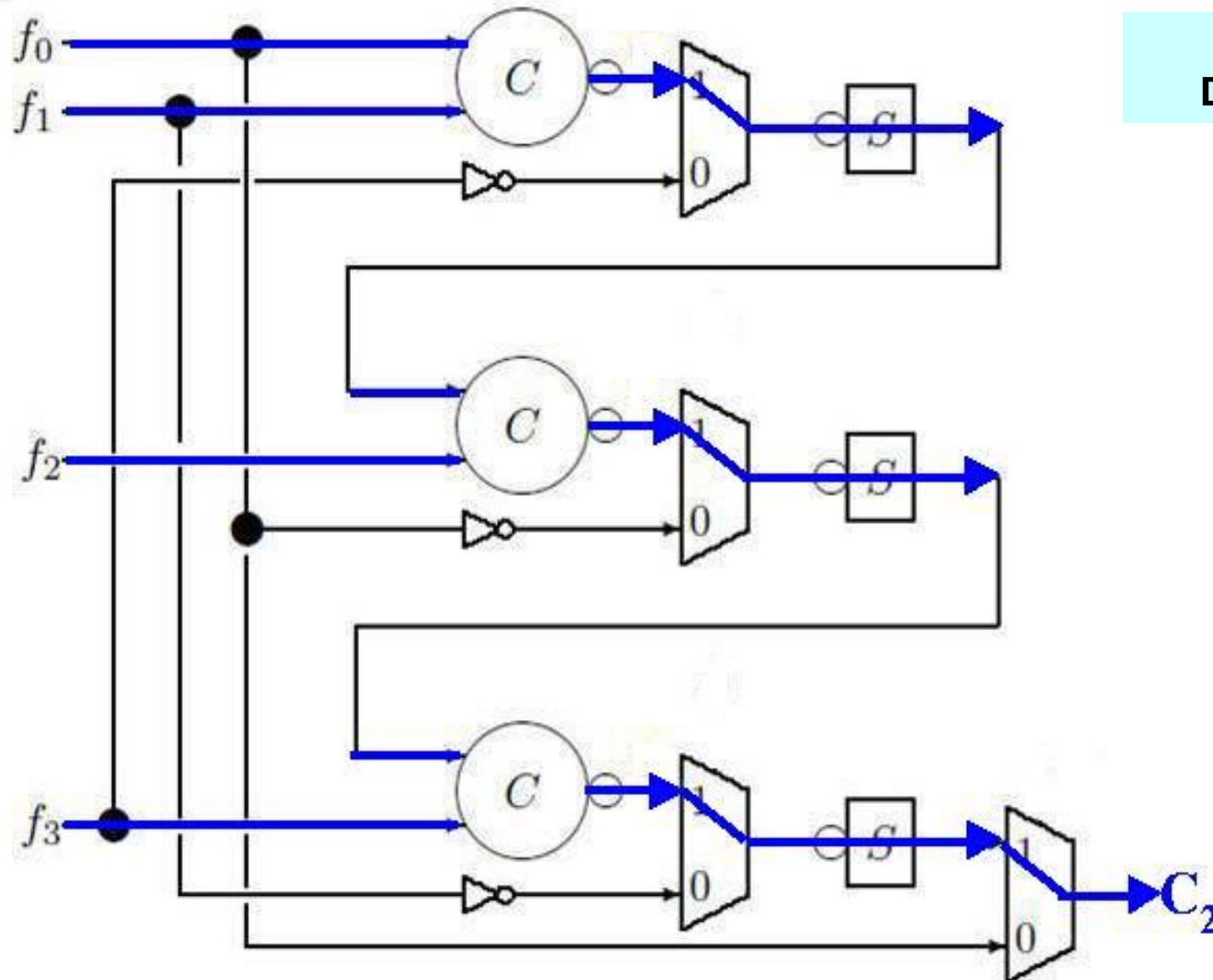
Basic Error-Correction Unit Cascaded C-elements, $d_v=4$

Contribution 4: Variable Node in MCD Phase 1

Phase 1
Initialization Phase

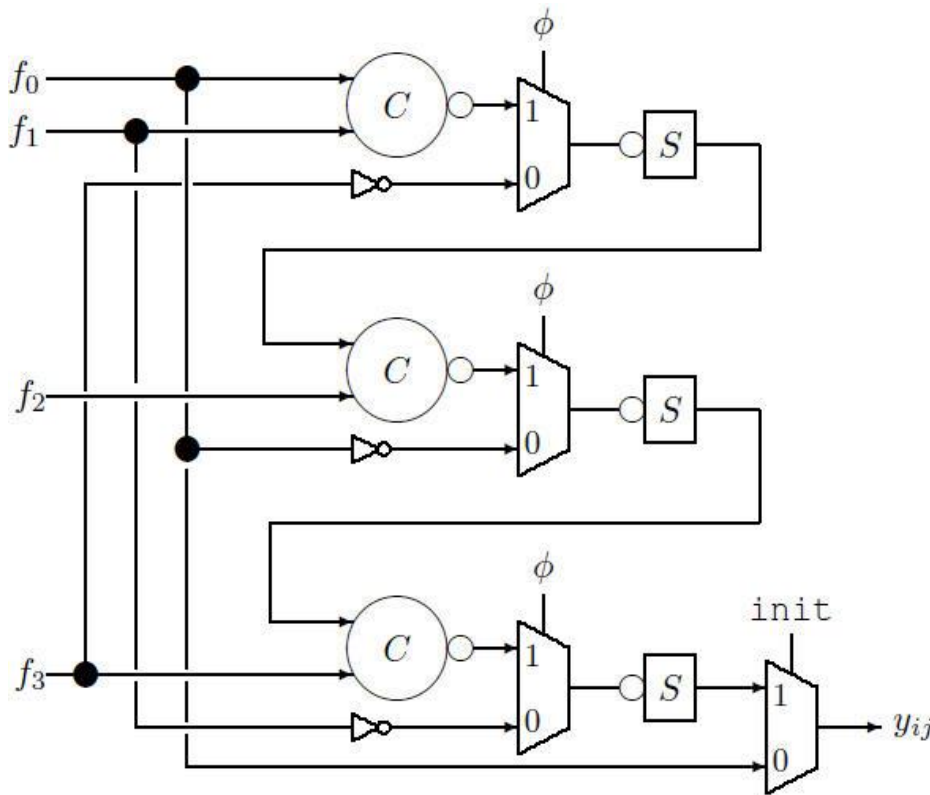


Contribution 4: Variable Node in MCD Phase 2

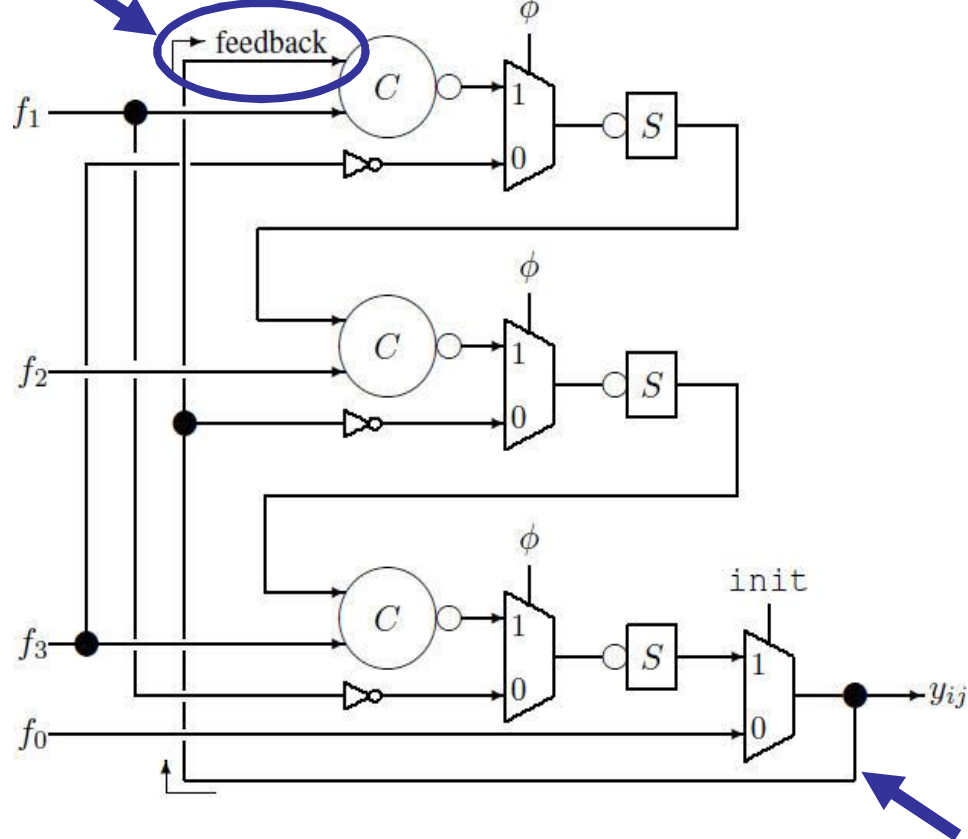


Iterative Phase 2
Driven by C-element

Contribution 4: MCDfb – feedback mechanism



MCD variable-node architecture, $d_v=4$

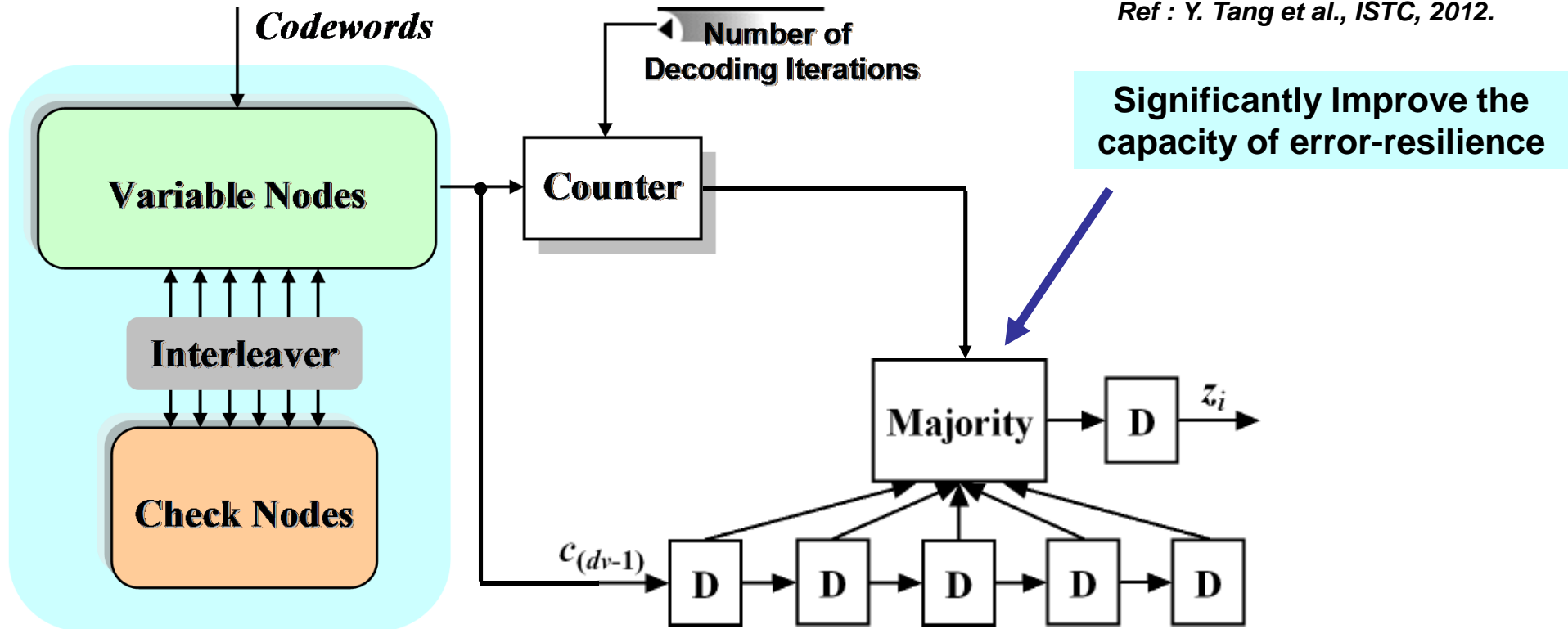


MCDfb variable-node architecture, $d_v=4$

A feedback mechanism employed to suppress internal fault event.

Contribution 4: Space-Time Technique – Improving BER

Ref : Y. Tang et al., ISTC, 2012.



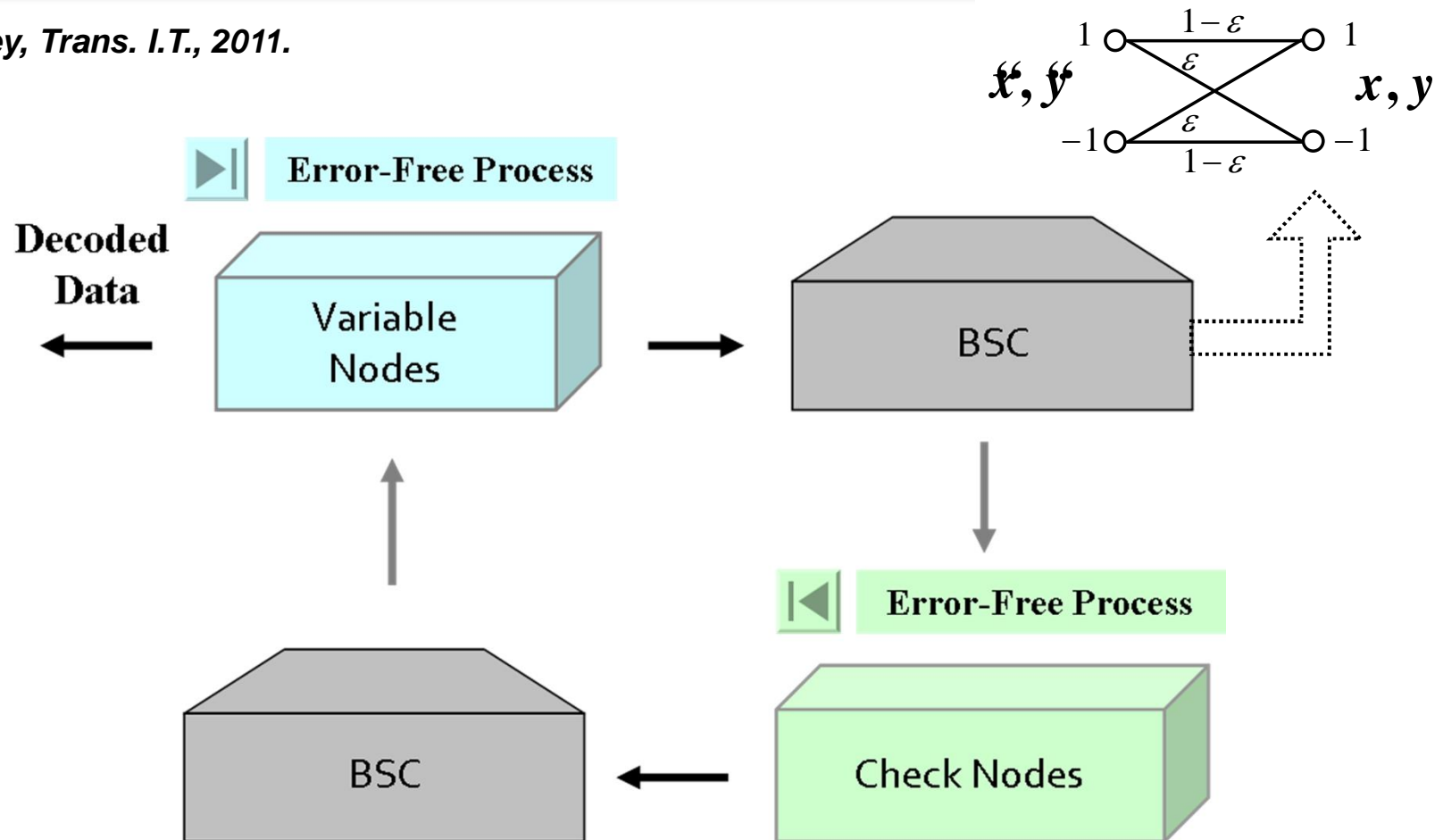
Message-Passing Decoder with Space-Time Redundancy Technique. For instance, the Majority unit can be a 3-of-5 voter.

Outline

- Introduction
- Formal Models of Embedded ECC
- Proposed Approaches
- **Experimental Results**
 - MCDB BER Performance Under a Faulty Process
 - Comparisons Between MCD and GBF
 - The Improvement of Decoding Performance by Space-Time Technique
- Conclusion & Future Perspectives

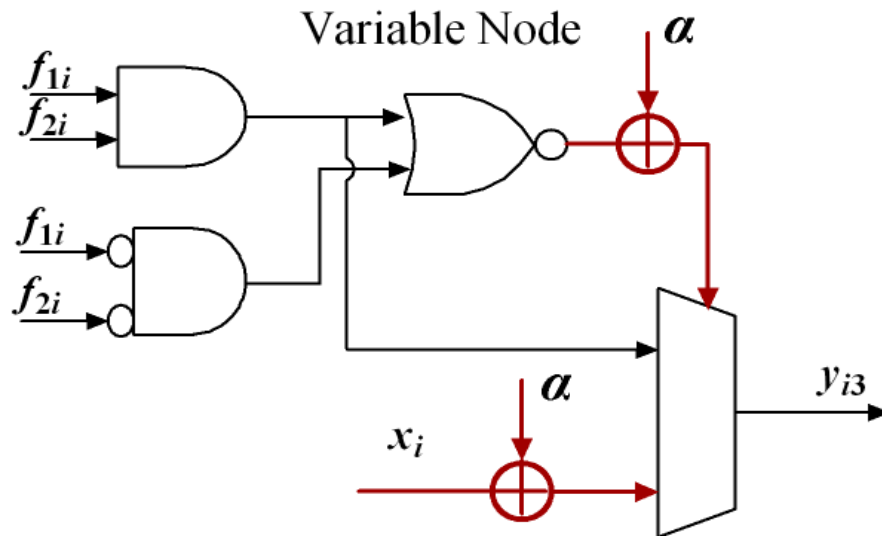
Error Model

Ref : L. Varshney, *Trans. I.T.*, 2011.

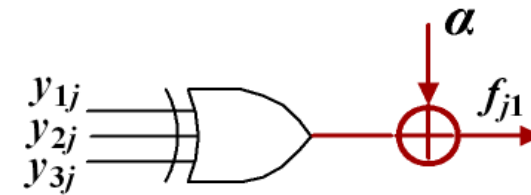


MCD BER Performance Under a Faulty Process

➤ Gallager Bit-Flipping Decoding Method (GBF)



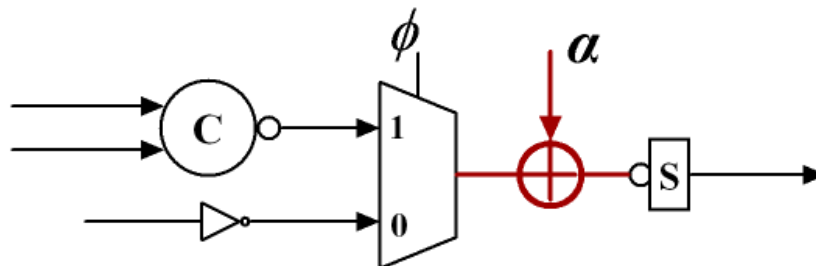
Parity-Check Node



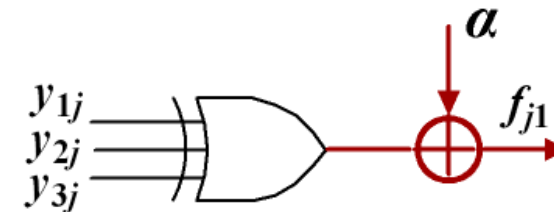
➤ Injected into critical path in RTL

➤ MCD

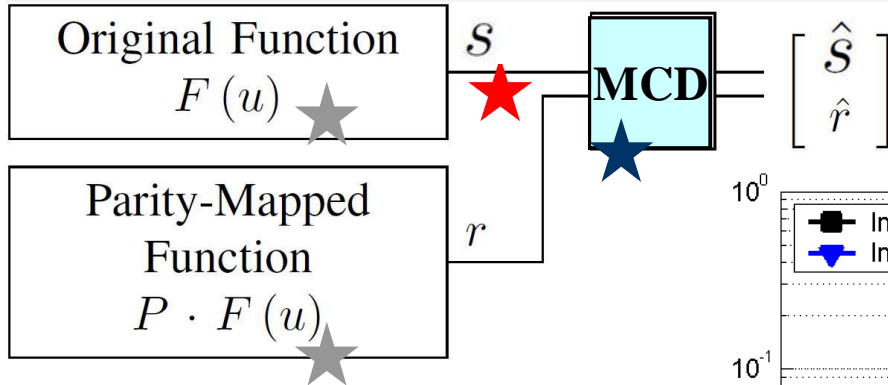
C-element of Variable Node



Parity-Check Node

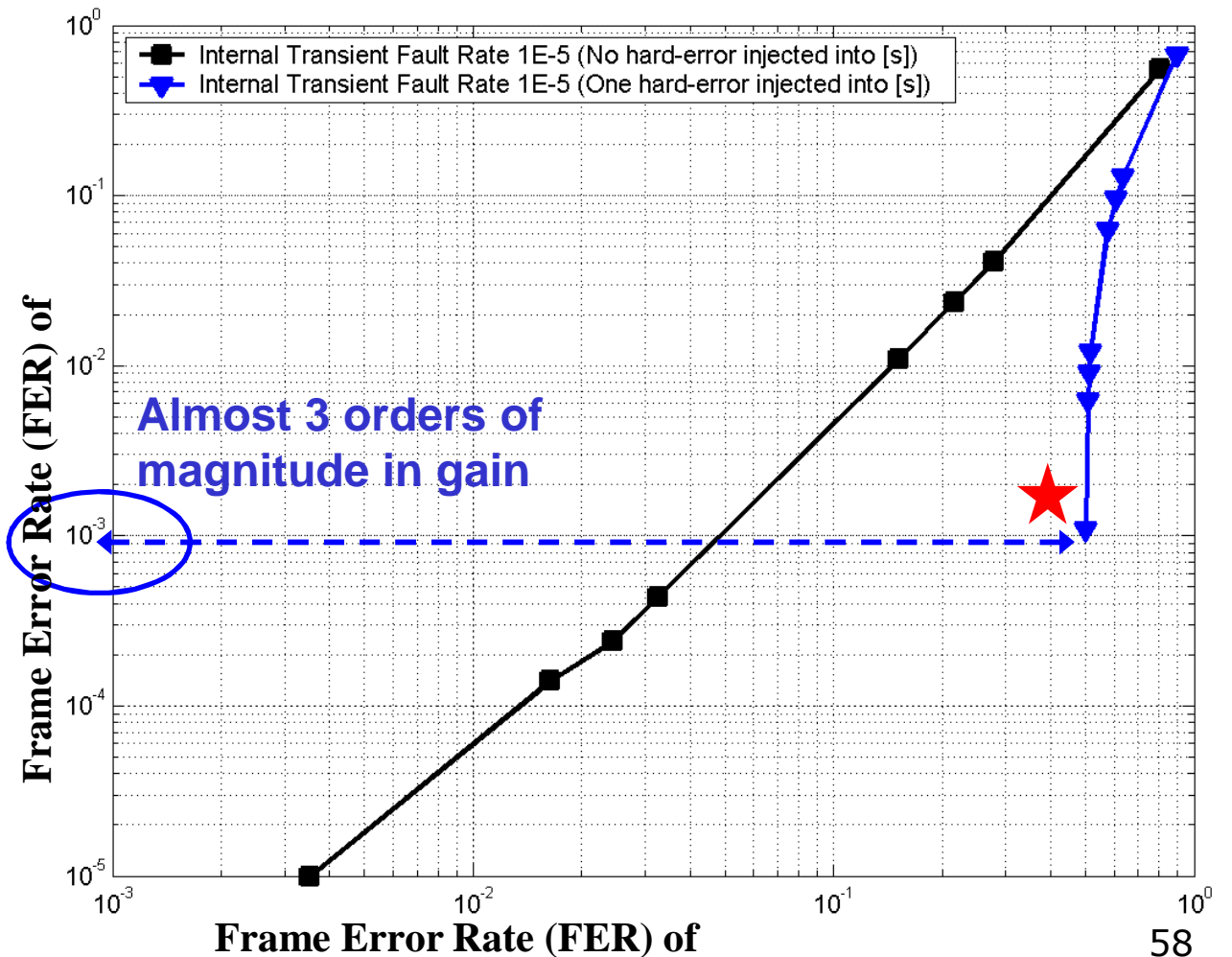


MCD BER Performance Under a Faulty Process



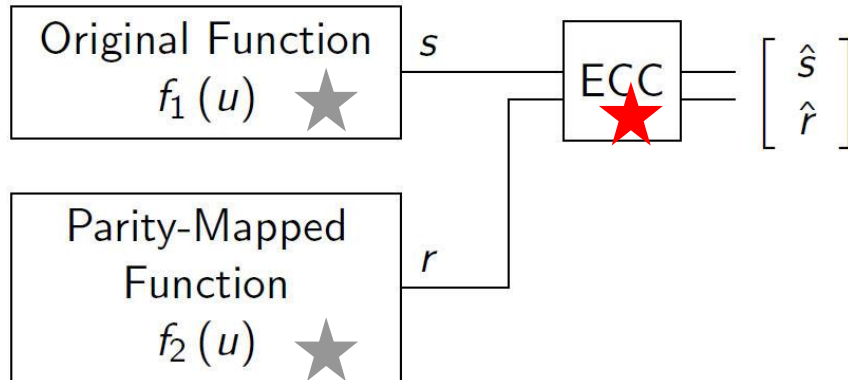
Simulation Results for rate (3,6) LDPC codes based on MCD architecture, 2 iterations.

- ★ Error occurred in Function
- ★ Single Hard-Error Injection
- ★ Internal Fault Injection (Error Rate 10^{-5})



Comparisons Between MCD and GBF, (3,6) LDPC codes $N=64$

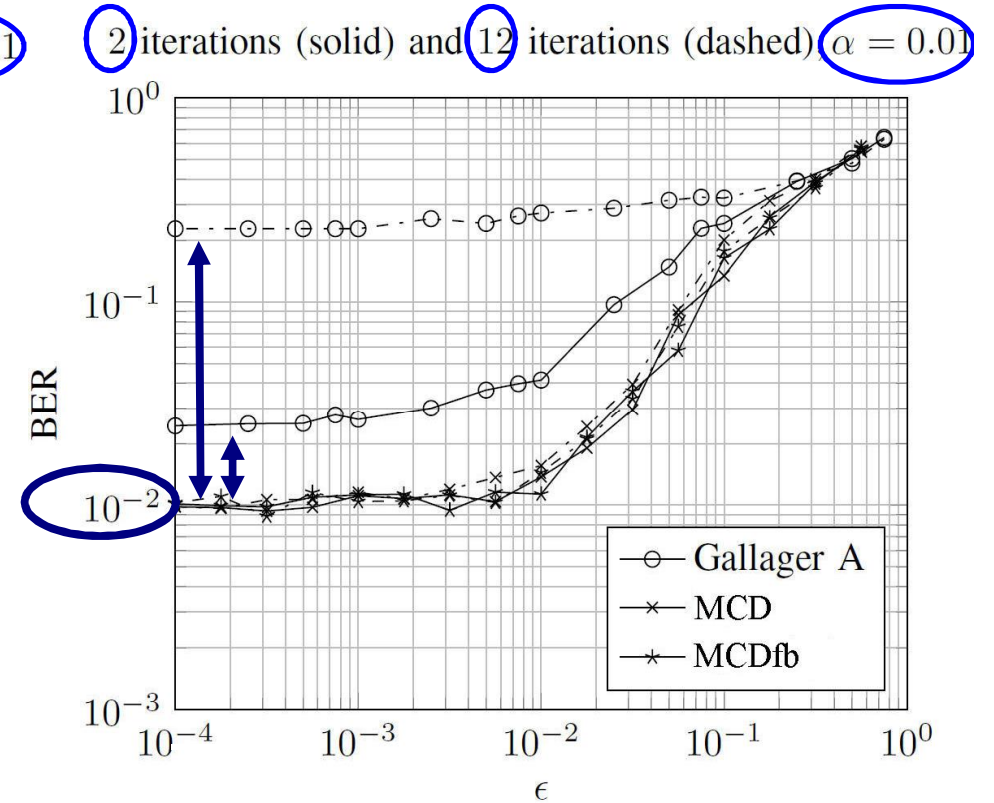
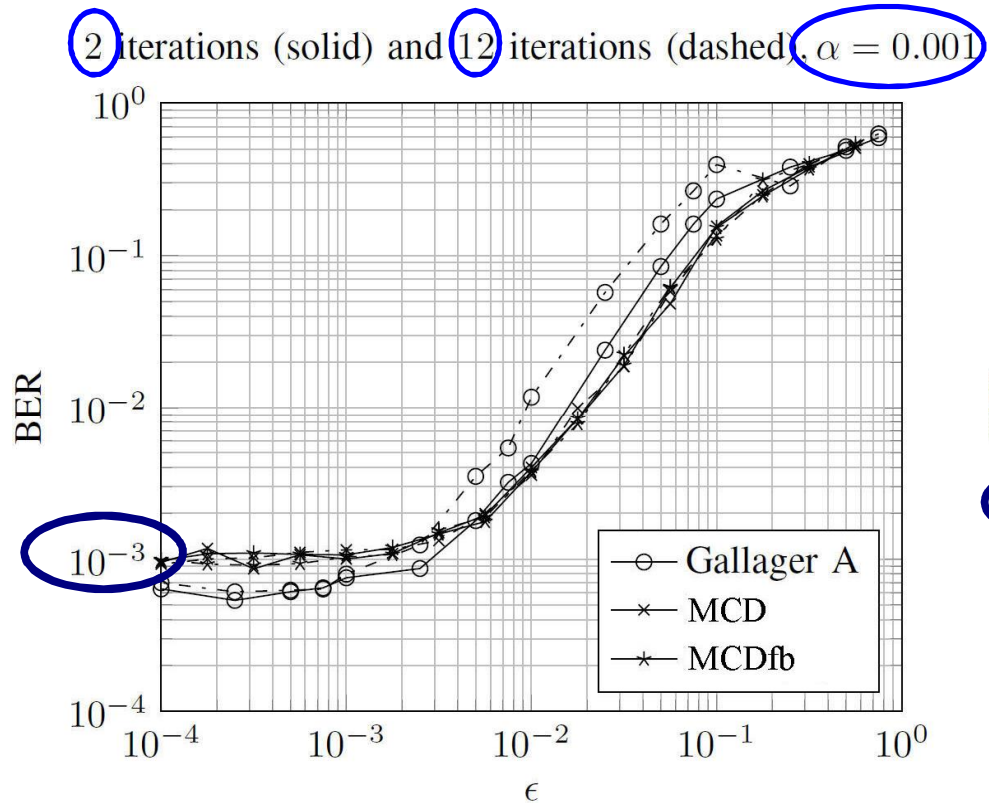
Ref : Y. Tang et al., ISTC, 2012.



★ **Error occurred in Function: *Epsilon***

★ **Internal Fault Injection: *Alpha***

Comparisons Between MCD and GBF, (3,6) LDPC codes $N=64$



“ Gallager-A performance worsens with increased iterations when internal faults rate is high.

“ But MCD does not exhibit this degradation.

Decoding Performance Improvement by Space-Time Technique

**WITH Space-Time
Redundancy**

Internal Fault Injection: *Alpha*
 $(3,6)$ LDPC code of length 64 over a BSC parameter of *Epsilon*

Ref : Y. Tang et al., ISTC, 2012.

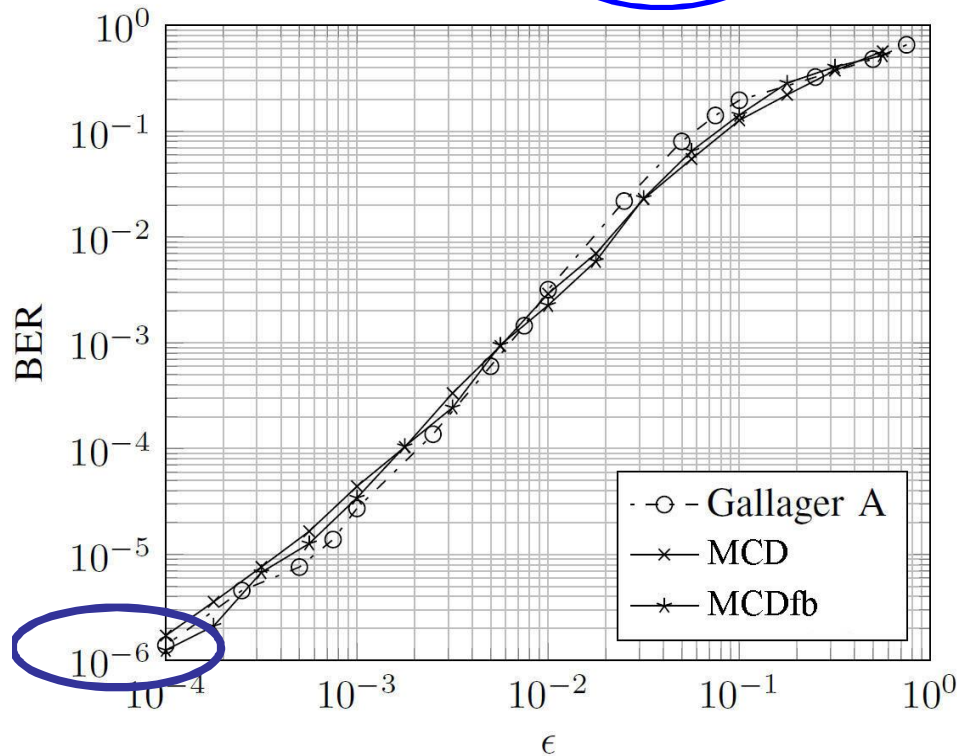
Decoding Performance Improvement by Space-Time Technique

WITH Space-Time Redundancy

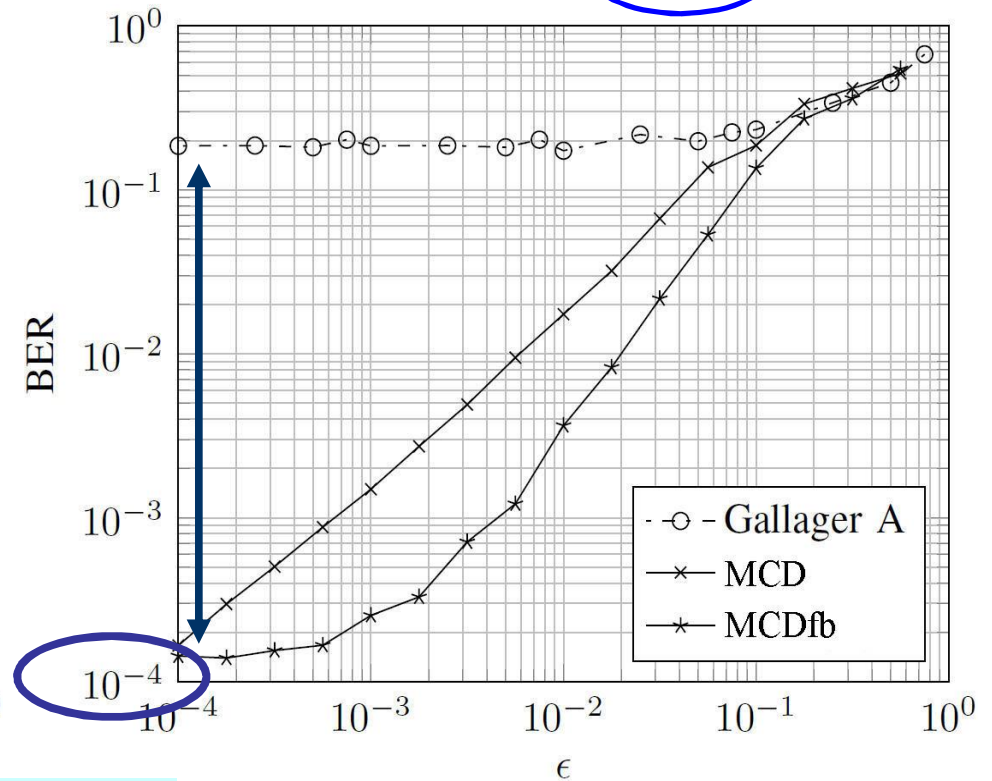
Internal Fault Injection: *Alpha*

$\epsilon(3,6)$ LDPC code of length 64 over a BSC parameter of *Epsilon*

25 iterations $\alpha = 0.001$



25 iterations $\alpha = 0.01$



"The majority voter significantly improves the decoder's BER performance."

"Feedback-Mechanism is helpful as well."

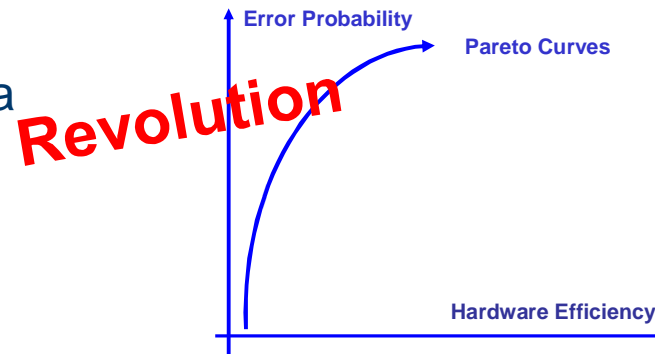
Outline

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- Architectural Approaches
- Experimental Results
- Conclusion & Future Perspectives

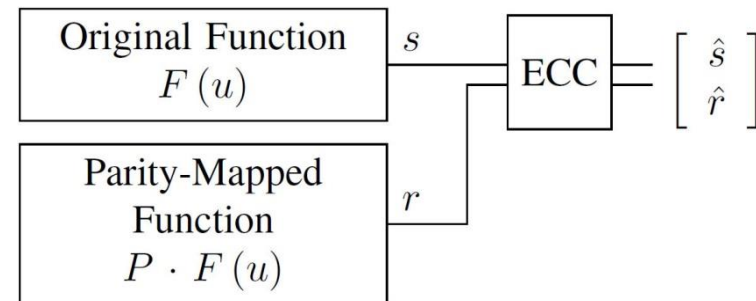
Conclusion

- Objective : to design an efficient error-resilient architecture by ECC
- Contribution :
- An arithmetic method (Fast computation & Error-resilience)

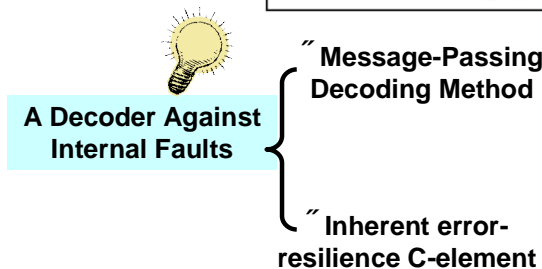
- Reliability-Efficiency Criteria



- A fault-tolerant system for computation

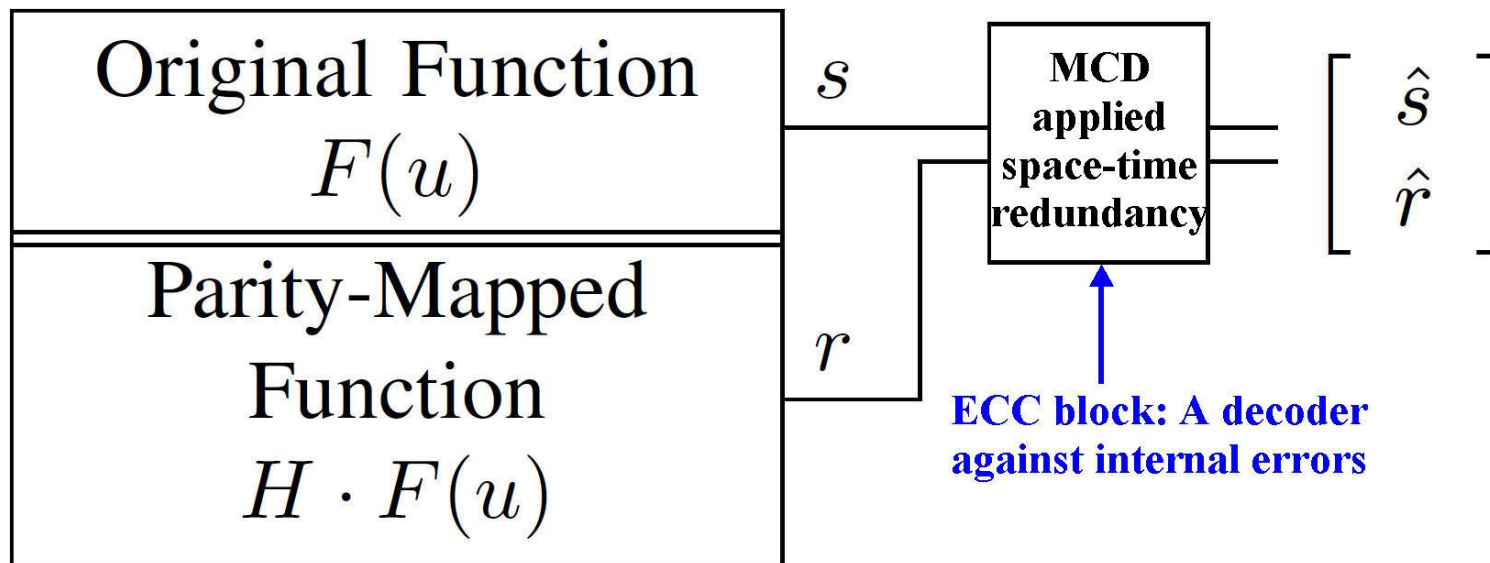


- A decoder against internal faults



Future Perspectives

- For MCD, high code rate, such like a (3,4) code, would be worth to try.
- Moreover, simulating a 3-of-5 voter based space-time technique.
- Approaching the proposed reliability criterion as a precise error model.
- Implementation of the cDMR & MCD & Space-Time Technique.



Future Directions

- **Collaboration** (2011. June - 2012) with LEFT-Lab directed by Dr. Chris Winstead, Utah State University, Logan, Utah, USA.
- **Continued Work** . Application of an ANR Project on this subject
 - Collaboration with, LEFT-Lab, *Dr. Winstead*
 - . Ongoing thesis work, *Pr. Jégo*
 - . Asynchronous Decoder to decrease power consumption



List of Publications

- *Yangyang Tang, Emmanuel Boutillon, Chris Winstead, Christophe Jégo, and Michel Jézéquel*
öMuller C-element based Decoder (MCD): A decoder against transient faults,ö
In Circuits and Systems (ISCAS), 2013 IEEE International Symposium on, May 2013. } 2013
- *Chris Winstead, Yangyang Tang, Emmanuel Boutillon, Christophe Jégo, and Michel Jézéquel*
öA space-time redundancy technique for embedded stochastic error correction,ö
Turbo Codes and Related Topics (ISTC), 2012 7th International Symposium on, August 2012.
- *Yangyang Tang, Chris Winstead, Emmanuel Boutillon, Christophe Jégo, and Michel Jézéquel*
öAn ldpc decoding method for fault-tolerant digital logic,ö
In Circuits and Systems (ISCAS), 2012 IEEE International Symposium on, May 2012, pp. 3025-3028. } 2012
- *Yangyang Tang, Gopalakrishnan Sundararajan, Chris Winstead, Emmanuel Boutillon, Christophe Jégo, and Michel Jézéquel*
öTechniques and prospects for fault-tolerance in post-CMOS ULSI,ö
21st International Workshop on Post-Binary ULSI Systems (ULSIWS), May 2012.
- *Yangyang Tang, Emmanuel Boutillon, Christophe Jégo, and Michel Jézéquel*
öHardware efficiency versus error probability in unreliable computation,ö
Signal Processing Systems (SiPS), 2011 IEEE workshop on, October, 2011 } 2011
- *Yangyang Tang, Emmanuel Boutillon, Christophe Jégo, and Michel Jézéquel*
öA new single-error correction scheme based on self-diagnosis residue number arithmetic,ö
Design and Architectures for Signal and Image Processing (DASIP), 2010. } 2010
- *Emmanuel Boutillon, Yangyang Tang, Cédric Marchand, and Pierre Bomel*
öHardware Discrete Channel Emulator,ö
High Performance Computing and Simulation (HPCS), 2010.

Acknowledgement



“(2010. Jan - 2013. Jan) This work was supported by the funding from East China Institute of Technology and Lab-STICC, Lorient.



“(2011. June - 2012. June) This work was also supported by the US National Science Foundation under award ECCS-0954747 and CCF-0916105.

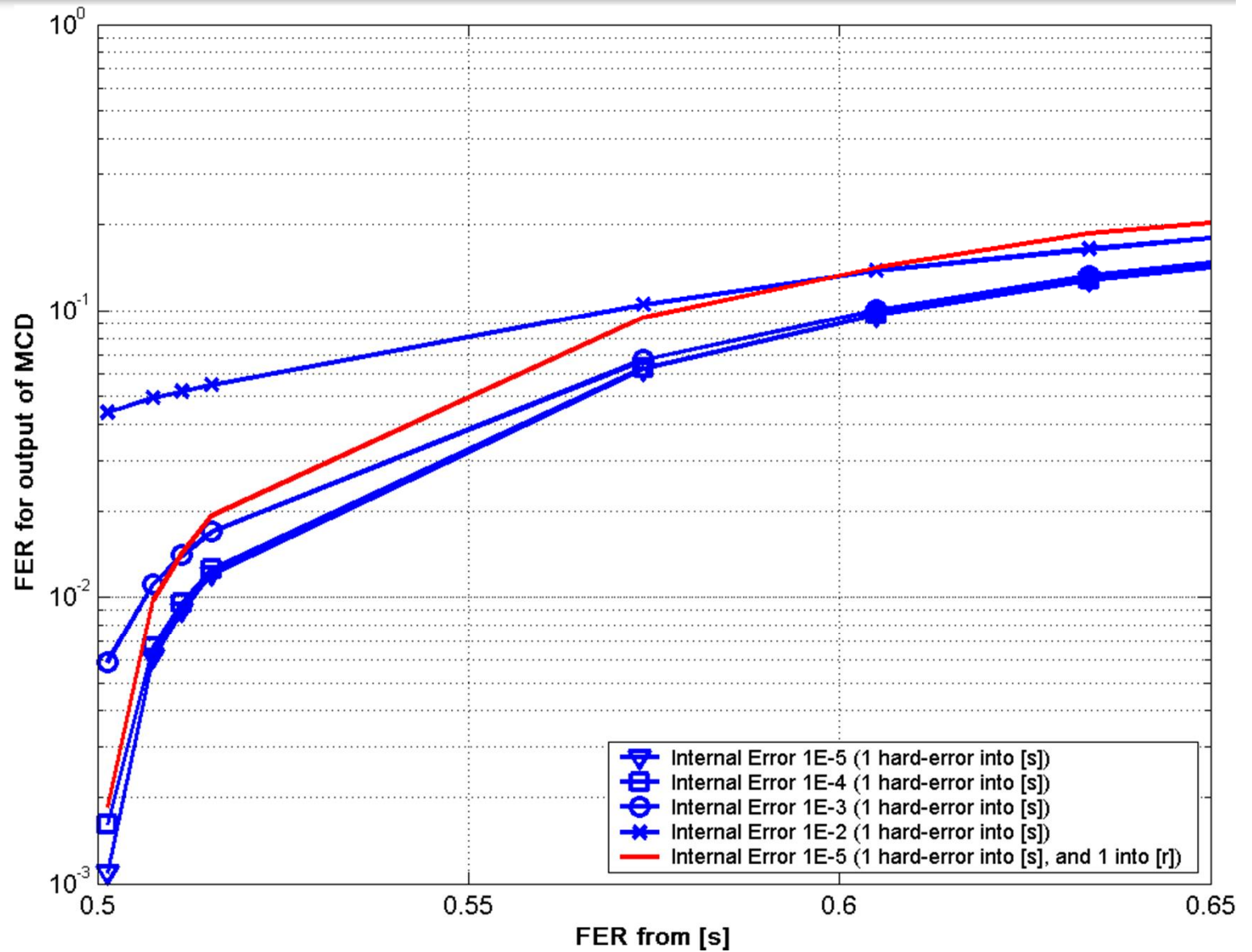
Questions

Thank You For Your Attention!

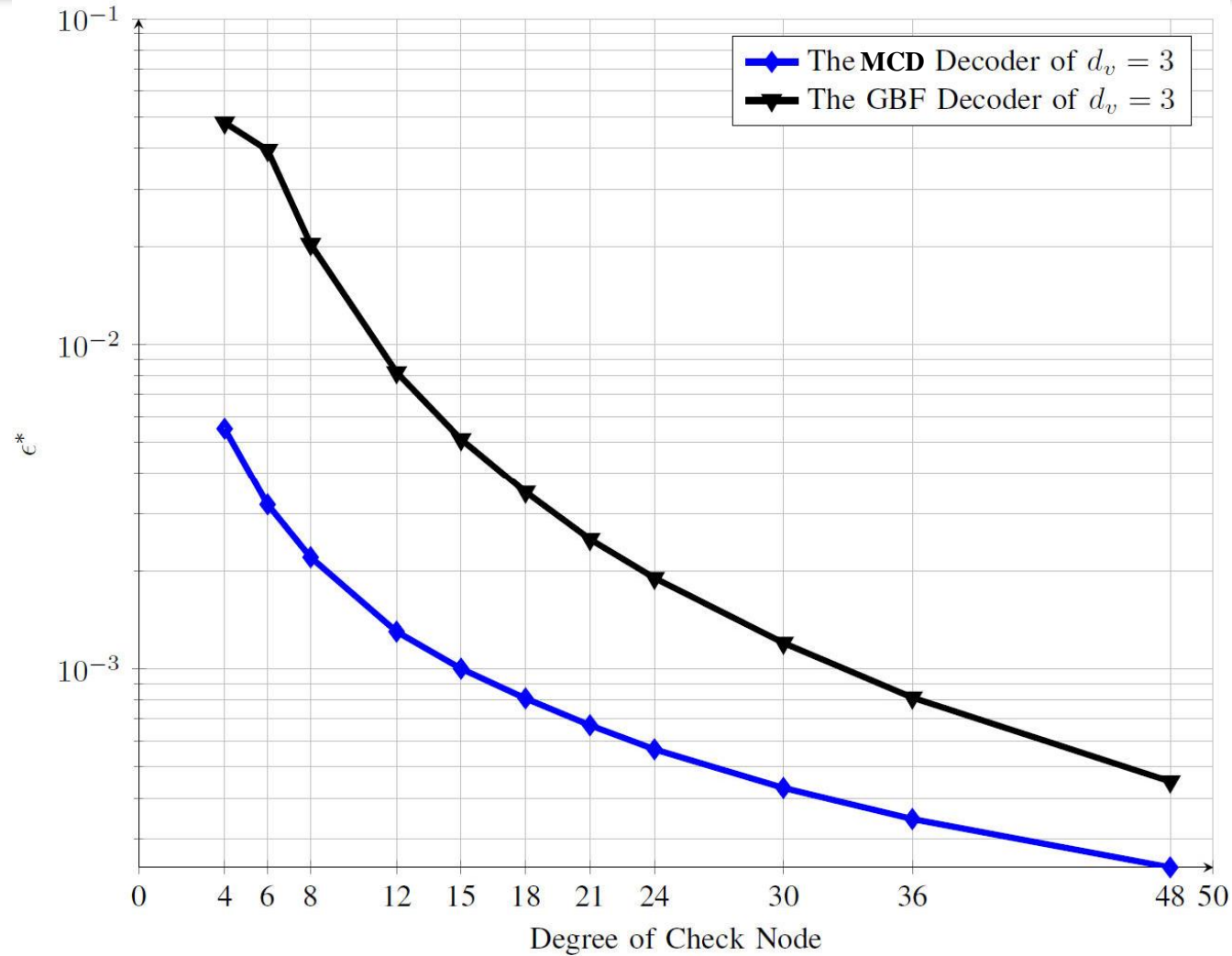


Back-Ups: FERs by Applying cDMR

(3,6) LDPC code of length 64 under 2 iterations



Back-Ups: Threshold Determinations



: BSC Channel
Parameter

Threshold of channel parameter over BSC for GBF (Gallager's Bit-Flipping method) and MCD, if the decoder process is error-free.

Back-Ups: Threshold Determinations

	GBF	MCD			
	(d_v, d_c)	(d_v, d_c)	(d_v, d_c)	(d_v, d_c)	(d_v, d_c)
	(3,6)	(3,6)	(3,12)	(3,24)	(3,48)
α^*	0.0082	0.0303	0.0096	0.0032	0.0011
ϵ^*	0.0126	0.0690	0.0199	0.0059	0.0011

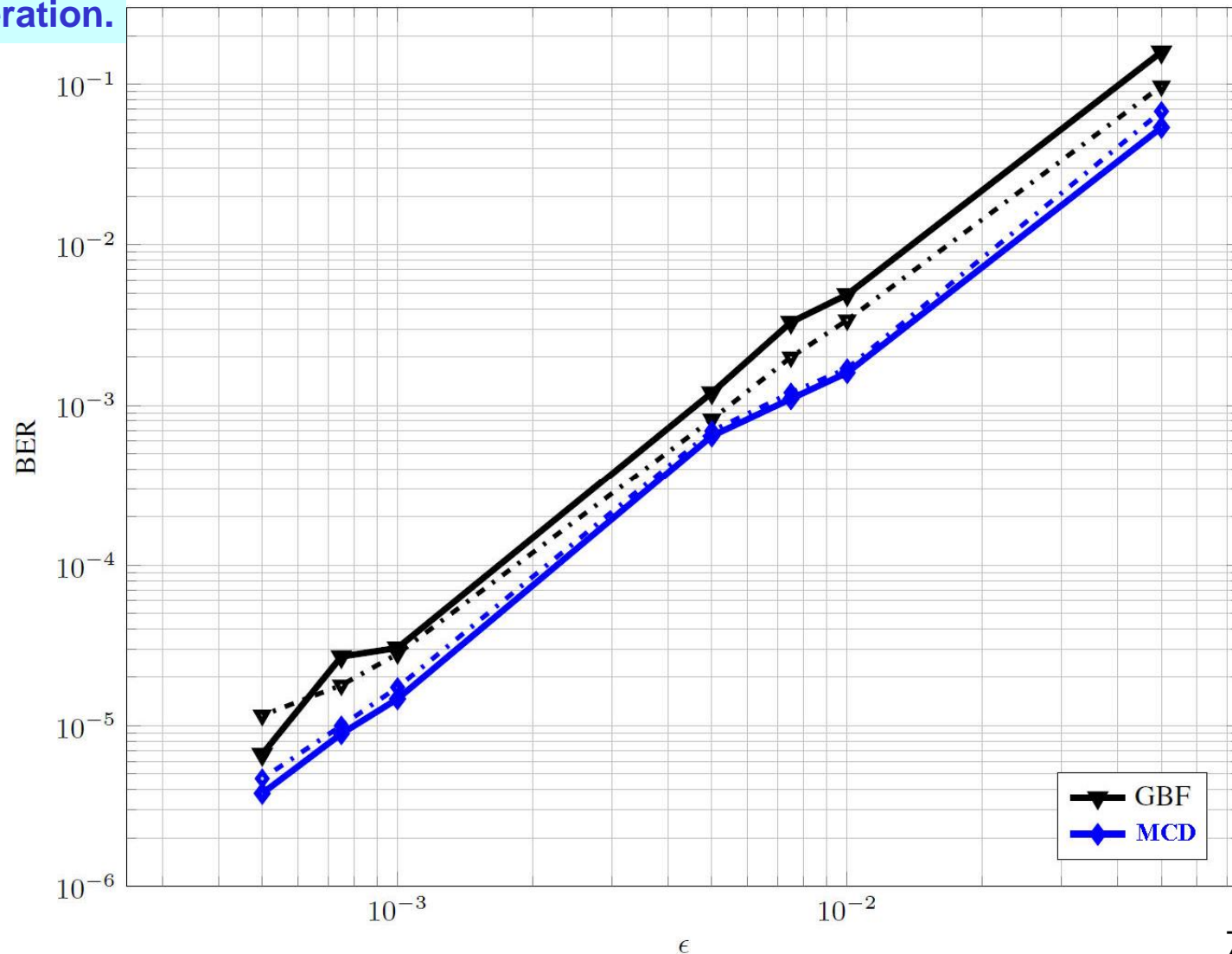
- : **Decoder Internal Transient Error Rate**
- : **BSC Channel Parameter**

Tab. 3 Maximal Parameter α^* and Maximal Parameter ϵ^* are determined when it is beneficial to use the decoder under a faulty decoder process.

Good Candidate for the ECC of cDMR: Error-Free Process

dash lines for 2-iteration,
solid lines for 20-iteration.

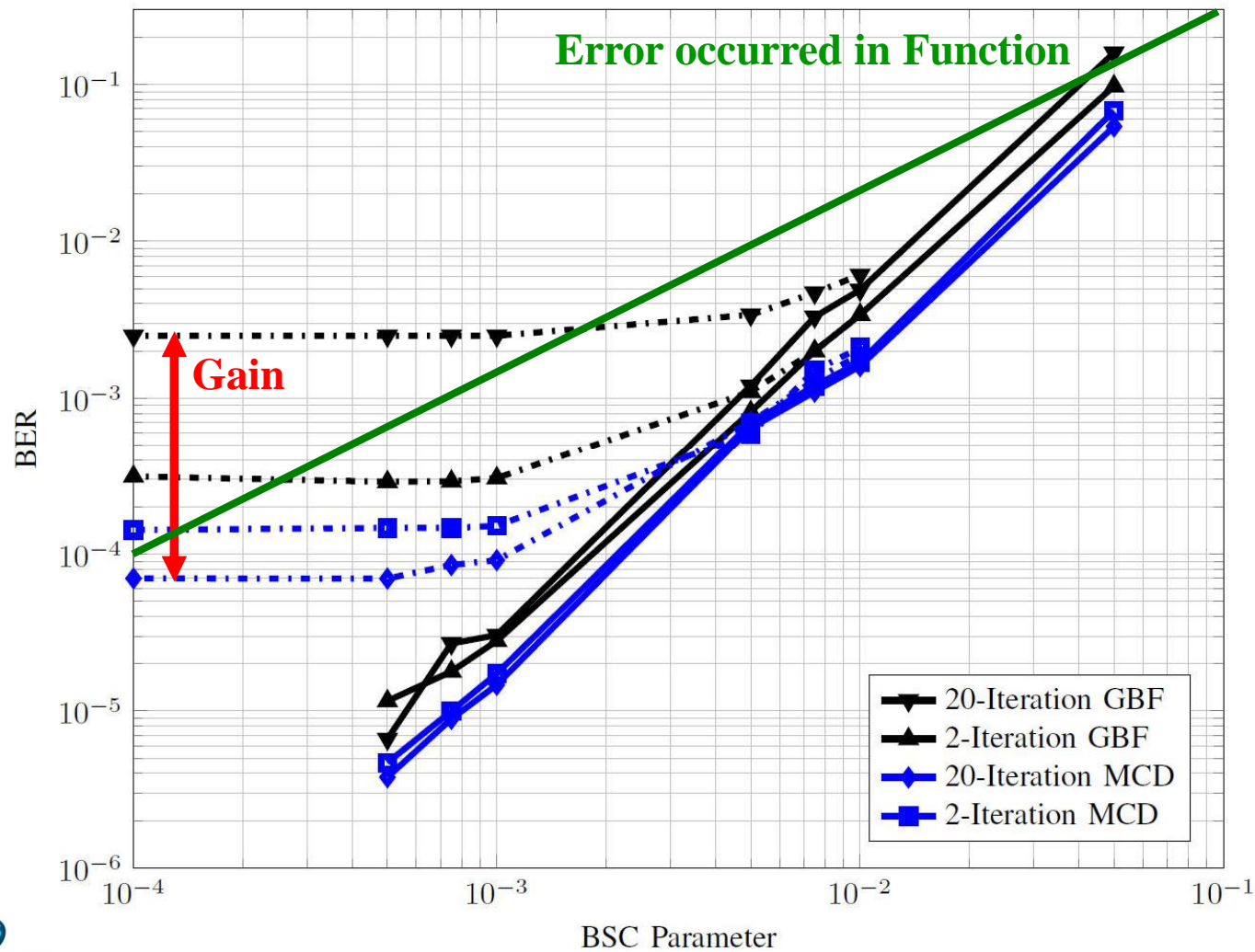
Ref : Y. Tang et al., ISCAS, 2013.



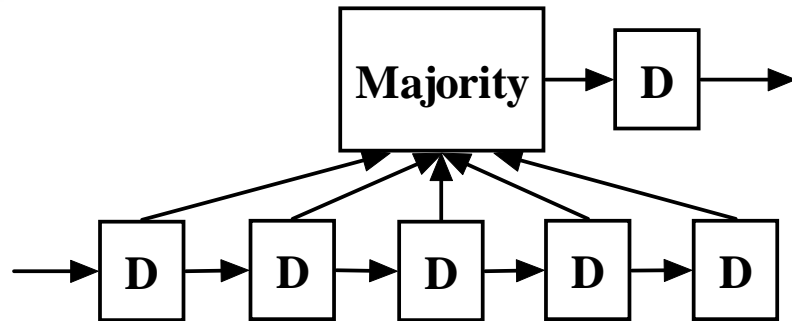
Good Candidate for the ECC of cDMR: Erroneous Process

Ref : Y. Tang et al., ISCAS, 2013.

Error-Free, (solid) and Faulty Decoding, internal fault rate 0.0001 (dashed)

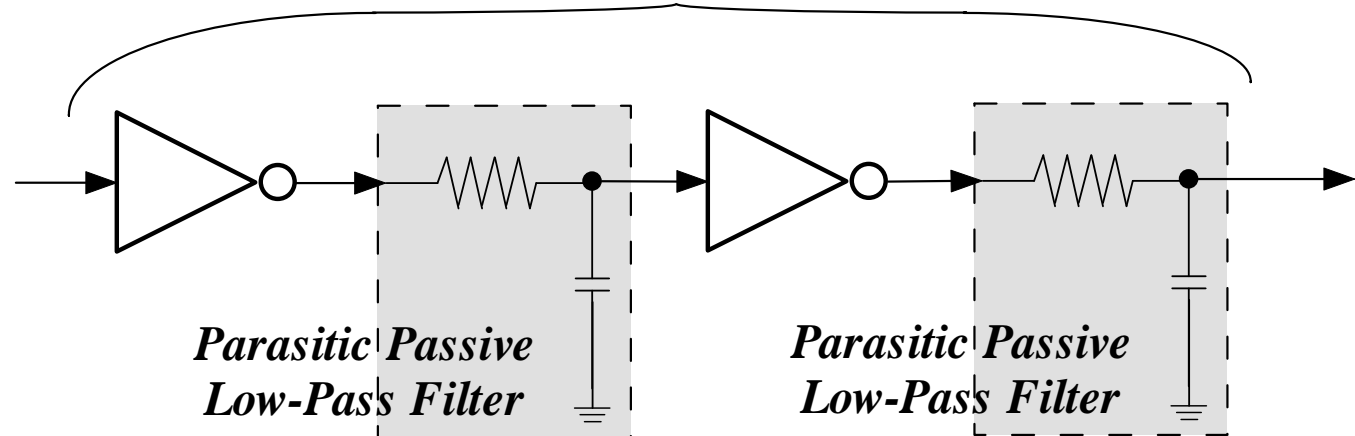


Solution 4: A Reliable Way to Implement S-T Technique



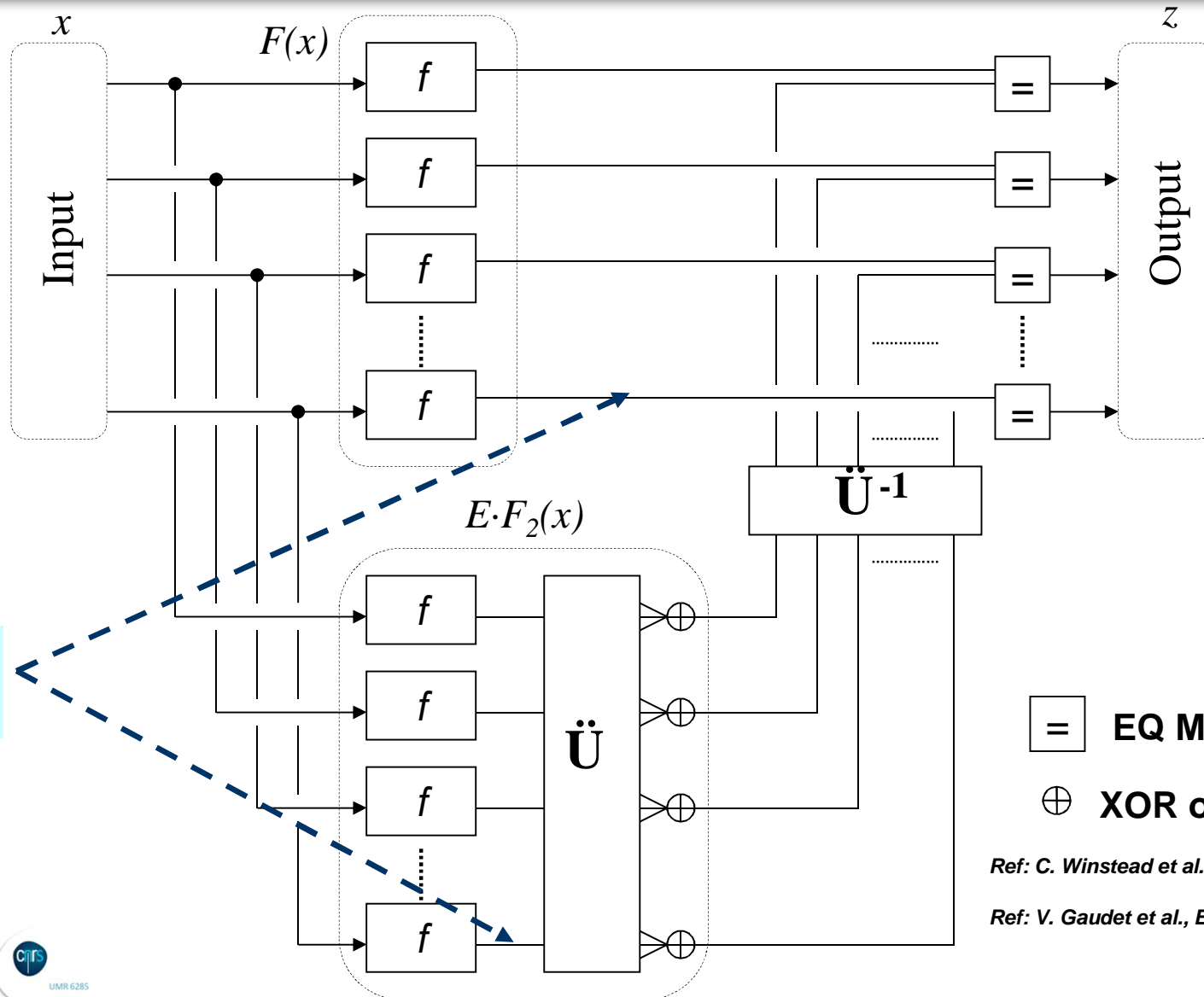
Easy Implementation for
Space-Time Redundancy Unit

Micro-Scale Circuit



This approach may be implemented, for instance, with the native R-C parasitics in a large-size output buffer.

Solution 4: Example – LDPC-coded Fault Compensation Technique (LFCT)



**Correlate
d Errors**

= EQ Module

\oplus XOR operation

Ref: C. Winstead et al., TCAS-II, 2009

Ref: V. Gaudet et al., E.L., 2003

Contribution 2: Experimental Results – Pareto Curves

Synthesis Results from XILINX Virtex 5 in Terms of [Slice(#)/Clock Freq.(mhz)].
(*N*: order of filter, *l*: bit length of input)

MA Filter Sizes (<i>N</i> , <i>l</i>)	MA-I (16, 5)	MA-II (32, 6)	MA-III (64, 7)	MA-IV (128, 8)
Strategies	Slice/Fre.	Slice/Fre.	Slice/Fre.	Slice/Fre.
The Original	[94, 424]	[128, 417]	[171, 410]	[232, 403]
S-TMR	[249, 405]	[383, 398]	[508, 300]	[681, 309]
BRRNS Based	[477, 353]	[843, 263]	[1133, 232]	[1686, 227]
ARQ-Modulo-4	[148, 349]	[205, 312]	[241, 300]	[383, 298]
ARQ-Modulo-8	[168, 349]	[237, 312]	[276, 300]	[408, 298]
ARQ-Modulo-16	[191, 349]	[277, 312]	[322, 300]	[426, 298]
ARQ-DMR	[207, 349]	[302, 312]	[409, 300]	[517, 298]

“ *a* unity area
 “ *t* clock cycle = 1 (due to the pipeline design)

Solution 1: RRNS – Single Error-Correction

“ Redundant RNS (RRNS)
 ⇔ RNS + Redundancy

| Error-Correction and Carry-Free properties |

$$\textcircled{21} + \textcircled{17}$$

Redundant Moduli

[4, 5, 7, 9, 11]

Redundant Residues

$$\boxed{1, 1, 0, 3, 10} + \boxed{1, 2, 3, 8, 6}$$

$$\begin{array}{c} \boxed{2, 3, 3, 2, 5} \\ \boxed{\textcircled{1}, 3, 3, 2, 5} \end{array}$$

“ Consistent-Checking (Detecting)

$$\times \quad \begin{array}{c} a_1, a_2, a_3, a_4 \\ 3, 0, 1, \triangle 0 \end{array} \quad (a_4=0?)$$

$$\underline{3} + \underline{0} * 5 + \underline{1} * 5 * 7 = \textcircled{38}$$

Error-Free

Solution 1: RRNS – High Latency

“ Redundant RNS (RRNS)
 ⇔ RNS + Redundancy

| Error-Correction and Carry-Free properties |

$$\textcircled{21} + \textcircled{17}$$

Redundant Moduli

$$\boxed{4, 5, 7, 9, 11}$$

Redundant Residues

$$\boxed{1, 1, 0, 3, 10} + \boxed{1, 2, 3, 8, 6}$$

$$\boxed{2, 3, 3, 2, 5}$$

Soft-Error

$$\boxed{2, 3, 3, 2, \textcircled{6}}$$

“ Consistent-Checking (Detecting)

$$a_1, a_2, a_3, a_4$$

$$1, \underline{3}, 0, 1, \triangle (a_4=0?)$$



Five Iterations at worst is needed to locate the error

Solution 1: Proposed Work – Bidirectional RRNS (BRRNS)

Integer X

$$\textcircled{21} + \textcircled{17}$$

'' (RRNS)

Moduli Set m_i

$$\boxed{4, 5, 7, 11, 13}$$

$$\boxed{4, 5, 7, 9, 11}$$

Residue Vector r_i

$\boxed{1, 1, 0, 10, 8} + \boxed{1, 2, 3, 6, 4}$

 $\boxed{2, 3, 3, 5, 12}$
 $\textcircled{1} \boxed{3, 3, 5, 12}$

Four of them for each checking

a_1, a_2, a_3, a_4, a_5

Consistent-Checkings

$(a_4=0 ?)$

$$a_1, a_2, a_3, a_4$$

$$\underline{1}, \underline{3}, \underline{3}, \underline{1}$$

$$a_1, a_2, a_3, a_5$$

$$\underline{1}, \underline{3}, \underline{3}, \underline{3}$$

$(a_5=0 ?)$

Recover Integer X from

Bi-Directional

Information

or

Redundancy

a_1, a_2, a_3

$$\underline{1} + \underline{3} * \underline{4} + \underline{3} * \underline{4} * \underline{5} = \textcircled{73}$$

a_4, a_5

$$\underline{5} + \underline{3} * \underline{11} = \textcircled{38}$$