A New Performance Evaluation Metric for Sub-Optimal Iterative Decoders

Ashwani Singh[1,2], Ali Al-Ghouwayel[1], Guido Masera[2], *Member, IEEE* and Emmanuel Boutillon[1], *Member, IEEE*

Abstract

In this letter, a new metric for fast and efficient performance evaluation of iterative decoding algorithms is proposed. It is based on the estimation of distance between probability density function (pdf) of the symbol log likelihood ratio (LLR) of optimal and suboptimal iterative decoding algorithms. We apply the notion of entropy to evaluate this function. The metric is tested on data sets from the different sub optimal algorithms for the duo binary turbo codes used in WiMax(802.16e) application and the (251,502) Galois Field (2^6) LDPC codes. Experimental results confirm that the values of the proposed metrics correlate well with the BER performance of the suboptimal implementation of the iterative decoding algorithm.

1 Introduction

LDPC codes and Turbo codes are among the known near Shannon limit codes that can achieve very low bit error rates for low Signal-to-Noise Ratio (SNR) applications [1],[2]. Efficient implementations with emphasis on small

[1]Authors are with Lab-STICC, Université Européenne de Bretagne, CNRS, UBS, BP 92116 56321 Lorient, France. Email: {*firstname.lastname*}@univ-ubs.fr.

[2]Authors are with VLSI Lab, Dipartimento di Elettronica, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129, Torino, Italy. Email: {*firstname.lastname*}@polito.it.

area, low power consumption and high throughput are of emerging importance. The achievement of such requirements often implies the adoption of sub-optimal choices and simplifications that affect code performance. Due to the large number of options to be tested, efficient methods for performance evaluation are of great interest.

The principle of Bit Error Rate estimation with the Monte-Carlo (MC) simulation is well known: generate a codeword, add some gaussian noise with a given standard deviation (given by the SNR), perform a given number of iterations of the decoding algorithm, then from the probability of symbol obtained (of Log-Likelihood Ratio), take a decision. Finally, if uncoded and decoded codewords differ, compute the number of error. This process is iterated a given number of time. If one looks at the set of final distributions of probability before decision and the final BER, a huge amount of information has been discarded. The question arise if it is possible to take into account the information before decision to improve the BER estimation?

In [3] it was shown that use of LLR values for soft decision simulations offers practical advantage of numerical stability over the conventional MC simulations. In this paper, we propose to use the value of probability before decision in a different application. As symbol LLRs are a tool to express symbol probabilities in iterative algorithms, similarty between pdfs of LLRs at the end of certain number of iterations for the two cases of an optimal and a sub optimal version of algorithm could be an effective and quick method to determine the performance of the sub optimal version relatively to the optimal one. Our project is then to find a metric between two pdf distributions so that, metric and performance degradation are well related.

However, the task of finding a significant metric between two LLR distribution is not trivial. Classical distribution distance defined in [4] does not give any significant correlation. The use of a Manhattan distance between two pdf (sum of absolute values of probability differences) does not also lead to good correlation. This can be explained by the fact that, from a decoding point of view, a probability of a symbol value of 10^{-6} and 10^{-12} are rather different, which is not the case when Manhantan distance is used. These considerations bring us to search a metric that takes into account both absolute difference and ratio of magnitude. At this point, a metric derived from the entropy definition of Shannon [5] was tested with success. The information entropy *H(X)* of a discrete random variable X that can take on possible values $x_1...x_n$ is given as:

$$H(X) = -\sum_{i=0}^{n} p\left(x_i\right) log_2 p\left(x_i\right) \tag{1}$$

where $p(\mathrm{x}_i) = P_r(\mathrm{X}{=}\mathrm{x}_i)$ is the probability mass function of X and entropy relates to the representation of information by quantifying its uncertainty.

2 The Distance Metric Definition

In a Non-binary iterative decoding algorithm (Turbo or LDPC code) exchanged messages can be represented as Log Likelihood Ratio (LLR) vectors. A q element probability vector $P = (p_0, p_1, \ldots, p_{q-1})$ is a vector of real numbers such that $p_i > 0$ for all i and $\sum_{i=0}^{q-1} p_i = 1$. The LLR vector associated to P is $\Lambda = (\lambda_0, \lambda_1, \ldots, \lambda_{q-1})$ with $\lambda_i = log\frac{p_i}{p_0}, i = 0, \ldots, q-1$.

Symbol probability as a function of LLR values is expressed as follows:

$$p_i = \frac{e^{\lambda_i}}{e^{\lambda_0} + e^{\lambda_1} + \cdots\cdots + e^{\lambda_{q-1}}} \quad (2)$$

where λ_i is the symbol LLR for i=0,1...,q-1. In order to quantify the impact of sub-optimal iterative decoding algorithms on error performance, we apply the concept of entropy to the databases D and $\tilde{D}$ composed by two sets of N q element vectors, each one corresponding to symbol probabilities p_i^n and $\tilde{p}_i^n$ of optimal and sub optimal decoding cases respectively. Extending the entropy equation (1) we define distance d in the form :

$$d(D, \tilde{D}) = \frac{\sum_{n=0}^{N-1}\sum_{i=0}^{q-1}\left(|p_i^n - \tilde{p}_i^n|\right)\left(log_2 |p_i^n - \tilde{p}_i^n|\right)}{\sum_{n=0}^{N-1}\sum_{i=0}^{q-1} p_i^n log_2 p_i^n} \quad (3)$$

The system model is shown in Figure 1: the extrinsic probabilities being fed to the distance evaluation block belong to optimal and sub-optimal databases, D and $\tilde{D}$ respectively. The distance metric that we use is only significant if the two distributions are close. For example, lets consider, the two distribution (1,0,0,0) and (0,1,0,0) for a duo-binary turbo-code: from a decoding point of view, the result is different but the distance is equal to 0.

At a given signal to noise ratio (SNR) we have the couples (Δ_{BER}, d) where Δ_{BER} is defined as:

$$\Delta_{BER} = log_{10}\left(\frac{BER_{sub}}{BER_{opt}}\right) \quad (4)$$

BER_{sub} and BER_{opt} correspond to the bit error rates for suboptimal and optimal algorithms respectively at a given SNR. The relationship between

Δ_{BER} and $d(D,\tilde{D})$ can be classified into following possibilities :

Case1 $(Excellent - Situation)$: Find d and function ζ so that

$$\Delta_{BER} = \zeta \left\{ d\left(D, \tilde{D}\right) \right\} \tag{5}$$

around a common point and in a interval of interest for making design choices.

Case2 $(Usefull - Situation)$: Find d so that the relation order of Δ_{BER} and d is respected. This means that if two design choices, 1 and 2, result into suboptimal algorithms with performance given by $\Delta_{BER1} < \Delta_{BER2}$, then definition in equation(3) will calculate an higher distance for choice 2 : d(2)>d(1).

3 Experimental Results

Above mentioned *Case 1* and *Case 2* are subsequently established in the following experiments. A duo binary turbo code used in WiMax(802.16e) application (block length K=960 and rate=0.333) and the (251,502) Galois Field (2^6) LDPC code are used.

3.1 WiMax Turbo Optimal Quantization of Channel Input

Fixed point arithmetic and quantization result in additional noise in the turbo decoding system. As the rounding off noise is fixed for a given structure, increasing the signal level to quantizer could result in better performance. However it cannot be increased too much because it may cause overflow as the dynamic range of quantizer is exceeded. Thus a optimal scaling factor α for recieved symbol is to be found which results in the best error performance of the decoder [6]. In order to validate our distance metric we evaluate Δ_{BER} varying the scaling factor α. Similar experiment is performed using the proposed distance metric. To numerically obtain the Δ BER we use channel input representation with large number of bits, thus making it a near floating point representation. BER values obtained for this floating point representation of the algorithm is used as the reference value (BER_{opt} in equation (4)).

Figure 2 illustrates the variation of couple (Δ_{BER}, d) for different scaling factors α. The value of α varies from 0.6 to 2.4 with step 0.2. The correlation curves are plotted for different *Eb/N0* and different code rates. The BER values are of the order of 10^{-3} and 10^{-4} for the *Eb/N0* values of 0.77dB and 0.87dB respectively. The number of bits N used for Monte Carlo simulation are 100 times higher than for the distance metric simulations. It can be seen that that couple (Δ_{BER}, d) gives the same optimal value of scaling factor α at 1.6 for code rate R=0.333 and at 1.2 for R=0.5, thus validating the *Case 1* mentioned previously.

3.2 WiMax Turbo Extrinsic Bit-Width Optimization

In serial, deterministic interleaver based or network on chip (NOC) based implementation of turbo decoders, size of the extrinsic memory, complexity of the interleaver and the communication resources of the network on chip greatly increase with the bit width of the extrinisic information. In [7] it was shown that least significant bit (LSB) drop-append combined with most significant bit (MSB) clipping can be an useful method for countering these effects. We utilise this bit width optimization method to establish the correlation between BER performance and proposed distance metric. The suboptimal database corresponds to symbol probabilities in LSB drop-append and MSB clipped version of the algorithm, while algorithm with 8 bit fixed point representation for the extrinsics is assumed to be optimal.

The correlation plots between Δ_{BER} and distance metric for suboptimal algorithms (shown by the dots in the curves) corresponding to 1, 2 and 3 LSBs drop append and 1, 2 MSB clip respectively is presented in Figure 3. The two curves correspond to different *Eb/N0*. The number of bits N used for simulation for distance metric simulations are lesser by a magnitude order of 100 compared to the Monte Carlo simulations performed to obtain the BER values. We can observe that for a given *Eb/N0* the correlation order is always respected between the bit width optimized sub optimal algorithms: in other words, Δ_{BER} and distance both increase when moving from one fixed point representation to a less accurate one. The correlation order also holds true across different *Eb/N0*.

3.3 LDPC GF(2^6) Case

The experiment were performed over an LDPC code (251,502) in GF (64). The optimal algorithm is considered where 64 messages are sent from each VN to CN while the sub optimal algorithms are related to sending lower number of messages (n_m) like 8,16,24 and 32 etc [8]. Using the optimal algorithm, we have generated a set of $N=$ 100*502 ,64 element vector. The first set corresponds to the intrinsic probability values of 100 frames sucessfully decoded with 20 iterations. The second corresponds to the extrinsic probabilty values representing the decided symbol probabilities at the variable nodes. After processing the set of intrinsic probabilities with the suboptimal algorithm using different numbers of messages, distance between the optimal and suboptimal algorithm for each n_m is evaluated.

In Figure 4 correlation between Δ_{FER} and distance for these suboptimal algorithms (shown by the dots in the curves) is depicted. Slope of the curve provides the quantitative correlation between the proposed distance metric and FER simulations albeit with faster simulation time.

4 Conclusion

We present a novel error performance assessment metric for sub optimal iterative decoding algorithms. It takes into account LLRs measured at the end of certain iteration to estimate how far is the pdf of the suboptimal symbol probabilities from the optimal symbol probabilities. We extended the concept of entropy to evaluate this distance. Experimental results confirm that the values of the proposed metric correlate well with corresponding BER performance analysis of the sub optimal iterative algorithms giving a significant improvement in terms of simulation time by at least a factor of 100. The work provides us a practical tool to quickly assess the performance of suboptimal iterative decoding algorithm and once a sub optimality domain of interest has been obtained, further accurate analysis can be performed using more classical approaches.

We know that other tools of the information theory can be used for our project (like mutual information, EXIT chart and so on) but we didn't find yet a usefull way of using it for our problem. This question is still open.

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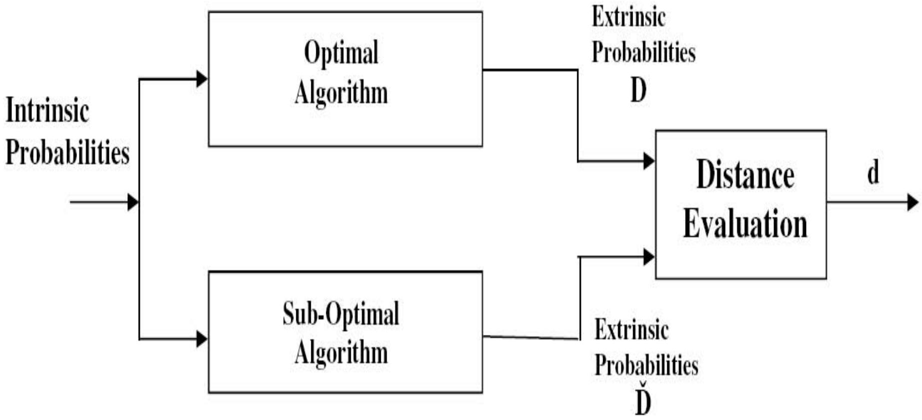

Figure 1: Model for Distance Evaluation System. Extrinsic probabilities belong to optimal and sub-optimal databases, D and $\tilde{D}$ respectively.

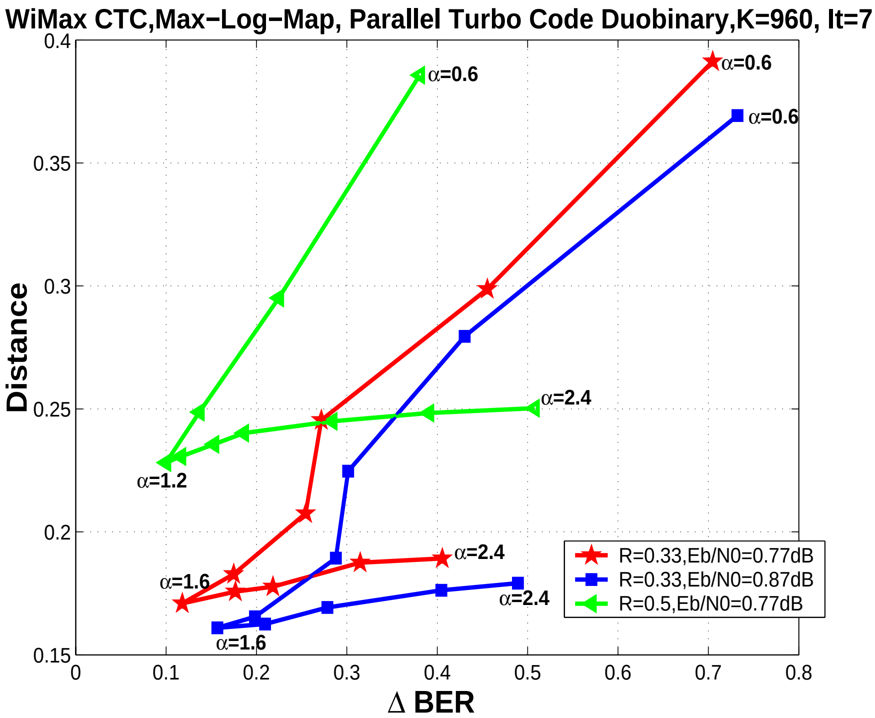

Figure 2: Correlation curve for Δ_{BER} and Distance variation with α. The correlation curves are represented for *Eb/N0* of 0.77 dB for code rates 0.33 and 0.5 and *Eb/N0* of 0.87 dB for code rate 0.33. The value of α varies from 0.6 to 2.4 with step 0.2.

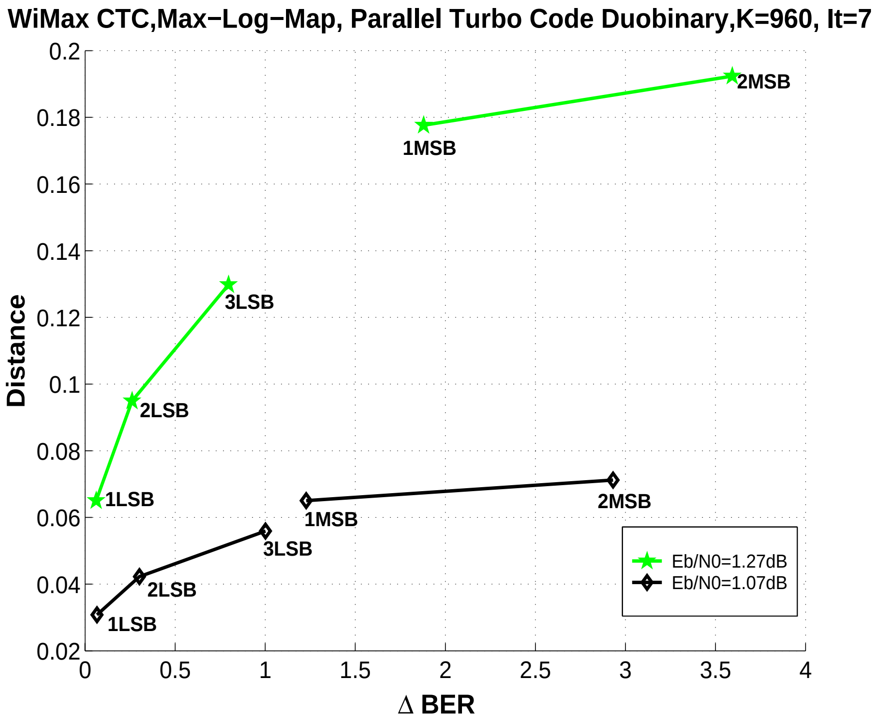

Figure 3: Correlation between Δ_{BER} and Distance for different fixed point representation of extrinsic information. Different representation shown by dots correspond to 1, 2, 3 LSB drop-append and 1, 2 MSB clip case.

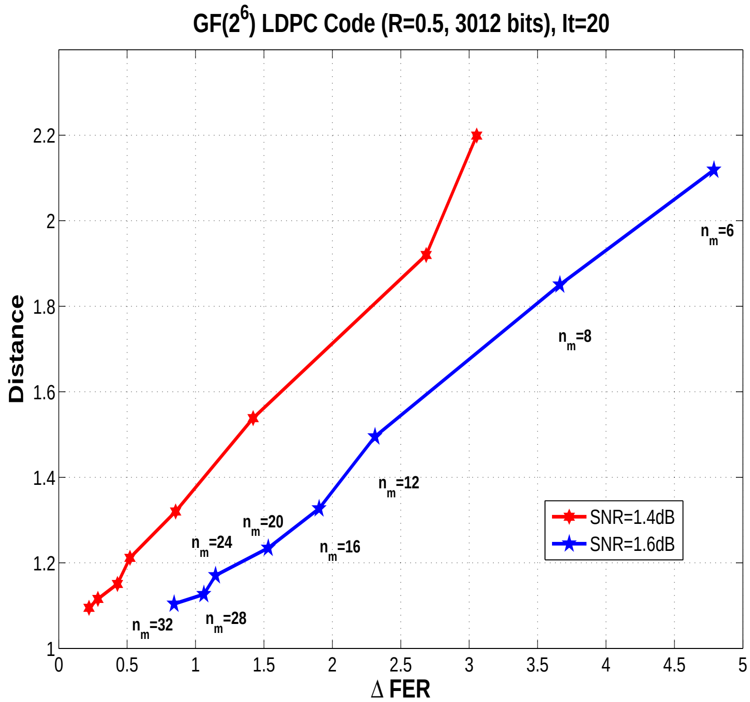

Figure 4: Correlation between Δ_{FER} and Distance for optimal and sub algorithms for different message length n_m. The value of n_m varies from 6 to 32. The correlation is represented for SNR= 1.4 dB and SNR= 1.6 dB.